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Predicting the Mathematical Abstraction Processes Using the Revised Bloom's Taxonomy: Secondary School 7th Graders[1]

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In this study, it was investigated whether it would be possible to observe abstraction processes of secondary school 7th graders using the Revised Bloom's Taxonomy. For this purpose, eight students participated in the study. The study was conducted at a state secondary school in Turkey. Purposeful sampling method was used in the selection of students and different students were examined by their achievement levels. The research was modeled as a case study and the data were obtained through interviews. Therefore, the data were collected through an interview form developed by the researchers. The collected data were analyzed according to descriptive analysis method. The findings show that the abstraction process differs according to the dimensions of the taxonomy. Accordingly, it was determined that a student who abstracts information should behave at least at the application level in the cognitive level and at least at the conceptual knowledge level in the knowledge dimension. It was also considered that the Revised Bloom's Taxonomy categorized the cognitive mechanisms required by abstraction processes thoroughly. Supporting this study with quantitative data is suggested so that the findings may become more significant.

Keywords: mathematical abstraction, mathematics education, equations, Revised Bloom's Taxonomy, APOS theory

Introduction

Meel (2003) mentioned about the developmental processes of the concept of understanding included in mathematics literature and examined this process in three parts. 'Understanding' in pre-1978 was generally considered equal to the development of relations within context while performing problem solving and algorithmic operations. Skemp's (1986) views were effective in the development of the concept of post-1978 'understanding', and this concept was examined and explained in four parts as relational, instrumental, logical and symbolic. Today, constructivist conceptualization for 'understanding' is at the forefront. According to this perspective, understanding is constructed by shaping mental objects and making sense of the relationships between these objects (Meel, 2003). This perspective also summarizes the development of mathematical understanding. Mathematical understanding is a dynamic process that develops with abstractions (Dubinsky, 1991). This dynamic process continues to develop in a progressive form by updating itself especially with the change of theories.

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Along with the changes in learning theories, especially the studies conducted in recent years have generally revealed how learning developed rather than the level of learning (Dreyfus, 2020; Kidron & Monaghan, 2009; Monaghan & Ozmantar, 2006; Presmeg, 2020; Van Es, Cashen, Barnhart, & Auger, 2017). In the current study, it was aimed to depict how the concepts related to the sub-learning domain of equation were structured. The developmental processes of these concepts were examined by two different structures: APOS theory as a theory of abstraction (Dubinsky, 1991) and the Revised Bloom's Taxonomy-RBT (Krathwohl, 2002). Both structures have meanings in accordance with the philosophy of constructivist teaching. In addition, carrying out analyses in accordance with both structures is a challenging process. It is an important example for researchers, teachers or prospective teachers who are interested in the subject. For the purpose of the research, answers for the following problems were searched:

What is the possibility of observing abstraction processes of the secondary school 7^{th} graders using RBT?

- What are the abstraction processes of the secondary school 7^{th} graders?
- What are the dimensions of knowledge and cognitive level of abstraction processes of the 7^{th} graders according to RBT?

Theoretical Framework

Mathematical Abstraction. The concept of abstraction has been discussed and investigated for centuries. Abstraction, which had first been explored in Aristotle's time, later became a topic of interest by various philosophers. Lane (1999) stated that the axiomatic development of abstraction started with Hilbert and had completed this development to a great extent by 1945. The idea of abstraction in these years was based on some assumptions as follows; (1) objects were represented by categories, (2) they were independent of context, (3) were distinctive features of further steps in further development of thought (Van Oers, 2001). These assumptions indicated that abstraction is mostly a high-level operation and time-and-place independent.

Abstraction is versatile and complex concept. Given the versatility of this concept, it is possible to see various definitions in the literature. The concept of abstraction has been defined and evaluated by many educators and psychologists from different perspectives (Ohlsson & Regan, 2001). Although there has been no consensus on the concept, the relevant literature revealed that the only point agreed about abstraction is that this concept allows the individual to assess his/her own from different perspectives. Hampton (2003) stated that abstraction should contain three types of information: information about which dimensions of situations are related (such as the color of a key), information about the values of dimensions that reliably predict how we should behave (color dimension) and information about the range of variability of predictable values. Hampton (2003) suggested that these three types of information be stored for abstraction apart from the fact that other information should not be stored. Thus, it was thought that a greater degree of abstraction would be achieved only by storing important and relevant information and expelling the other. For this, the researcher dealt with the concept of triangle: by selecting the general elements of a triangle and ignoring the details of the triangles, we can abstract the representation of the triangle that we can imagine in our imagination.

Abstraction is a process like mathematics that deals with the individuals' mental activities. This makes abstraction one of the important issues in mathematics. Thus, mathematical abstraction becomes an important skill for individuals. In the studies on mathematics education, the concept of abstraction has been evaluated from different perspectives (Frorer, Manes, & Hazzan, 1997; Tall, 1999). Although this concept has been a controversial issue, there have been some views that it is possible to probe the concept from different perspectives and even to offer a rich meaning (Hazzan & Zazkis, 2005). Ferrari (2003) argued that abstraction is a fundamental process for mathematics. According to him, abstraction is related to the formation of new mathematical concepts, but it is difficult to show this relation. In order to achieve this, it would be necessary to consider the developmental process of mathematics. Only then abstraction could be seen as a fundamental step in the formation of new concepts. On the other hand, Yılmaz (2011) argued that mathematical abstraction is the process of extracting the essence of the features that form the basis of a mathematical concept, removing its relation to real world objects and generalizing it to wider practices.

Revised Bloom's Taxonomy. Taxonomy means classification and all of the rules used in this classification (Turkish Language Association, 2019). Basbay (2008) defined taxonomy as a concept indicating that the desired behaviors are graded from easy to difficult, from concrete to abstract, from simple to complex, as a precondition for each other. Anderson et al. (2001) declared the purpose of this progressivity as providing educators with the opportunity to examine the learner's eyes, helping the combination of cognitive processes by trying to solve instructional problems and creating a comfortable communication environment by leading assessment.

The structure aiming at classifying the objectives incrementally was developed by Bloom et al. in 1956. First, it was mentioned in the book 'Taxonomy of Educational Objectives, The Classification of Educational Goals, Handbook I: Cognitive Domain', and then this structure took its place in the literature as 'Bloom's Taxonomy'. Benjamin Bloom stated that with the structure he developed, he specifically aimed at developing the coding system by designing learning objects in a hierarchical order (Marzano & Kendall, 2006, p. 2).

Various criticisms have been raised against this structure, which has been in use since 1956. Research findings such as Bloom's Taxonomy's consisting of one dimension (Furst, 1994), claiming that the hierarchical order is not correct since the synthesis step is more complex than the evaluation step (Krietzer & Madaus, 1994), discussing that adherence to the hierarchical structure has negative consequences and limits the researchers on this subject (Ormel, 1979; Seddon, 1978), being inadequate for all kinds of subjects (Fairbrother, 1975) triggered the idea of revising the taxonomy (see Bumen, 2006; Marzano and Kendall, 2006, p. 8). Anderson et al. (2001) attributed the revision of Bloom's Taxonomy to two reasons: (1) To enable educators to re-focus on the original Bloom's Taxonomy and (2) To reflect the developments about development and learning psychology, teaching methods and techniques, assessment and evaluation since 1956.

A group of researchers interested in Classical Bloom's Taxonomy (CBT) first came together in 1995 and examined numerous research results by conducting a meta-analysis study (Krathwohl, 2002). Anderson et al. (2001) regularly met each year, revising the original taxonomy 45 years later and re-published under the name "A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives" (Anderson, 2005; Forehand, 2012).

The most striking feature of the RBT is that it has been transformed from a one-dimensional structure to a two-dimensional structure (Anderson et al., 2001). In explaining this change, Krathwohl (2002) stated that educational target expressions should be examined. According to him, target expressions included both course subjects and desired behavior. Therefore, these expressions consist of (a) noun or noun phrases, (b) verb or verb phrases. In CBT, noun and verb elements were given under the same title. Unidimensionality of the classical taxonomy was deemed insufficient to reflect the relevant educational objective. Therefore, this situation was regarded in the new taxonomy and the necessity of expressing them as separate situations was emphasized. In other words, name was the basis of knowledge dimension while the verb was the basis of the cognitive process dimension. Thus, the original one-dimensional taxonomy was designed as two-dimensional (Krathwohl, 2002). As a result, the new taxonomy is represented by two dimensions consisting of four sub-levels under the knowledge dimension and six sub-levels under the cognitive processes (Table 1).

Table 1. Dimensions of the Revised Bloom's Taxonomy

	The Cognitive Processes					
The Knowledge Dimensions	1. Remember	2. Understand	3. Apply	4. Analyze	5. Evaluate	6. Create
A. Factual Knowledge						
B. Conceptual Knowledge						
C. Procedural Knowledge						
D. Metacognitive Knowledge						

While the knowledge dimensions form the rows, the cognitive processes form the columns of the table. Rows indicate names or phrases; columns show verbs or verb phrases. The intersection of each level of knowledge and cognitive processes constitute cells. These cells are mainly used to categorize student behavior. It should be noted that it is possible to display each behavior in more than one cell (Amer, 2006; Anderson, 2005; Krathwohl, 2002). It can be considered that this feature provides flexibility for educators, researchers and program developers. In addition, Amer (2006) stated that taxonomic knowledge and cognitive processes have various purposes and advantages. According to the researcher, this new structure provides analyzing objectives of a syllabus or units by presenting clear, short and visual representation, helping teachers not to confuse objectives and activities and to reveal the relationship between learning - teaching activities and evaluation.

Methodology

Model

This study was modeled in accordance with case study. According to McMillan and Schumacher (2010, p. 344), a case study is defined as in-depth analyses of a single unit or subject. Such studies are those in which details of a particular subject are tried to be identified and explained thoroughly (Stake, 2010). In the current study, the abstraction processes of the students were regarded as cases, and detailed analyses of these cases were given.

Participants

The research was carried out with 8 students in a state secondary school (7th grade) in Turkey. Since the students' thinking processes were examined in this research, their characteristics of being able to represent the whole class in terms of achievement were taken into consideration as well as their personal characteristics such as being able to express themselves clearly and to have enough motivation to complete the activities. Therefore, purposeful sampling method was used in the selection of students. Purposeful sampling method is a qualitative sampling method used when in-depth information about an event or phenomenon is needed (Yin, 2011, p. 88). The students' achievement levels were defined as high, good, medium and low. In the study, it was deemed necessary to select students from different achievement levels not for reflecting different interpretations in the section of conclusion, but for diversifying the participants of the research. The findings obtained in this respect were not compared in terms of students' achievement. The students' school achievement grades, mathematics achievement grades and teachers' opinions were effective in determining their achievement levels. In terms of achievement, Fatih and Talha had high, Ezgi and Okan had good, Islim and Harun had medium, Sena and Nur had low achievement levels. The students' real names were not used. In addition, the principle of volunteering was adopted in the selection of the participants, and necessary information was provided to ensure the confidentiality of the data.

Data Collection Tool

In order to collect the data of the study, an interview form was prepared by the researcher with two open-ended questions. With this form, it was aimed to reveal and examine the knowledge and thoughts the students had regardless of checking on. Therefore, special attention was paid to ensure that the students were able to express themselves easily. In addition, the scenarios were prepared in a way to provide students the opportunity to observe the stages of action, process, object and schema on the subject.

The topics of the scenarios in the interview form were related to the 7th grade sub-learning area of equation. The choice of this subject was not accidental since subject selection is important for examining the abstraction processes (Mitchelmore & White, 2007). Algebra has frequently been a preferred subject in the analysis of abstraction processes (Dubinsky, 1991). For the data collection tool, firstly, different scenarios were created in line with the outcomes of the sub-learning area of equation, and views of one faculty member, two research assistants and two mathematics teachers were used for the evaluation of the scenarios. Two scenario cases among these scenarios were chosen consecutively, and the appropriateness of language used in the scenarios was assessed by a Turkish language teacher. The draft form was applied to 31 seventh graders at the same school. As a result of the pilot application, necessary revisions were made, and the form was finalized (Appendix 1).

Data Collection Process and Analyses

A room was allocated to the researcher by the school administration for the interviews. The room was technically ready before the interview. The interviews were recorded with a camera focusing on the worksheets on which the students only noted their thoughts. Each interview lasted approximately 50 minutes. The records of the interviews were transcribed and converted into written texts. In addition to the verbal and written data obtained from the students, non-verbal communication was also observed during the interviews. The data were analyzed by descriptive analysis. Descriptive analysis is summarizing and interpreting data in terms of predefined categories (Simsek & Yıldırım, 2011, p. 224). In the data analysis, the structure consisting of the components of Action (A), Process (P), Object (O), Schema (S) put forward by Dubinsky (1991) and the cognitive and knowledge dimensions of the RBT proposed by Anderson et al. (2001) were used as categories.

Analysis of the Abstraction Process. In analysis of the abstraction process, the structure consisting of the components of Action (A) Process (P) Object (O) Schema (S) proposed by Dubinsky (1991) was used. APOS theory argues that there is a close relationship between the nature of mathematical concepts and the development of mathematical concepts in an individual's mind, and it illustrates the abstraction processes of an individual (Dubinsky, Weller, McDonald, & Brown, 2005). There are cognitive mechanisms related to each component of this structure. These mechanisms have been identified as internalization, coordination, encapsulation, generalization and reversal (Dubinsky, 1991). These cognitive mechanisms were determinant in deciding under which category the data would be evaluated. The decision-making process of each category in the evaluation of APOS theory was managed as follows.

Students who show *action-level* behavior find it necessary to give an external clue such as a formula for the use of numbers. In addition, calculating the values of the function given with algebraic expressions for the concrete values of the independent variables is also a behavior to be evaluated at action level. Once the action is repeated and reflected, the student can now internalize the action as a *process*. At this level, the student can describe and reflect on the steps of the process and even think the other way around. Moreover, while performing these, it does not need to do these steps explicitly. The individual can structure processes by coordinating or reversing previously created processes. For example, at the process level, the learner does not need an external clue such as a formula to perceive a function, can interpret the values of dependent variables for one or more values of the independent variable. When the individual reflects the processes applied to a certain process, becomes aware of the process in general, performs transformations (be it actions or processes), he can now think of this process as an *object*. For example, being able to apply a derivative on a function is at this level of behavior. Finally, in the framework of abstraction, the level of *schema* is involvement in the organization of objects. That is, it is the thematization of a diagram to an object. For example, functions can be shaped within groups, operations can be performed or controlled on these groups.

Analysis of RBT. In the data analysis process, the cognitive and knowledge dimensions of the RBT, proposed by Anderson et al. (2001), were employed as a category. There are keywords in both dimensions of the RBT. These keywords played an important role in the analyses.

In the analysis of the cognitive dimension, the decision of each step was evaluated with the following features. Behavior of recognizing and remembering each small unit in the equation subject was evaluated as the *remember* step. If students can adapt or transfer new information to different formats, it is evaluated at the *understand* step. For example, behaviors such as interpreting information, making small conclusions, comparing, explaining are the determinant behaviors of the understand step. On the other hand, using algorithms has been considered as the *apply* step. In other words, a student evaluated at this stage can use the principles in solving the problem and realize what is desired. In the *analyze* step, it is important to break down information and understand the place of these pieces in the whole. In other words, a student who distinguishes important units for solution in the information community included in the research scenarios and knows how to organize these information units, behaved in the analyze step. In the *evaluate* step, the student checks the results he obtains, identifies contradictory situations, criticizes positive and negative judgments and can make decisions based on certain criteria and standards. Finally, the criterion for the *create* step is the student's process of creating a new and original product. The creativity of the student evaluated at this step is at the forefront.

In the analysis of the dimension of knowledge, on the other hand, *factual knowledge* includes the basic information of the student and the information that he/she must know. For example, term information, symbols, written or unwritten definitions related to the subject of equations, are factual information. The fact that some mathematical concepts represent more complex structures distinguishes this information from factual information. This is expressed as *conceptual knowledge*. Conceptual knowledge is organized information forms that contain classifications and relationships within comprehensive and organized knowledge bodies. For example, knowledge of classifications and categories, or knowledge of principles and generalizations, are the units of this step. *Procedural knowledge*, on the other hand, is the knowledge of skills and algorithms related to a particular subject, the knowledge of techniques and methods related to a particular subject, and the knowledge of criteria in determining the use of appropriate methods. Finally, *metacognitive knowledge* means that a person is aware of his/her own cognition and having information about it means having general knowledge about cognition. This category includes information about general strategies used for different learning tasks, identifying which strategies are effective, and self-knowledge.

Results

Findings about Abstraction Process

Analyses of the students' abstraction process were presented in this section. The data regarding the stage of each student's abstraction process according to their achievement was given in Table 2.

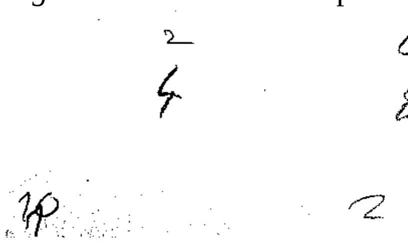
Table 2. Abstraction Processes of the Students

Student	Achievement status	Action	Process	Object	Schema
Fatih	high		√		
Talha	high		√		
Ezgi	good		√		
Okan	good		√		
Islim	middle			√	
Harun	middle	√			
Sena	low	√			
Nur	low	-			

As seen in Table 2, Fatih, Talha, Ezgi and Okan exhibited behavior in the process, İslim exhibited behavior in the object, Harun and Sena exhibited behavior in the action category. The reasons for the category under which students' behaviors are evaluated were explained through dialogues. For example, a section from the interview with Fatih, who showed behavior in the process category, is as follows:

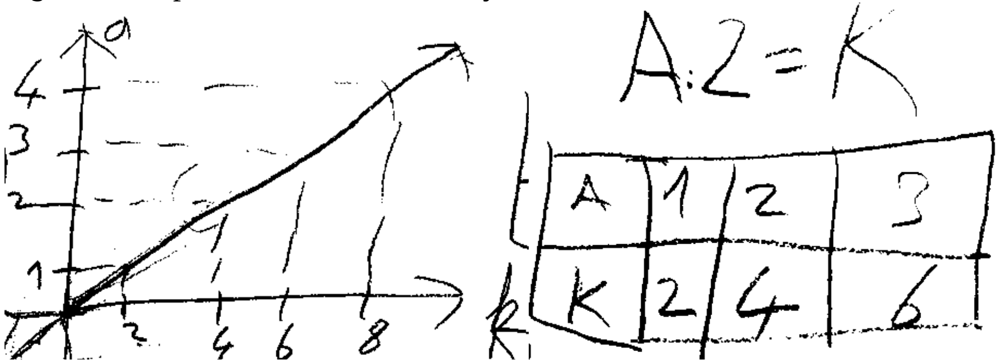
...
15A: He buys it for 8 liras and sells it for 2 liras.
16F: I think I have been confused, if the purchase price is 8 liras, if we sell it with 25% liras, the selling price would be 10 liras (He expresses these operations without using pen and paper)... 25% means 1/4, ¼ of 8 liras is 2, we can find 10 liras when we add them together.
17A:… So what do you say about the relationship between a and k?
18F: I say it’s proportional, so there’s no other pattern that's always doubled, so it's proportional.
19A: If I say a and k, I would express this relationship in one sentence…
20F: So, as k is 2 times of a, it is directly proportional.

Figure 1. Fatih’s Status Representation



...
29A: How can we do it?
30F: maybe $a = 2k$.
31A: You’ve said “maybe”, how can you be sure?
32F: I can do it by value… Here, k is 2 times of a, when a is 1, so when we sell 1 piece, k –that is profit - is 2 liras, when we sell 2 pieces, it is 4 liras… (deletes the equation in $a = 2k$) … (actually, Fatih knows that the equation is wrong, but he still does not fully realize and gets confused). Yes, $a.2 = k$, okay now, we can write the equation like this… We can be sure of its accuracy by giving value… Yes it is compatible with the table. That is right.
33A: Why have you just offered not to write equality?
34F: These are proportional or interdependent at the time, so then I realized that I had to write.
35A: What are you trying to explain by making an equation?
36F: Equation is a mathematical term consisting of numbers with unknown ones. So, teacher, we don’t know how much it is sold, it will be easier, so we will easily show that 2 times as much profit will be get.

Figure 2. Graphics and Table Drawn by Fatih

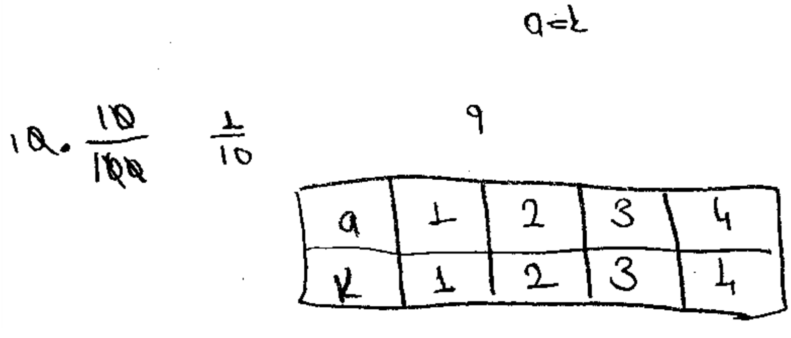


Fatih could create the variable information within the framework of the process stage by interpreting the variables within the scenario, repeating his transactions and reflecting the information units he had obtained (16F, 18F, 20F). Based on the information units that had been formed, it was seen that the student tried to reach small generalizations such as being able to show the relationship between variables as algebraic expression as a pattern rule and wider generalizations based on this (32F, 36F). Fatih's 36F statement was an indication that he was trying to generalize.

As can be seen from Table 2, Islim was evaluated in the object stage. Some of the data for this student was as follows:

15A: What is the relationship between them?
16I: There is something between them, rising two by two. I mean it multiples as it is but how can I make? Is it x^2 ? Say 2 for x, x^2 4, say 3 it is 9, this is not true; say $x.2$, 1 for x it is 2, ııı say 2 it is 4, say 3 it is 6, thus I guess it's OK.
17A: What is x, what is 2?
18I: x is a number such as a, i.e., the number of increase, 2 refers to how many times it progresses. So we can say a (it changes x and writes a).
19A: So what does $a.2$ give us?
20I: The expression of $2.a$ here gives the relationship between a and k, which is k.
21A: So how can we write?
22I: I can write $2a = k$.
23A: So can you continue?
24İ: When you make a 10% discount, it will be 9 liras? (Continues without waiting for approval) How much profit? According to the previous price of 1 lira loss, but according to the purchase price it is still profitable - 1 lira.
…

Figure 3. Status Representation of Islim

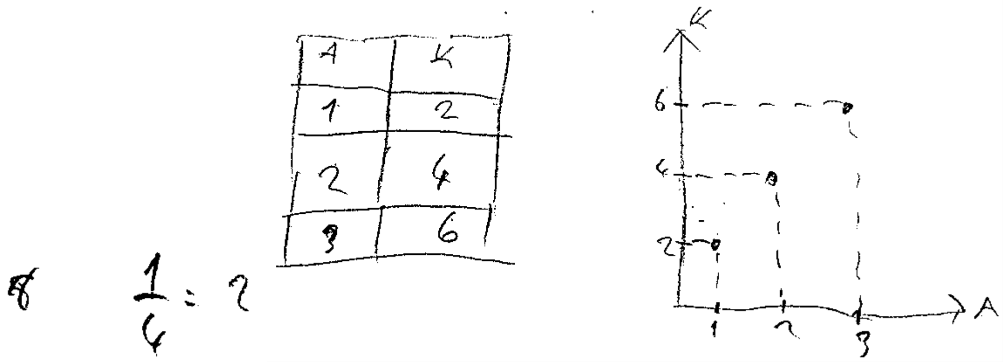


Islim started the scenario with a variable (x) she determined, and she changed these variables as required in the last case within the frame of the scenario (16I, 18I). She was able to interpret and reflect information units such as variable, equality and pattern accurately. It was understood that the student followed a definite and complete sequence such as creating a table and finding the general rule in order to form an equation, and then wrote and explained the equation related with this rule (20I, 22İ, 24İ, 26İ). In the last case, she tried to summarize the situation by associating the purchase, profitable and discount sale prices retrospectively and by comparing the graphs of the two equations on a single graph (Figure 3). After all these explanations, it can be claimed that Islim was able to group and objectify the data obtained and to organize and associate these objects.

Finally, it was found that Harun and Sena acted in the action stage (Table 2). Therefore, it can be stated that these students are at the most basic step of the abstraction process.

...
19A: What does it mean that the graph is like this?
20H: The more we sell, the higher is our profit.
21A: Well, we don't know how much we have sold, can we say something more general about the relationship between a and k?
22H: (thinking quietly) I don't know…

Figure 4. Harun's Status Representation



...
27A: And what is done here?
28H: 10%, that is 1/10 discount, which is 1 lira, so it will be 9 liras.
29A: OK.
30H: Each time, for example, we have sold 2 pieces, they are 18 liras, and our discount is 10% - 2 liras. In other words, it increases one by one....
31A: Can you summarize what you mean?
32H: The discount is 1 lira if we have sold 1 piece.
...

Harun was able to answer the questions in the scenario, but he did not intend to use algebraic expressions for this (Figure 4). In general, he could perform simple mathematical operations and make sense of each piece of information, but could not synthesize these units (20H, 22H, 28H, 30H, 32H). This situation shows that the student had inadequate abstraction skill. Nur could not be assessed in any categories related to the abstraction process. This can be understood directly from the data of the interview with the student.

...
15A: How a relationship did you find?
16N: There is an unknown here, that is, the relationship a and k has been made with an unknown equation, as algebraic expression.
17A: Nur, what do you mean by making an equation?
18N: I mean the relationship between the sold parts and the profit obtained by making an equation in which there are unknown things, as shown here a and k.
19A: Which one is the equation?
20N: We write a here as the profit, bought from bulbs, the company bought bulbs of 8 liras, each bulb 25% plus - that is 25% profit, we can write k for it, this is an unknown (meanwhile, she is taking notes on the worksheet).
21A: So, is this an equation?
22N: Yes.
23A: You said that a and k are unknown, can't you find the values of a and k?
24N: It is not stated how much we sold, so we can't find.
25A: Can't we comment?
26N: We can't do as we couldn't find.
...

Throughout the interview with Nur, it was noticed that the student got confused and could not distinguish concepts such as variable, algebraic expression and equation. For example, it was seen that she considered the things such as profit and loss symbolically only in a statement. That is to say, she claimed that the representations representing profitable sales such as $a + k$ were indeed an equation in which relations between variables were put forward (16N, 18N, 20N, 22N, 24N, 26N). It was found that Nur could not even use the clues given by the researcher effectively and could not create even the small units of information necessary for abstracting the concept of equation.

Findings about RBT

After analyzing the students' abstraction processes on making equations, the data were reevaluated by considering the knowledge and cognitive dimensions of RBT. The data obtained were given in Table 3.

Table 3. Representation of Students' Levels according to RBT

	The Cognitive Processes					
The Knowledge Dimension	1. Remember	2. Understand	3. Apply	4. Analyze	5. Evaluate	6. Create
A. Factual Knowledge	Nur					
B. Conceptual Knowledge			Okan-Harun-Sena			
C. Procedural Knowledge			Ezgi	Talha		İslim
D. Metacognitive Knowledge						Fatih

It can be stated that Fatih could make sense of variables, express the relationship between numbers, show this relationship in more general way and reflect it for new situations, organize the units of information he had obtained to create new information and embody it as a whole. That the student even explained why he did not use the concepts that he did not/could not use was an indication of his awareness of the subject. Fatih’s awareness and his knowledge of his own cognition revealed that he was at the level of metacognitive knowledge according to RBT. In addition, the behaviors mentioned above indicated that the student was at the level of creation according to the cognitive processes. After these explanations, Fatih was considered to be in the cell D6. Ezgi, on the other hand, could find the numbers representing the relationship and express the relationships between these numbers. However, she could not evaluate these objects holistically and generalize them. It can be said that Ezgi knew how to do something and could smoothly perform and carry out what was required in a certain situation. These competences indicated that the student be in the C3. It was revealed that Islim was successful enough in formulating and interpreting the equation, but she especially followed an order to do so. This was an indication that the student was acting according to a certain procedure. It was seen that the student firstly obtained, without hesitation, small information units for the new structure she wanted to create, and then embodied them by associating this information with each other and reflecting them to new situations and reprocessed these objects to form larger structures. This revealed that the student could put the pieces together to form a new product. After these explanations, it was suggested that the student be evaluated in the cell C6. On the other hand, Sena did not try to see the whole picture by combining the ideas she put forward in the abstraction process. She only examined the units of information she obtained, but did not intend to assess them as a whole. Therefore, the student was unable to analyze the parts she created. It was thought that Sena behaved in the cell B3. Moreover, she knew the concepts in the scenario and used them appropriately. Okan and Harun behaved similar to Sena, and these students were in the same cell. It can be mentioned that Talha took all the necessary steps to put forward the big picture on the subject of the equation but could not evaluate these steps holistically. Considering that he could divide the information he had obtained, express his relationship with the whole and decide which operations to be performed, Talha was evaluated in the cell C4. Finally, during the interviews with Nur, it was seen that the student could only perform the necessary mathematical operations and could not express the relationship between the numbers obtained. In addition, it can be stated that the student did not understand the concepts related to the subject and usually needed an external clue at each step. As the student was able to recognize the concepts related to the subject and use the symbols related to the subject, she was evaluated in the cell A1.

Discussion

In the research, firstly, the abstraction processes of the students were illustrated. In the light of the detailed evaluations about the abstraction process, it can be stated that some important points have become prominent. The first of these is that the students' abstraction processes of a certain concept were versatile. While these preferences led some students to the final goal, others could not reach the desired point. This gives us important feedback about the abstraction process. Some students like Ezgi made progress more easily by using visual forms, but some other students like Islim and Talha could achieve this only with algebraic expressions. The results of the studies conducted by Ozmantar and Monaghan (2007) and Sezgin Memnun (2010) support the statement that the students' abstraction processes may differ.

It was also found that the use of visual elements had positive effects in the process of abstraction. It was revealed that the students like Ezgi, Sena and Talha, who tried to abstract the concept of equation, were able to understand and generalize the relations between numbers after they had seen the figures such as graphs and tables. In addition, the fact that the students wanted to reconsider the information units they could not understand after using the visuals showed that they motivated the students positively. However, it was also concluded that Okan and Nur used these images only to express the relations between the numbers and that they could not go one step further. Therefore, it can be said that using only these images in the abstraction process is not sufficient, but they promote the creation of the related concept. In fact, this result of the study is similar to the results of the studies carried out on the importance of visualization in the process of abstraction (Cetin & Top, 2014; Kabael, 2011; Yilmaz, 2011).

One of the results of this study was that the communication skills of the students should be sufficient in examining the abstraction processes. In this process, students' inability to express themselves fully causes inadequate comments about the skills that are aimed to be observed. Talha and Ezgi's inability to fully express what they wanted to express and especially Ezgi's choice to be silent during the interview prevented the analyses of these students' abstraction processes. Therefore, the students' being able to express their ideas as they are becoming more important in the analysis of abstraction processes. These situations may be possible not only at the communication level of the students but also during the process. For instance, Sena and Nur's inability to shape algebraic understandings decreased their willingness to continue the interviews. This caused the students to take a back seat and restricted the analyses of abstraction processes. Therefore, these students started the process with similar willingness but could not continue with the same motivation. Herskowitz, Schwarz, and Dreyfus (2001) stated that a student's ability to express himself well is related to his psychological state, and it is important for illustrating the student's abstraction process.

The students' abstraction processes were analyzed according to RBT, too. As a result of the analysis, it was found that the students exhibited behavior at the 3^{rd} level of cognitive processes and at the level of conceptual and operational knowledge of the knowledge dimension. In addition, as the cognitive level increased, the level of knowledge increased, too. Zorluoglu, Kizilarslan, and Sozbilir (2016) suggested that 67% of the outcomes in cognitive processes dimension are in understanding level and 59% of the outcomes in the knowledge dimension are in conceptual knowledge. They also stated that there were no outcomes regarding the step of creating.

Analysis of Nur's abstraction process indicated that the student was not evaluated even at the stage of action. In this case, it can be stated that the student did not even fulfill simple cognitive mechanisms. On the other hand, the analysis regarding Nur according to RBT, showed that she was at the 1st level cognitively, and at the level of factual knowledge. In other words, it can be stated that in the abstraction processes, even the stage of action predicts higher-order behaviors rather than simple behaviors. Even if the transformation of action constitutes the first step of abstraction, it is not a simple transformation (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Dubinsky, 1991). At the stage of action, the individuals encounter a new mathematical concept, and conceptual knowledge stirs with the transformation of existing mental or physical objects. The individuals follow cognitive steps to perform this transformation, and the process is not simple.

Harun and Sena were found to be at the first stage of the abstraction - action. According to RBT, both of these students were evaluated at the 3rd level and conceptual knowledge level. This finding became even more meaningful when the data regarding Nur were evaluated. In other words, it can be stated that the cognitive behaviors of the first step of abstraction process should be evaluated starting from the application level (level 3) according to the RBT. Kieran (2004) and Garcia-Cruz and Martinon (1997) associated the basic step of students' abstraction of a particular concept with the adequacy of their understanding of the relevant concept.

On the other hand, the data obtained from Ezgi, Talha, Fatih and Okan were evaluated in the stage of process in terms of abstraction. Among these students, Okan and Ezgi were at the 3rd level, Talha was at the 4th level and Fatih was at the 6th level. In addition, Okan was at conceptual knowledge level, Ezgi and Talha were at operational knowledge level and Fatih was at metacognitive knowledge level. Finally, while Islim was in the stage of object in terms of abstraction, she exhibited behavior at the 6th level of cognitive processes and procedural knowledge of the knowledge dimension of the RBT. These findings together with the findings in the previous paragraph showed that it would not be wrong to suggest that the abstraction process be evaluated at least at the level of conceptual knowledge. Accordingly, it was determined that a student who abstracts information should behave at least at the application level in the cognitive level and at least at the conceptual knowledge level in the knowledge dimension. It was also supposed that the RBT categorizes the cognitive mechanisms required by abstraction processes in more detail. In other words, it was noticed that even smaller pieces of information found places.

Conclusions

The use of models such as APOS theory in revealing students' abstraction processes is complex and difficult. An attempt has been made to reduce this difficulty and also to make the pictures of students' abstraction processes visible. This attempt is to test the utilization of the RBT in revealing the abstraction processes. In other words, the cognitive mechanisms required by the abstraction process were compared with the dimensions of the taxonomy and the structure of the abstraction process was tested through taxonomy. The results obtained are thought to categorize the cognitive mechanisms required by the abstraction processes in a more detailed way, with the analysis made with RBT.

Implications

In this study, a qualitative study was conducted with eight students and the aim of the study was achieved with this sample. Quantitative study, on the other hand, can provide various advantages over qualitative research in terms of sample size. For example, reaching more students can contribute to the reliability of the results obtained. Therefore, carrying out this study with a greater number of students can support these results.

Abstraction processes are not easy to study. This becomes more apparent especially as the age level decreases. However, sub-dimensions of the RBT may provide clearer data for revealing cognitive characteristics even in early age levels. Examining the stages of the formation of a concept in a student's mind can provide several advantages, particularly for teaching subjects that are difficult to understand. Therefore, it is recommended that teachers examine such studies and benefit from the practices.

Limitations

Abstraction and the study of cognitive processes are comprehensive and hence a lot of data is obtained. This situation limits the researchers in terms of the number of samples. In fact, increasing the number of participants makes important contributions to the results of this study.

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Appendix 1. Interview Form

COMPANY SCENARIO

Imagine working in the sales department of a light bulb company. Your company produces bulbs for 8 lira each and plans to sell each bulb with a 25% increase. Your main task is to prepare a document showing the relationship between the number of parts sold and the profit made by finding the profit this company made for 1 month (a, number of parts sold; k, profit made).

a. Do you think there is a relationship between the number of parts sold and the profit made, can you explain if so?
b. How can you show off this relationship?
c. You handed over the company's 1-month report to the manager and the manager was not satisfied with the sales and demanded a 10% discount on the sales price. And you will again prepare a document that relates the number of parts sold to the profit made. What would you consider doing?

CONSTRUCTION SCENARIO

Kemal wants to make use of his summer vacation to get his school allowance. For this, he starts a job in a construction company. Kemal's task is to take the sacks full of sand from the ground floor to the second floor of the building. Your task, on the other hand, is to find out how many sacks Kemal carries and how long he did this work in total, and correlate the elapsed time with the remaining sacks. The equation that gives the time (t) during transportation and the number of bags not transported (a) is:

$$a = 12.(4 - t)(t, hour)$$

a. Do you have any idea how many sacks Kemal started with?
b. How long did it take Kemal to carry the sacks?
c. Prepare a form showing the relationship between the elapsed time and the number of unmoved bags.