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Front Pages

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Athens Journal of Sciences

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President's Message

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The current issue is the third of the twelfth volume of the *Athens Journal of Sciences (AJS)*, published by Natural & Formal Sciences Division of Athens Institute.

Gregory T. Papanikos
President
Athens Institute

Athens Institute for Education and Research
A World Association of Academics and Researchers

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20-24 July 2026, Athens, Greece

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- **Dr. Haiduke Sarafian**, Head, Natural Sciences Unit, ATINER & Professor of Physics and Endowed Chair of John T. and Paige S. Smith Professor of Science, Pennsylvania State University, USA.

Important Dates

- Abstract Submission: **16 December 2025**
- Acceptance of Abstract: 4 Weeks after Submission
- Submission of Paper: **22 June 2026**

Social and Educational Program

The Social Program Emphasizes the Educational Aspect of the Academic Meetings of Athens Institute.

- Greek Night Entertainment (This is the official dinner of the conference)
- Athens Sightseeing: Old and New-An Educational Urban Walk
- Social Dinner
- Mycenae Visit
- Exploration of the Aegean Islands
- Delphi Visit

Conference Fees

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Athens Journal of Sciences- Volume 12, Issue 3, September 2025 – Pages 151-170

Wave Energy Potential in the Mediterranean Sea: Assessment, Challenges, and Future Prospects

By Tarek M. El-Geziry[*], Amna S. Dabbous[±] & Dina M. Hendy[°]

As the global demand for sustainable and low-carbon energy sources intensifies, the United Nations' Sustainable Development Goals (SDGs) advocate for renewable, sustainable, and environmentally friendly energy sources. Ocean wave energy, a predictable and stable renewable resource, presents a significant opportunity to diversify the energy mix, enhance energy security, and mitigate climate change. Despite having a moderate energy flux compared to open oceans, the Mediterranean Sea offers a unique opportunity for harnessing wave energy. This is mainly attributed to the stable sea conditions, lower installation risks, and proximity to coastal populations. However, realizing this potential requires overcoming challenges related to technology durability, high capital costs, environmental impact, and fragmented regional policies. Through a qualitative synthesis of recent resource assessments, this paper provides an overview of the Mediterranean Sea's wave energy capacity, highlighting the performance and deployment of wave energy converters (WECs) in various locations within the basin. This study also explores the role of WECs, their technological advancements, integration with hybrid renewable systems, and emerging pilot projects in Mediterranean countries. The study highlights the need for regional cooperation, innovation in wave-hybrid systems, and climate adaptation strategies to harness wave energy as a viable contributor to the Mediterranean's sustainable energy transition.

Keywords: *Mediterranean Sea, wave energy resource, SDGs, WECs, assessment*

List of Abbreviations

Hs………………………………	Significant Wave Height
IPTA……………………………	Innovative Polygon Trend Analysis
ITA……………………………..	Innovative Trend analysis
NDCs…………………………...	Nationally Determined Contributions
OWC…………………………...	Oscillating Water Column
PTO…………………………….	Power Take-off
REWEC3……………………….	Resonant Wave Energy Converter 3
SDGs……………………………	Suitable Development Goals
TW……………………………...	Tera Watts
UN………………………………	United Nations
WECs…………………………...	Wave Energy Converters

Introduction

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The world's energy demand is currently expanding in order to meet the world's ambitious development goals. Between 2003 and 2030, energy consumption is expected to increase by an annual average of 2.0% (AbdElhafez et al. 2012). Since the late 1800s industrial revolution, traditional fossil fuel energy supplies, such as coal and oil, have been primarily used in development processes (Avagyan 2021). Traditional resources are responsible for over ⅓ of global greenhouse gas emissions (Mondal et al. 2022), causing catastrophic climate change. Additionally, increased demand hastens the depletion of fossil fuel reserves. The United Nations (UN) adopted the Sustainable Development Goals (SDGs) to guarantee a more sustainable future for humankind. The goals were established in 2015, and significant implementation is scheduled to be completed by 2030. The proposed SDGs aim to address the pressing global challenges of poverty, inequality, and climate change while ensuring environmental sustainability. Figure 1 displays the SDGs framework, which allows for monitoring of Agenda 2030 implementation and accomplishments. As a result, academics and politicians are interested in exploring renewable, sustainable, and environmentally friendly energy resources. Renewable energy promotes energy security in various sectors, including transportation, the environment, buildings, and industry, in line with the SDGs (Olabi et al. 2023).

Renewable energy resources include wind, sun, geothermal energy, salinity gradient, tides, ocean wave energy, biofuels, and biogas. Renewable energy systems are becoming less costly and more effective, and their share of total energy consumption is growing (Mohammed et al. 2019). Energy from renewable resources accounted for one-third of the total worldwide power capacity built in 2019 (IRENA 2020).

Wave energy, derived from the movement of ocean waves, represents an untapped potential in the renewable energy sector. Unlike solar and wind energy, which are highly variable and weather-dependent, wave energy is more predictable and stable due to its relationship with ocean currents and wind patterns. As the world moves toward decarbonisation, wave energy presents a valuable opportunity to diversify the renewable energy mix, enhance energy security, and contribute to climate change mitigation.

Among the UN-SDGs, SDG 7 (Affordable and Clean Energy) and SDG 13 (Climate Action) are critical, especially in the context of renewable energy. Renewable energy plays a significant role in contributing to several SDGs because it can help countries reduce their dependency on fossil fuels, promote cleaner energy, and create sustainable industries (Jaiswal et al. 2022).

As a renewable energy resource, the contribution of ocean wave to fulfilling SDGs can be highlighted in the following points according to priority:

- SDG 7: Affordable and Clean Energy

Wave energy is a sustainable and reliable source of energy for coastal and island communities. It can complement existing renewable sources, such as solar and wind energy, to offer a more consistent energy generation solution. Countries with high energy demands and reliance on imported fossil fuels can use wave energy to diversify their energy mix, improve energy security, and reduce the environmental footprint of their energy production (Falcão 2010).

Figure 1. *Sustainable Development Goals (SDGs) of the United Nations*

Source: https://www.un.org/sustainabledevelopment/blog/2015/12/sustainable-development-goals-kick-off-with-start-of-new-year/#

- SDG 13: Climate Action

Wave energy contributes to climate change action by reducing greenhouse gas emissions. By shifting from fossil fuels to renewable energy sources, such as wave energy, countries can significantly lower their carbon emissions. The integration of wave energy converters (WECs) into the energy grid also supports the global effort to meet the targets of the Paris Agreement on climate change by enabling cleaner, more sustainable energy generation (IEA-OES 2021).

- SDG 9: Industry, Innovation, and Infrastructure

The development of wave energy technologies has promoted innovation in energy systems, manufacturing, and infrastructure. Investments in WECs drive technological advancements, creating new industries, provide employment opportunities, and foster economic growth in regions with significant wave energy potential (IEA-OES 2021).

- Wave energy also aligns with SDG 14: Life Below Water because it offers an opportunity for low-impact energy generation that minimizes harm to marine ecosystems (Fallah Shayan et al. 2022).
- Furthermore, exploring wave energy resources supports SDG 8: Decent Work and Economic Growth by creating opportunities for jobs in the manufacturing, installation, and maintenance of WECs (Foteinis 2022).

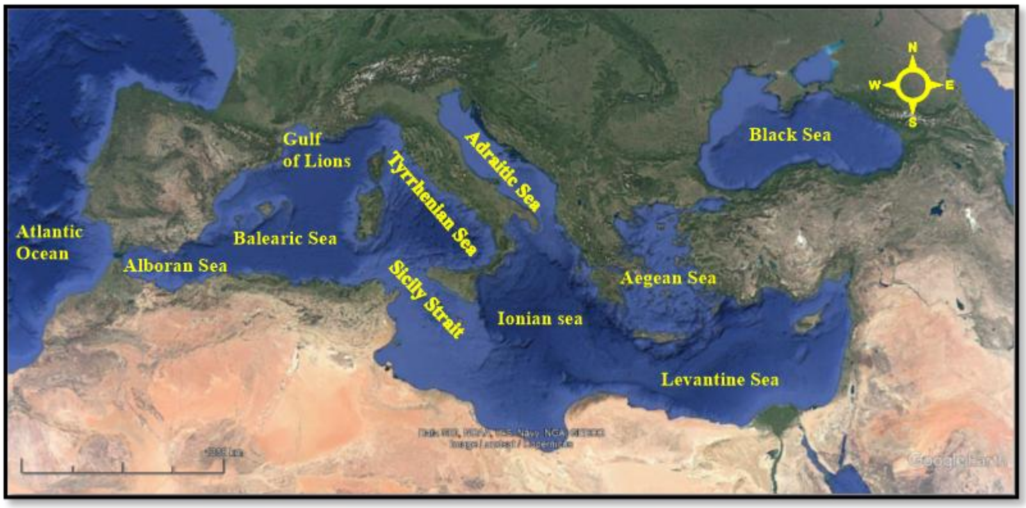
Although established renewable energy sources like solar, wind, biomass, geothermal, and hydropower, are widely utilized, there is an urgent need to diversify the energy mix to effectively mitigate climate change. Wave energy, an abundant yet underutilized resource, has emerged as a promising candidate to complement existing renewable energy sources and reduce dependence on fossil fuels.

According to Mørk et al. (2010), the global gross wave energy resource is estimated to be 3.7 terawatts (TW), with regions such as the North Atlantic and the Mediterranean Sea exhibiting considerable wave energy potential. These areas are home to some of the highest wave power densities, ranging from 10 to 40 kW/m. High-energy regions, such as the Atlantic coastlines of Europe, specifically the United Kingdom, Portugal, and Ireland, exhibit more significant wave energy fluxes, reaching up to 70 kW/m in certain locations (Kalogeri et al. 2017). However, these areas pose operational challenges due to harsh weather conditions, which can hinder the survivability of WECs. Semi-enclosed seas, such as the Mediterranean Sea, offer a viable alternative due to their moderate wave energy levels and reduced risk of extreme events.

The Mediterranean Sea (Figure 2) characterized by an average wave energy power lower than that of the Atlantic, has nonetheless garnered attention for its potential to support sustainable energy projects. Studies have highlighted several promising locations, including the northwestern Mediterranean, southern Italy, and parts of Greece, where wave power levels range from 7 to 15 kW/m (Dialyna and Tsoutsos 2021). Their findings suggest that although the Mediterranean may not rival the Atlantic in raw energy potential, its relatively stable conditions could facilitate cost-effective and reliable wave energy exploitation.

This paper provides a comprehensive qualitative overview of the wave energy landscape of the Mediterranean Sea. The primary objectives of this study are to assess the existing wave energy capacity, analyze the performance and deployment experiences of various WECs across different Mediterranean locations, and identify the key challenges that hinder their widespread adoption. The paper also explores the integration of wave energy with other renewable sources, such as solar and winds, to enhance energy resilience and grid stability in the region. The novelty of this work lies in its interdisciplinary framework, linking wave energy development to broader climate action strategies, socioeconomic benefits, and sustainable coastal infrastructure. By addressing current knowledge gaps and identifying regional opportunities and challenges, this study contributes to advancing wave energy as a viable component of the Mediterranean's sustainable energy future.

Figure 2. *Mediterranean Sea and its Sub-seas*



Source: Authors by using Surfer16® Software

Global Climate Action and the Role of Renewable Energy

While climate change remains a pressing global challenge, notable advancements have been made in mitigation and adaptation strategies, particularly through the adoption of renewable energy systems. These successful advancements often stem from a combination of technological innovation, supportive policy frameworks, and increased public and private investment.

One significant area of success lies in the rapid advancement and deployment of renewable energy technologies, particularly solar and wind power. The global renewable power capacity built in 2019 accounted for one-third of the total worldwide power capacity (IRENA 2020). The drastic decrease in the cost of solar photovoltaic and wind energy over the past decade has made them economically competitive with, and often cheaper than, fossil fuels in many regions (Osman 2023). This cost reduction, driven by research and development, economies of scale, and supportive government incentives (such as feed-in tariffs and tax credits), has led to a significant increase in their adoption (Rydehell et al. 2024). This demonstrates the power of sustained investment and policy support in transforming energy landscapes.

Furthermore, international agreements and frameworks, though sometimes criticized for their pace, have played a crucial role in galvanizing action. For instance, the Paris Agreement established a global framework for climate action, with countries committing to nationally determined contributions (NDCs) to reduce greenhouse gas emissions (Siriwardana and Nong 2019). The focus on achieving the SDGs, particularly SDG 7 (Affordable and Clean Energy) and SDG 13 (Climate Action), has also provided a common agenda for governments, industries, and researchers to pursue cleaner energy solutions.

Another area of success is the growing recognition and implementation of circular economy principles (Velenturf and Purnell 2021). Although not a direct energy source, the circular economy indirectly contributes to climate change mitigation by reducing overall energy consumption and associated emissions (Knäble et al. 2022). This involves reducing waste, reusing materials, and recycling, thereby decreasing the demand for virgin resources and the energy required for their extraction and processing (Velenturf and Purnell 2021, Knäble et al. 2022).

However, a critical analysis of these successes also reveals important caveats. The pace of renewable energy adoption, while impressive, is still not sufficient to meet the ambitious targets required to limit global warming to 1.5°C above pre-industrial levels. Significant challenges remain in grid integration, energy storage, and ensuring grid stability with a high penetration of intermittent renewables (Ergun et al. 2025). The successes achieved often represent the initial steps, and the true test lies in scaling these solutions globally and equitably.

In the context of the Mediterranean region, several successful initiatives demonstrate progress toward decarbonisation and resilience. Several Mediterranean countries have integrated climate goals into their national energy policies. For example, Greece and Italy have implemented strategies to increase the share of renewables, aligning with SDG 7 and SDG 13. These policies have led to the expansion of solar, wind, and, more recently, wave energy pilot projects. EU-funded programs such as Horizon 2020 and regional initiatives under the Union for the Mediterranean have supported joint ventures in marine renewable energy. These programs facilitate technology transfer, data sharing, and capacity building, all of which are crucial for climate adaptation and low-carbon innovation. From a coastal protection point of view, projects in Spain and Italy have demonstrated how WECs can reduce shoreline erosion, offering dual benefits in the face of sea-level rise and intensifying storms, both consequences of climate change (Bergillos et al. 2019, Rodriguez-Delgado et al. 2019).

Despite these successes, progress remains uneven. Many Mediterranean nations still lack cohesive regulatory frameworks for marine renewables, and large-scale WEC deployment remains rare. Additionally, monitoring and evaluating the long-term climate benefits of wave energy projects are still in the early stages.

Wave Energy in the Mediterranean Sea

Resource Assessment

Extensive research has been conducted to evaluate wave energy potential across the Mediterranean basin, with particular emphasis on identifying high-energy hotspots and understanding seasonal variations. The wave energy potential in the Mediterranean region is highly variable, and influenced wind regimes, fetch length, and bathymetry. Studies using wave models and satellite data indicate that the western Mediterranean and specific hotspots like the Ligurian Sea and the Alboran Sea exhibit higher wave energy densities than the eastern Mediterranean (Acar et al. 2023). Seasonal variability is also significant, with peak energy observed during the winter months.

Arena et al. (2015) identified the northwestern Mediterranean, including the Alghero region (west of Italy), as the most energetic area, with an average wave power of 15.1 kW/m. Besio et al. (2016), utilizing 35 years of wave data, pinpointed regions between Sardinia, Corsica, northern Algeria, and the Balearic Islands as particularly promising, with an average wave energy potential of around 10 kW/m. The eastern Mediterranean has been the focal point of numerous studies due to its unique characteristics and moderate wave energy potential. Key locations, such as northern Tunisia, western Crete, southern Sicily, and the southern Ionian Sea, exhibit wave power levels ranging from 7.3 to 11.1 kW/m (Ayat 2013). Vicinanza et al. (2013) highlighted the western Sardinian coastline as a prime site, specifically the Porto Alabe and Torre del Porticciolo areas. Zodiatis et al. (2014) estimated the mean wave energy in the eastern Mediterranean as 10 kW/m. Similarly, studies in Sicily have identified the western coastline and the Strait of Sicily as regions with strong wave energy potential, with average wave power fluxes of 8 kW/m and 4-6 kW/m, respectively (Iuppa et al. 2015, Dialyna and Tsoutsos 2021).

Within this region, the Aegean Sea, Levantine Basin, and Libyan Sea have shown considerable promise for exploitation of wave energy, albeit with varying levels of seasonal and spatial variability.

The Aegean Sea, a semi-enclosed area with a complex coastline and numerous islands, offers a mix of opportunities and challenges for wave energy development. Studies by Ayat (2013) and others have identified regions such as southern Crete, Karpathos, and eastern Crete as having consistently high wave energy flux, with an average of around 8 kW/m during the winter months. Additionally, central areas of the Aegean, including the straits between islands such as Mykonos and Ikaria or between Crete and Kasos, exhibit localized energy peaks driven by strong winds and favorable topography (Porichis 2023). These regions are particularly appealing due to their relatively low variability compared to the central Aegean region, which experiences greater fluctuations. The Adriatic and Aegean Seas have promising but underutilized potential, averaging 3-7 kW/m (Lavidas and Venugopal 2017). Other promising locations include the Aegadian Islands, the northern Latium coasts, and specific nearshore areas, such as Argentario in Tuscany (Lo Re et al. 2019, Dialyna and Tsoutsos 2021). The regions near Crete and Karpathos demonstrate consistent energy levels of around 8 kW/m (Lavidas and Venugopal 2017). Seasonal studies in these areas emphasize lower variability compared to the central Aegean, suggesting their suitability for wave energy exploitation.

The Levantine Basin, encompassing the coasts of Cyprus, Israel, Lebanon, and Egypt, is another significant area of interest. Zodiatis et al. (2014) highlighted the western coastline of Cyprus and parts of the Israeli shoreline as high-potential zones for wave energy, with average wave power levels of around 6–7 kW/m. El-Sharkawy et al. (2017) evaluated the wave energy resource off Port Said along the eastern Egyptian Mediterranean coastline to be 4.5-6.2 kW/m. These sites are characterized by stable wave conditions and proximity to densely populated coastal regions, making them suitable for localized energy generation projects. Furthermore, the basin's relatively calm conditions reduce the risk of extreme weather events, enhancing the operational reliability of WECs.

The Libyan Sea, located to the south of Crete, also stands out as a key area for wave energy research. Lavidas and Venugopal (2017) assessed the wave energy flux in this region, reporting seasonal averages between 8 and 10 kW/m during winter

months, with reduced variability compared to other parts of the Mediterranean. This stability, coupled with its proximity to key maritime routes and emerging markets in North Africa, positions the Libyan Sea as a strategic location for future wave energy investments.
Overall, these studies demonstrate that although the eastern Mediterranean has lower wave energy levels than the Atlantic, it offers significant opportunities for sustainable energy development. The relatively stable wave conditions and strategic geographical location make this region attractive for localized energy production and integration into renewable energy systems.

Table 1 outlines some previous studies on wave energy potential across the Mediterranean Sea, showing the region of interest and type of data used.

Table 1. *Wave Energy Resource Assessment Studies in the Mediterranean Sea*

Geographical Region	Data Types	References
Mediterranean Sea (Entire)	Numerical Wave Models, Satellite Data; ERA5	(Liberti et al. 2013, Besio et al. 2016, Acar et al. 2023)
Libyan Sea	*In-Situ* Measurements; ERA5	(Lavidas and Venugopal 2017, Foti et al. 2022)
Aegean and Levantine Seas	Numerical Wave Models, *In-Situ* Measurements, Satellite Data	(Ayat 2013, Zodiatis et al. 2014, El-Sharkawy et al. 2017, Porichis 2023)
Greek Coasts	Numerical Wave Models, *In-Situ* Measurements	(Soukissian et al. 2011, Emmanouil et al. 2016, Porichis, 2023)
Sardinian Coasts	Numerical Wave Models, *In-Situ* Measurements	(Vicinanza et al. 2013)

Technological Advances in Wave Energy

Recent advances in wave energy conversion technologies have been significant. The development of various WECs, such as point absorbers, oscillating water columns (OWC), and overtopping devices, has led to successful pilot projects and deployments worldwide. The integration of these devices into coastal infrastructure, such as breakwaters and ports is a cost-effective and environmentally friendly solution (Vicinanza et al. 2013).

In terms of technological readiness, the most advanced WECs are those tested in real-world environments. For example, the Resonant Wave Energy Converter 3 (REWEC3) device, deployed in Civitavecchia, Italy, has demonstrated a substantial ability to generate electricity, with installed capacities ranging from 50 kW to 2500 kW, depending on the technology and location (Arena et al. 2015). Additionally, hybrid systems that combine wave energy with other renewable energy sources, such as wind and solar, have been identified as promising solutions to increase overall energy generation and provide more consistent energy outputs (Ferrari et al. 2020).

However, challenges arise when using the different technologies to harness wave energy. Wave energy technologies face hurdles such as high capital costs, durability in marine environments, and low efficiency under moderate wave climates. Point absorbers, oscillating water columns, and overtopping devices are among the technologies explored for the Mediterranean. In addition, potential impacts on marine ecosystems, such as noise pollution and habitat disruption, necessitate comprehensive environmental impact assessments. The Mediterranean's biodiversity and designation as a semi-enclosed sea intensify these concerns. Lastly, policy and regulatory barriers remain important challenges to harnessing energy from the Mediterranean wave regime.

Integration with Other Renewable Energy Sources

Recent research has explored the feasibility of combining wave energy with other renewables such as wind and solar. Such hybrid systems aim to enhance energy generation reliability and efficiency by leveraging the complementary nature of these resources. Hybrid systems have shown promise in maximizing energy output and stabilizing supply (Hassan et al. 2023). Several studies have evaluated the feasibility of integrated wave and wind energy systems across the Mediterranean. Ferrari et al. (2020) identified the Algerian coastline as an advantageous area for harvesting both wave and wind energy. Studies indicate that regions like the Strait of Sicily, the Gulf of Lions, and offshore areas near Sardinia and the Balearic Islands are particularly well-suited for integrated wave and wind energy harvesting (Kalogeri et al. 2017, Azzellino et al. 2019, Dialyna and Tsoutsos 2021). These locations exhibit consistent wave activity and favorable wind conditions, enabling stable and high-yield energy production.

In the eastern Mediterranean, Greece has emerged as a promising region for wave-wind hybrid systems. The Greek wave and wind potential, which underlies the largest wave power potential in the southern Ionian Sea and the western Cretan Sea - nearly 7 kW/m- was assessed by Emmanouil et al. (2016). In addition, locations such as southeastern Mykonos and northwestern Crete have been identified as optimal for combined exploitation (Ganea et al. 2017). Assessments of the Italian coastlines revealed viable sites for co-deployment, considering factors like depth, environmental impact, and accessibility (Azzellino et al. 2019). Their findings suggest that locations near Elba Island, the Aeolian Islands, and the southern Adriatic and Ionian Seas offer significant potential for such hybrid installations. These areas are strategically advantageous, considering environmental factors, sea depth, and accessibility.

Table 2 summarizes studies on hybrid wave-wind energy systems, highlighting key regions and the potential for combined energy harvesting.

Table 2. *Examples of Wave and Wind Energy Resource Assessments in the Mediterranean Sea*

Geographical Area	Wind and Wave Energy Hybrid System	Reference
Strait of Sicily	High potential for combined energy systems	(Kalogeri et al. 2017)
Greek Coasts	Optimized hybrid systems for energy generation	(Vasileiou et al. 2017)
Balearic Islands	Potential for co-deployment of wind and wave energy	(Azzellino et al. 2019)

These findings highlight the potential of hybrid renewable energy systems to enhance energy security and sustainability in the Mediterranean region. By integrating wave energy with other renewable sources, the Mediterranean region can optimize energy production, improve grid stability, and reduce reliance on fossil fuels. These synergies could also lead to innovative applications, such as powering desalination plants and providing clean energy to coastal communities.

Ongoing Trends of Wave-Climate

Climate change has a profound impact on wave energy resources, influencing key parameters such as wave height, period, and propagation direction. Factually, ocean waves respond complexly to climate change, which may have a direct impact on the variables involved in their genesis, transformation, and dissipation (Simonetti and Cappietti 2023, de Leo et al. 2024). Recent studies have examined how these trends might alter the viability of wave energy projects in the Mediterranean (e.g., Morim et al. 2019, Acar et al. 2023, De Leo et al. 2024). Sierra et al. (2017) explored future wave climate scenarios for Menorca, Spain, revealing a potential decline in wave energy during the autumn and winter, coupled with spatial variability in the summer. In contrast, studies on the Moroccan Mediterranean coast have predicted relatively stable wave energy resources under future climate scenarios, indicating minimal disruption to energy harvesting potential. A projected increase in wave power due to longer wave periods along the Calabria coasts was concluded through analysis of wave-climatology data in the Italian seas (Foti et al. 2022). Their research also indicated positive trends in wave energy potential for most Italian coastal regions, excluding the Adriatic and Ligurian Seas. Similar trends were identified by Casas-Prat et al. (2022), who estimated changes in the wave parameters in the northwestern Mediterranean. Their findings suggest seasonal variations, with winter waves becoming more energetic while summer conditions remain stable. De Leo et al. (2024) reviewed studies that used multi-decadal datasets from numerical models, satellite observations, and *in-situ* measurements to draw up trends in ocean wave climates within the Mediterranean Sea, focusing mainly on significant wave height (Hs). Negative trends were more commonly observed, although often not statistically significant. Positive and significant trends are notable in the western Mediterranean, particularly in the Gulf of Lion and the Tyrrhenian Sea. The authors recommended the need for rigorous, high-resolution studies to address the gaps in understanding and provide actionable insights for climate change adaptation in the Mediterranean region. Acar et al. (2023) investigated long-term trends in wave power in the Mediterranean Sea using data from the ERA5 reanalysis spanning 60 years (1962–2021) by employing classical statistical approaches such as the Mann–Kendall test and innovative methods, including the Innovative Trend Analysis (ITA) and Innovative Polygon Trend Analysis (IPTA), to evaluate annual and seasonal changes in wave power. Notable increasing trends in areas like the Libyan Sea suggest promising zones for renewable energy exploitation through WECs. However, this variability emphasizes the need for careful planning to address potential risks and optimize the sustainability of wave energy installations. The use of advanced methods such as ITA and IPTA is encouraged for nuanced analyses of wave energy trends in future studies.

Overall, the findings of wave-climate studies emphasize the need for adaptive strategies in wave energy system design to account for evolving wave climate conditions. By anticipating these changes, developers can ensure the long-term viability and resilience of wave energy projects in the Mediterranean.

Wave Energy and Other Considerations

The exploitation of wave energy in the Mediterranean involves multidimensional considerations, including coastal protection, socioeconomic benefits, environmental impacts, and integration with desalination technologies.

Wave energy converters have the potential to serve dual purposes by generating clean energy while protecting coastal areas from erosion. Bergillos et al. (2018) investigated the role of WECs in safeguarding the Guadalfeo deltaic coast in southern Spain and demonstrated their effectiveness in reducing erosion in vulnerable regions. Similarly, different configurations of WEC installations in low-energy seas were evaluated by Foteinis (2022), emphasizing the ability of deployed WECs to enhance coastal sustainability while mitigating the high capital costs typically associated with wave energy projects. The socioeconomic implications of wave energy development are particularly significant for Mediterranean countries. Lavidas (2019) explored the opportunities for WECs in Greece, where the relatively moderate wave energy potential could be advantageous due to reduced risks of extreme weather events. His study highlighted the potential for job creation, economic growth, and energy security, and advocated for policy support to foster wave energy adoption in regions like Greece.

Environmental assessments of WEC installations have focused on minimizing their ecological footprint. For instance, Corsini et al. (2015) examined the deployment of nearshore WECs on Ponza Island, Italy, and identified their low environmental impact while producing sustainable energy. Additionally, Buscaino et al. (2019) studied the acoustic emissions of WECs in Mediterranean shallow waters, and found that their impact on marine life was minimal and could be managed effectively with proper planning and monitoring.

Desalination is a critical application of wave energy in water-scarce Mediterranean regions, but its high energy demands pose economic and environmental challenges. Viola et al. (2016) demonstrated the feasibility of WEC-powered desalination systems in Sicily, Italy, highlighting their potential to provide a sustainable water supply.

Similarly, Hwang and Kiung (2017) assessed the use of point absorbers for wave energy extraction around Sicily, and integrated this technology into existing desalination plants. These studies suggested that coupling wave energy with desalination offers a practical solution to address both energy and water needs in the Mediterranean.

While wave energy shows promise, several challenges remain, including high capital costs, technological maturity, and environmental concerns. The scalability of wave energy projects requires further investigation, particularly in terms of long-term durability and the economic feasibility of large-scale deployments. Additionally, the integration of WECs into national grids must be carefully managed to ensure grid stability and to prevent over generation during peak production periods. Future research should focus on reducing the costs of WECs, improving their efficiency, and developing hybrid systems that combine wave energy with other renewable sources, such as solar and wind. Increased investments in research and development, along with supportive government policies, will be critical in advancing wave energy technology and making it a commercially viable energy source.

Installed Capacities and Performances of Wave Energy Converters (WECs)

Overview of Installations across the Mediterranean Basin

The Mediterranean basin hosts several pilot and operational WEC installations, reflecting diverse wave energy harnessing approaches. Despite its moderate wave climate, technological advancements have facilitated deployment in key regions:

- **Egypt** (Salah et al. 2022): While still in nascent stages, Egypt has initiated studies of its wave energy potential along its Mediterranean coastline, particularly near Alexandria. Preliminary pilot projects aim to test oscillating water column (OWC) technologies, targeting energy outputs of 50-75 kW. These efforts align with the country's broader renewable energy strategy.
- **Israel** (Dialyna and Tsoutsos 2021): Coastal installations, such as Eco Wave Power's project in Jaffa Port, use innovative float-based WECs. These systems have achieved capacities up to 100 kW, with scalability plans are underway.
- **Italy** (Alfano 2023): Projects such as the ENEA Wave Energy Pilot near Pantelleria have demonstrated the potential of point absorber technologies. Recent evaluations indicate energy outputs averaging 80-100 MWh annually for moderate wave climates.
- **Greece** (Pompodakis et al. 2024): The Aegean Sea features small-scale installations, including the Poseidon Platform, which integrates wave and solar energy. These systems have high-capacity factors during winter months, reaching 35% efficiency.
- **Spain** (Carreno-Madinabeitia et al. 2024): along the Bay of Biscay, the Mutriku Breakwater wave plant was integrated into a breakwater structure

and utilizes oscillating water column (OWC) technology. Inaugurated in July 2011, it has a capacity of 296 kW from 16 turbine units. By October 2022, the plant had delivered 2.7 GWh of energy to the grid, demonstrating the viability of wave energy in the region.

These installations represent significant steps towards the commercialization of wave energy in the Mediterranean Basin. They provide valuable data on device performance, grid integration, and environmental impact, paving the way for future large-scale deployments.

Performance Analysis

Vannucchi and Cappietti (2016) highlighted the following performance metrics for WEC installations in the Mediterranean basin:

1. **Capacity Utilization**: Efficiency ranges from 20% to 40%, depending on the location and wave climate.
2. **Durability**: Long-term operations emphasize the importance of corrosion-resistant materials and adaptive maintenance strategies.
3. **Economic Viability**: Subsidies and incentives have played a significant role in supporting initial deployments, particularly in regions with higher energy costs.

Challenges in Harnessing Wave Energy

Technological Challenges

Developing efficient and durable WECs is a primary technical hurdle. Wave energy technologies face hurdles such as high capital costs, poor durability in marine environments, and efficiency under moderate wave climates. Point absorbers, oscillating water columns, and overtopping devices are among the technologies explored for the Mediterranean. A review by McLeod and Ringwood (2022) highlighted that the irregular, low-frequency nature of ocean waves poses challenges to conventional energy conversion systems, necessitating specialized designs for effective energy capture. Optimization of WEC layouts and power take-off (PTO) systems is crucial for maximizing energy extraction. Recent research has focused on enhancing PTO parameters and site selection procedures to improve efficiency. For instance, Mehdipoura et al. (2023) introduced a novel hybrid algorithm that achieved a 3.31% increase in power output, demonstrating the potential of advanced optimization techniques. While the Mediterranean's semi-enclosed nature offers a less extreme environment than open oceans, ensuring the durability of WECs against storms and long-term wear remains a concern (Dialyna and Tsoutsos 2021).

Environmental Concerns

Potential impacts on marine ecosystems, such as noise pollution and habitat disruption, necessitate comprehensive environmental impact assessments. The Mediterranean's biodiversity and designation as a semi-enclosed sea intensify these concerns (Buscaino et al. 2019). Allocating maritime space for WECs must consider existing uses such as fishing, tourism, and shipping lanes to minimize conflicts and ensure sustainable development.

Economic Viability

The economic feasibility of wave energy in the Mediterranean (Pompodakis et al. 2024) is influenced by the following:

High Initial Costs: The development and deployment of WECs involve substantial upfront investments, which can be a barrier to entry.

Market Competition: Wave energy must compete with more established renewable sources like wind and solar, which currently benefit from lower costs and more mature technologies.

Table 3 summarizes studies that discuss the economic impact, job creation, and energy security benefits of deploying wave energy converters in Mediterranean countries.

Table 3. *Socioeconomic Benefits of Wave Energy in the Mediterranean Region*

Country/Region	Socioeconomic Benefits	Reference
Greece	Job creation, economic growth, energy security	(Lavidas 2019)
Italy	Coastal protection, energy independence	Bergillos et al. (2018)
Cyprus	Sustainable water supply (desalination)	(Viola et al. 2016)

Policy and Regulatory Barriers

Efforts are underway to develop supportive regulatory frameworks and incentives to attract investment and facilitate the integration of wave energy into national energy strategies. A fragmented regulatory framework and a lack of region-specific policies hinder the deployment of wave energy projects. The varied economic and political landscapes of the Mediterranean countries further complicate collaborative initiatives.

Future Prospects

Technological Innovations

Advances in WECs, materials, and hybrid systems can enhance efficiency and reduce costs. Floating WECs and multi-purpose platforms for integrating wave, wind, and solar energy are promising. Materials science advancements, energy collection techniques, and power take-off systems are projected to increase efficiency and lower operational costs. Furthermore, digitalisation and artificial intelligence can improve the performance prediction, remote monitoring, and adaptive maintenance of WECs in complicated marine environments.

Regional Cooperation

Transboundary collaboration is essential for large-scale deployment. Initiatives like the Union for the Mediterranean's renewable energy agenda can play a pivotal role in fostering joint ventures and knowledge sharing. Future progress depends on improved regional cooperation, such as standardised permission frameworks, grid integration standards, and maritime spatial planning. Cross-border research and demonstration activities will also aid in the development of WECs, with funding from the EU and foreign sources. Policy instruments such as feed-in tariffs, green procurement rules, and innovation incentives are critical for lowering investment risk and attracting private sector participation.

Climate Change Considerations

Climate change may alter the wave energy patterns in the Mediterranean. While some studies have predicted an overall decrease in wave energy due to weaker winds, localized increases in extreme wave events could offer new opportunities. Future wave energy projects must undergo comprehensive environmental impact studies to ensure minimum harm to marine biodiversity and fisheries. These projects should be integrated into multi-use coastal zones so that they can give additional advantages like coastal preservation and habitat enhancement. In addition, coupling wave energy with sustainable desalination technologies can address both energy and water security concerns in arid Mediterranean regions.

Discussion

This detailed analysis highlights the considerable, yet frequently under-valued, potential of wave energy in the Mediterranean Sea. While generally regarded as an area with low energy flux when compared to open oceans, our research, confirmed by current literature, identifies particular hotspots with high wave power density. The detailed assessment of wave energy potential across diverse Mediterranean regions, which incorporates current data, lays a solid platform for future site-specific developments and resource allocation. The study's findings are consistent with recent research highlighting the western Sardinian coast, Aegean Sea, and parts of the Libyan and Levantine Seas as viable hotspots for wave energy exploitation.

This work stands out for its integrated perspective, which links spatial resource variability to SDG alignment, environmental issues, and socioeconomic results. For example, while technical promise alone may not justify large-scale investment in some places, combining wave energy with desalination or coastal protection infrastructure might increase economic feasibility and social acceptance.

The progress in Wave Energy Converter (WEC) technology within the Mediterranean is particularly noteworthy. The evolution of WEC designs—particularly hybrid and nearshore-integrated systems—demonstrates increasing adaptability to the Mediterranean's specific wave climate. Pilot projects in Spain, Greece, and Italy have demonstrated that innovations like overtopping devices and oscillating water columns can be beneficial even in low-energy situations. However, widespread deployment is hampered by high capital costs and the requirement for long-term endurance in corrosive marine environments. This technological advancement is a result of ongoing research and development efforts, reflecting important triumphs witnessed in other renewable industries such as solar and wind power, where persistent innovation and scale have brought down costs and improved adoption rates.

Beyond the technical feasibility, the socioeconomic benefits associated with wave energy deployment in the Mediterranean are substantial and multifaceted. The capacity for significant job creation across the entire value chain—from research and development to manufacturing, installation, operation, and maintenance—presents a compelling case for national and regional investment. This potential for green job creation aligns perfectly with broader sustainable development goals and can help foster economic growth in coastal communities. Furthermore, by providing a stable and predictable source of renewable electricity, wave energy can markedly enhance energy security, reducing the region's reliance on volatile fossil fuel markets and contributing to macroeconomic stability. The environmental advantages, highlighted by Life Cycle Assessment data, solidify wave energy's role as a clean alternative, offering a pathway to significantly reduce carbon emissions in the energy sector.

However, realizing the full potential of wave energy in the Mediterranean necessitates addressing several critical challenges. While evolving, policy and regulatory frameworks often lack the long-term consistency and clarity required to attract the substantial private investment needed for large-scale projects. The upfront capital costs of WEC technologies, though decreasing, still represent a barrier, necessitating innovative financing mechanisms and targeted incentives. Grid integration, particularly for island nations or remote coastal areas, poses technical and infrastructural challenges that require strategic planning and investment. Moreover, ensuring public and local community acceptance through transparent engagement and benefit-sharing mechanisms is paramount for successful deployment.

Despite its contributions, this study acknowledges certain limitations. Data variability among regional models, limited validation of long-term trends, and the still-emerging body of Mediterranean-specific WEC performance data necessitate cautious generalizations. Further empirical research and in-situ experimentation are required to refine site selection and improve the techno-economic assessment of projects under real-world conditions.

Conclusion

In conclusion, the Mediterranean Sea, despite its moderate wave energy potential in comparison to high-energy oceanic regions, stands as a vital region for diversifying the global energy mix and advancing towards a sustainable future. This study provided a comprehensive assessment of wave energy resources across the basin, emphasizing the interplay between technological advancements, spatial variability, climate resilience, and regional policy dynamics. Harnessing wave energy potential within the Mediterranean basin requires a multi-faceted approach encompassing continued technological innovation, supportive regulatory environments, and collaborative efforts among Mediterranean countries. Exploration of the Mediterranean wave energy resources aligns strongly with the SDGs of the UN, particularly SDG 7 (Affordable and Clean Energy), SDG 13 (Climate Action), and SDG 9 (Industry, Innovation, and Infrastructure). The findings underscore that while wave energy fluxes in the Mediterranean generally range from 5 to 15 kW/m, specific hotspots such as the western Sardinian coast, the Aegean Sea, the Libyan Sea, and parts of the Levantine Basin show stable and viable energy levels for localized deployments. These regions are particularly favorable due to their milder sea states, reduced risks of extreme wave conditions, and proximity to densely populated coastlines. Future research should focus on detailed techno-economic analyses of integrated hybrid renewable energy systems (wave-wind-solar), exploring optimal WEC configurations for specific regional characteristics, and conducting comprehensive social acceptance studies. By strategically investing in and collaboratively developing its inherent wave energy resources, the Mediterranean can significantly contribute to global decarbonisation efforts, enhance regional energy independence, and foster sustainable economic development in line with the UN-SDGs.

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Systematic Review and Meta-analysis: The Application of Robotic and Drone Technology in Waste Management

By Tyrone Bright[*]

This systematic review and meta-analysis aims to evaluate the application of robotic and drone technology in waste management, specifically focusing on image recognition and deep learning for waste identification and categorisation. Due to urbanisation and the rapid growth of modern cities, ecosystems face challenges such as waste management. Therefore, there is a growing trend to develop innovative, efficient and environmentally sustainable solutions. These solutions include integrating advanced technologies such as robotic and drone technologies. It is coupled with artificial intelligence (AI), particularly image recognition and deep learning algorithms, which have emerged as promising tools to enhance waste management processes. The methodology of this review involves a comprehensive search of databases such as IEEE Xplore for studies published up to June 2024. The inclusion criteria are studies that describe the use of robotics and drones equipped with image recognition and deep learning capabilities in waste management settings. The primary outcomes assessed include the accuracy of waste identification, the efficiency of waste categorisation, and the overall impact on waste management practices. The results of this meta-analysis reveal significant advancements in the accuracy and efficiency of waste management processes facilitated by robotic and drone technologies. Image recognition and deep learning algorithms have shown high efficacy in identifying and categorising various types of waste, thereby optimising the sorting and recycling process. Furthermore, the application of these technologies has demonstrated potential in reducing human exposure to hazardous waste, improving the speed of waste processing, and enhancing the precision of waste data collection for better environmental management. This review discusses the implications of these findings for developing intelligent waste management systems. It highlights the role of robotic and drone technology in advancing sustainable waste management practices, addressing challenges such as waste segregation and recycling rates. Additionally, the review identifies gaps in current research. It suggests directions for future studies, emphasising the need for scalable solutions and integrating these technologies into existing waste management frameworks. Through this analysis, the review contributes to understanding how emerging technologies can be leveraged to improve environmental sustainability and waste management efficiency.

Keywords: *robotic technologies, drone technologies, waste management, deep learning, image recognition, artificial intelligence, waste categorisation*

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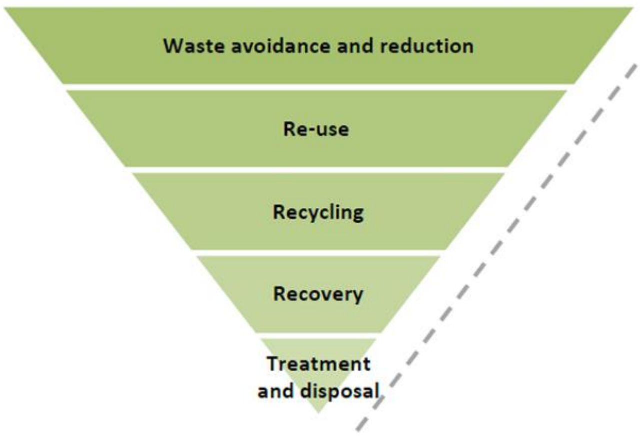
Introduction

The world is producing almost double the amount of plastic waste compared to two decades ago, with only 9% successfully recycled (OECD 2022). This is according to a recent report by the Organization for Economic Cooperation and Development (OECD) outlining climate action and statistics in developing and developed countries.

The European Union (EU) recently adopted a Circular Economy Plan that creates a framework for providing incentives for investing in close-loop material for critical raw material (CRM). This framework addresses the increasing demand for waste production and the potential shortfall of CRMs for European manufacturers by recovering and recycling electronic waste. Electronic waste, known as e-Waste, is the fastest-growing waste stream globally, with 53.6 million tons of e-Waste being generated globally in 2019 (Johnson et al. 2022).

With the significant increase in plastic waste and the recent adoption of the EU circular Economy Plan, waste management is important in developing and developed worlds. Waste management focuses on the control of waste streams. It is based on the hierarchy approach (see Figure 1). This approach consists of various categories for waste through its lifecycle and is arranged in descending order of priority. Primarily, waste should always be avoided and reduced. It would reduce the number of waste management systems required, which is the foundation of the hierarchy. If waste cannot be avoided, it should be recovered, reused, recycled, or treated. Disposal of waste should be a last resort. This research will focus on the recovered category of waste management.

Figure 1. *Waste Management Hierarchy (Sibeko & Adeniji 2020)*



With a focus on waste recovery, classification, identification, and sorting play fundamental roles in this process. Research shows that many countries still rely on manual waste classification, especially in third-world countries (Ferronato & Torretta 2019). It has significant drawbacks as it poses health risks to waste operators, it is costly and highly inefficient. With recent developments in machine learning, robotic technology, drone technology and artificial intelligence as outlined by Koskinopoulou et al. (2021), Bhavana et al. (2022), Bobulski & Kubanek (2021) and Liu et al. (2023), waste classification and identification have become a popular research area. A systematic review and meta-analysis were conducted to understand the state of research on waste identification, classification and sorting with advanced technologies.

This systematic review and meta-analysis aims to evaluate the effectiveness of robotic and drone technology in waste management, specifically focusing on image recognition and deep learning for waste identification, categorisation and sorting. There is a growing trend to develop innovative, efficient and environmentally sustainable solutions in this field of research. These solutions include integrating advanced technologies such as robotic and drone technologies. It is coupled with artificial intelligence (AI), particularly image recognition and deep learning algorithms, which have emerged as promising tools to enhance waste management processes.

The methodology of this review involves a comprehensive search of the IEEE Xplore database for studies published from 2019 to 2024. The inclusion criteria are studies that describe the use of robotics and drones equipped with deep learning and image recognition capabilities for waste management systems. The primary outcomes assessed included the accuracy of waste identification, the efficiency of waste categorisation, and the overall impact of these technologies on waste management practices.

This meta-analysis revealed significant advancements in the accuracy and efficiency of waste management processes facilitated by robotic and drone technologies. Image recognition and deep learning algorithms have shown high efficacy in identifying and categorising various types of waste, thereby optimising and enhancing the recovery process. Furthermore, these technologies have demonstrated the potential to reduce human exposure to hazardous waste, improve the speed of waste processing, and enhance the precision of waste data collection for better environmental management.

This review discusses the implications of these findings for developing intelligent waste management systems. It highlights the role of robotic and drone technology in advancing sustainable waste management practices, addressing challenges such as waste segregation and recovery rates. Additionally, the review identifies gaps in current research. It suggests directions for future studies, emphasising the need for scalable solutions and integrating these technologies into existing waste management frameworks. Through this analysis, the review contributes to understanding how emerging technologies can be leveraged to improve the sustainability and efficiency of waste management solutions.

Section 2 of the research will focus on the literature review to develop intelligent waste management systems with advanced technologies. Section 3 will outline the methodology of the systematic review and meta-analysis. Section 4 highlights the results of the systematic review and meta-analysis. The study's impact and future research direction will be discussed in sections 5 and 6.

Literature Review

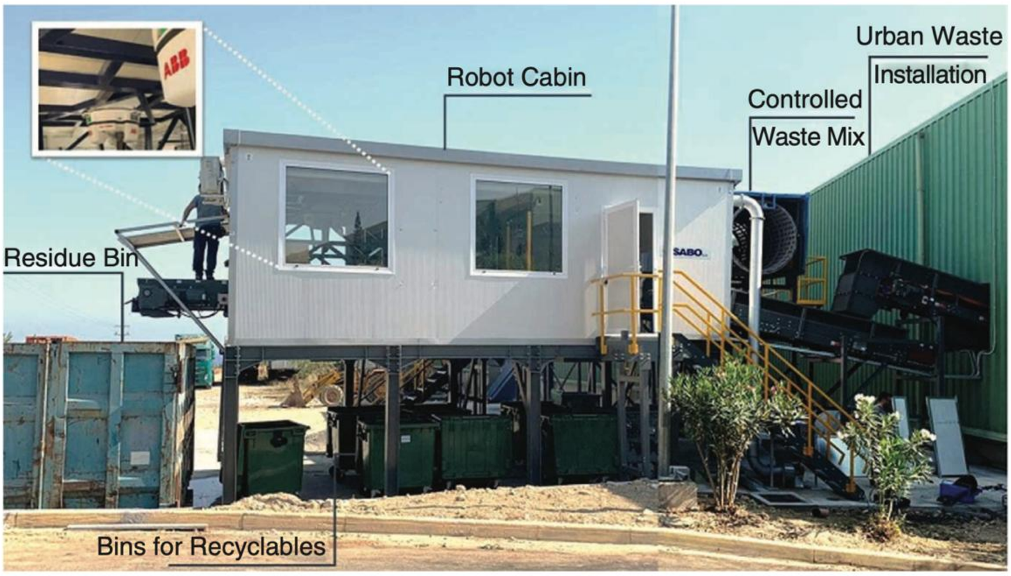
Waste management can be defined as processes and actions required to control waste. These processes include collecting, transporting, treating, monitoring and disposing waste. Since waste management consists of treating and disposing of waste, it has legal and regulatory frameworks that govern how, where and what waste can be disposed of. As such, waste management remains a manual and costly process in most parts of the world. It has presented various challenges to the industry, such as unsafe labour conditions, high costs and low efficiency. Therefore, there is a significant focus on researching more 'smart' and efficient methods to manage waste.

In recent decades, substantial research has been conducted on innovative and intelligent waste management processes. Initially, the focus was on using sensors in recycling plants to improve the sorting process and monitor waste movement. It included using magnetism, air columns and cameras to sort or identify waste throughout the waste management facility. This allowed waste to be sorted into various base categories, i.e. plastic, paper, metals, cardboard and solid waste. This has developed further through robotic technology, drone technology and advanced neural networks.

Researchers such as Joseph et al. (2023), Koskinopoulou et al. (2021), Tornero et al. (2022) developed advanced robotic systems that used sensor data and advanced neural networks for the identification and sorting/collection of waste. Joseph et al. (2023) leveraged a deep learning algorithm and robotic arm, known as E-Waste Intelligent Robotic Technology (EIRT), to detect, sort and classify waste. The deep learning algorithm used an EfficientNET-D2 architecture with waste classification, sorting, and pressure testing modules. The algorithm was trained using a TrashBox dataset, which included pictures of waste and e-waste. The performance matrix of the system showed that Accuracy classification was 82.32%, the precision of the model varied between 76,21 % to 91,14 %, and the F1 score ranged from 78.24% to 90.97%. The F1 score refers to the mean of the precision and recall value. It is used to gain a better understanding of model performance. The system was only tested in a controlled, closed environment where it identified objects in isolation. It was noted that while it displayed promising results, more work could be done to train the module. This would increase its ability to accurately classify features and waste faster.

Koskinopoulou et al. (2021) focused on developing an autonomous robotic system for categorising and sorting recyclables according to the material type. The autonomous robotic system was designed with a low-cost computer vision system based on deep learning technology. The autonomous robotic system was developed with a low-cost computer vision system coupled with deep learning technology. The system used an ABB IRB360 Delta robot, a ZED camera module, a vacuum gripper module and a Mask Regional CNN (R-CNN) network. Data from TrashNet and TacoNet were augmented to create a greater dataset for the network training. The composite system was tested and produced an average success rate of 91.8%. This indicated that the combination of rotation, translation, and scaling contributed to improvements in the Mask R-CNN performance. This system was implemented in an industrial research installation at the Crete MRF, as seen in Figure 2. The system was tested with multiple waste objects on a single conveyor.

Figure 2. *A Depiction of the Industrial Research Installation at the Crete MRF*



Starting from the right, two feeders place waste on a conveyor belt. The feeders are located above the belt to provide a controlled waste. The waste mixture is passed through the robot cabin, where the system identifies and separates the waste according to class. The separated waste is dumped into its respective recyclable bins for further processing. Any non-recyclable objects are disposed of in the residue bin.

Tornero et al. (2022) developed a sensory system using colour and depth images to detect instances of waste. The system could classify waste into four categories (plastic bottles, boxes, cans and glass bottles). It would determine the type of waste and the 3D coordinate for collection by the robotic arm. The system comprises an Autonomous Robot (Blue), an NVIDIA DGX-A100 GPU, an RGBD Camera (Intel Realsense d435i with LIDAR) and a YOLO detection algorithm. Data augmentation techniques were used with various versions of YOLO algorithms (YOLOv3, YOLOv3-SPP, YOLOv5s, YOLOv51, YOLOv51 Augmented). The system results highlighted that YOLOv51 Augments achieved the most efficient and promising option with precision parameter – 0.991, recall – 0.992 and rate of detection – 36 ms. This project was conducted in an open and dynamic environment with isolated objects. One of the shortcomings of the results is that it can't achieve the same performance when waste objects are piled together.

Jalali et al. (2023) developed a real-time waste detection and recycling delta parallel robot based on deep learning. The system focused on the use of two methods for waste detection. It compared multi-stage detection, which focused on localisation, segmentation and classification, to one-stage detection (YOLOv5).

Samsudeen et al. (2023) and Kumar et al. (2021) have developed drone surveillance models to detect waste. They use technology such as Geo-location and mapping, image stitching and deep learning to analyse the data collected. This data produces notifications and alert systems for waste collection and management. Kumar et al. (2021) developed a proposed advanced deep learning solution for solid waste detection using UAVs/drone technology. The main focus of the research was to provide a platform for waste in specific areas to be shared and viewed by anyone in the world. This would encourage transparency in municipals and NGOs that deal with waste management. The system's structure consisted of a drone, Rasberry Pi, Tensor flow and OpenCV. While the system achieved its main focus as a proof of concept, no results were recorded. The researchers highlighted that a shortcoming of the research was its inability to collect the waste once it was identified and the lack of application testing conducted on the research. While the system could capture and share data, it could not valorise the waste.

Samsudeen et al. (2023) developed a drone surveillance model using deep learning for plastic waste monitoring. The proposed system was built with a modified YOLOv8 object detection algorithm and an Nvidia Nano kit. CSPDarkNet53 was used as the dataset for feature extraction for the waste detection model. The researchers showed that the YOLOv8 outperformed comparative systems with a precision parameter of 98.6%, a recall parameter of 96.13% and a detection success rate of 99.3%. Researchers identified challengers by identifying very tiny plastic wastes of various types. Further work was identified using more advanced YOLO versions to yield better results. i.e. training the algorithm with large datasets and applying further data augmentation.

Researchers such as Rashida et al. (2022), Zhou et al. (2021) and Johnson et al. (2022) have researched various uses of deep learning techniques for object detection using computer vision modules. These modules include R-CNN (Regional Convolutional Neural Network), Single Shot Detectors (SSD) models, and You Only Look Once (YOLO) models. Each module can be developed differently based on the base network or architecture and the database set used to train and test the algorithm. Some modules are developed from a fundamental level, while others are developed through transfer learning.

Rashida et al. (2022) developed an automatic waste segregation system for ground operation. It was developed with a robotic arm, Raspberry Pi, ultrasonic sensors, infrared sensors, SSD-Mobile Netv2 module and a Garbage Classification dataset. The SSD-Mobile Netv2 module enabled a real-time object detection system through the Mobile Net feature extractor. It extracts high-level features from a streamed image and generates a feature map to detect the type of waste. It concurrently determines a bounding box around the waste to determine its location in the image. It achieved an average accuracy detection of 81%, with paper – 68%, plastic – 84%, glass – 85%, and metal 87%. The researcher highlighted possible improvements by implementing a motion algorithm using fuzzy logic.

An automated system for battery detection for close-loop battery recovery and resource efficiency through identifying and sorting E-waste was developed by Johnson et al. (2022). The solution, known as RoboCRM, integrated computer vision, robotics, artificial intelligence, and machine learning technology, is an ideal solution for hazardous waste handling and sorting. The system was built with an optical scanning system, robotic sorting system, Fast R-CNN classification and identification model, feature detection and identification system with a DenseNet121 (find tuned) dataset for training and testing. This development was purely theoretical and was based on research currently being conducted in the field. The researcher identified possible improvements to the system by utilising thermal imagery. This could improve the feature vectors extracted from the data and the categorisation accuracy.

Zhou et al. (2021) developed a novel segregation algorithm based on Improved YOLOv4. The researchers integrated the Ghost module to improve the network structure in the YOLOv4 architecture. A CSPDarkNet53 was used as the backbone of the architecture to reduce the number of network connections and enhance the network feature learning ability. The neck of the architecture featured a spatial pyramid pooling module and path aggregation network. The head accounted for the regression and the classification operations. The Ghost module was implemented in the backbone. It reduced the feature redundancies, therefore resulting in fewer loss errors, confidence errors and classification errors. The researchers used DToU NMS and the centre point distance between two frames, which improved the detection accuracy of overlapped and occluded targets. The accuracy of the YOLOV4 was 94.83% and 91.25%. This represented an increase of 2.29 % and 1.49 %. Therefore, the improved YOLOv4 architecture was feasible to increase the accuracy of the trash segregation algorithm.

Methodology

There has been significant production of research papers, i.e. conference or journal articles, that have been produced based on robotic and drone technology for intelligent waste management systems. This study investigates the combination of these topics and predefines the objectives to guide the researchers through the study. The study required the researcher to review the state of the research on the application of robotic and drone technology in waste management – specifically using deep learning and image recognition techniques to identify and categorise waste. The objectives of the study are to:

- Understand the state of research in robotic and drone technology for waste management using artificial intelligence – specifically on identifying and categorising waste using deep learning algorithms and image recognition.
- Perform a meta-analysis of the research to determine its efficiency.
- Highlight critical limitations of the study.
- Conclude on the impact and future research direction of the study.

Eligibility Criteria

This study was defined as a systematic review. Therefore, it is a review of a specific research question or topic of interest that aims to identify, evaluate, and summarise all available evidence in a structured approach (Moher et al. 2009). The study required a robust review of existing research that had been performed and produced research outputs. Research outputs were defined as conference papers or journal articles associated with accredited organisations/institutions. This was defined as the type of study.

The study intended to understand the latest scope and application of robotic and drone technology in smart waste management systems. This included research such as the use of advanced UAVs in the detection of municipal solid waste using deep learning and remote sensing, and robotic waste sorting technology powered by vision-based categorisation systems (Koskinopoulou et al. 2021, Youme et al. 2021).

Based on the research, the study will be able to understand the efficiency of different strategies, the performance of the systems and the future research direction of the study. Therefore, the following criteria were chosen based on:

- Accredited academic research papers published within the last five years (since 2019 and up to June 2024) from organisations or institutions such as IEEE. This was to ensure that relevant research and information were collected.
- All English studies were considered due to the constraints of the researcher.
- Studies that produced comprehensive results from a physical system. This allowed the researcher to analyse the performance data and conclude on the efficiency of the study.
- The study needed to identify, categorise or sort waste with drone or robotic technology. It was a core technology that had shown significant advancements in this research field and was fundamental to future research.
- Studies using Artificial intelligence, specifically image processing and deep learning techniques. This core technology has shown significant advancements in this research field and is fundamental to future research.

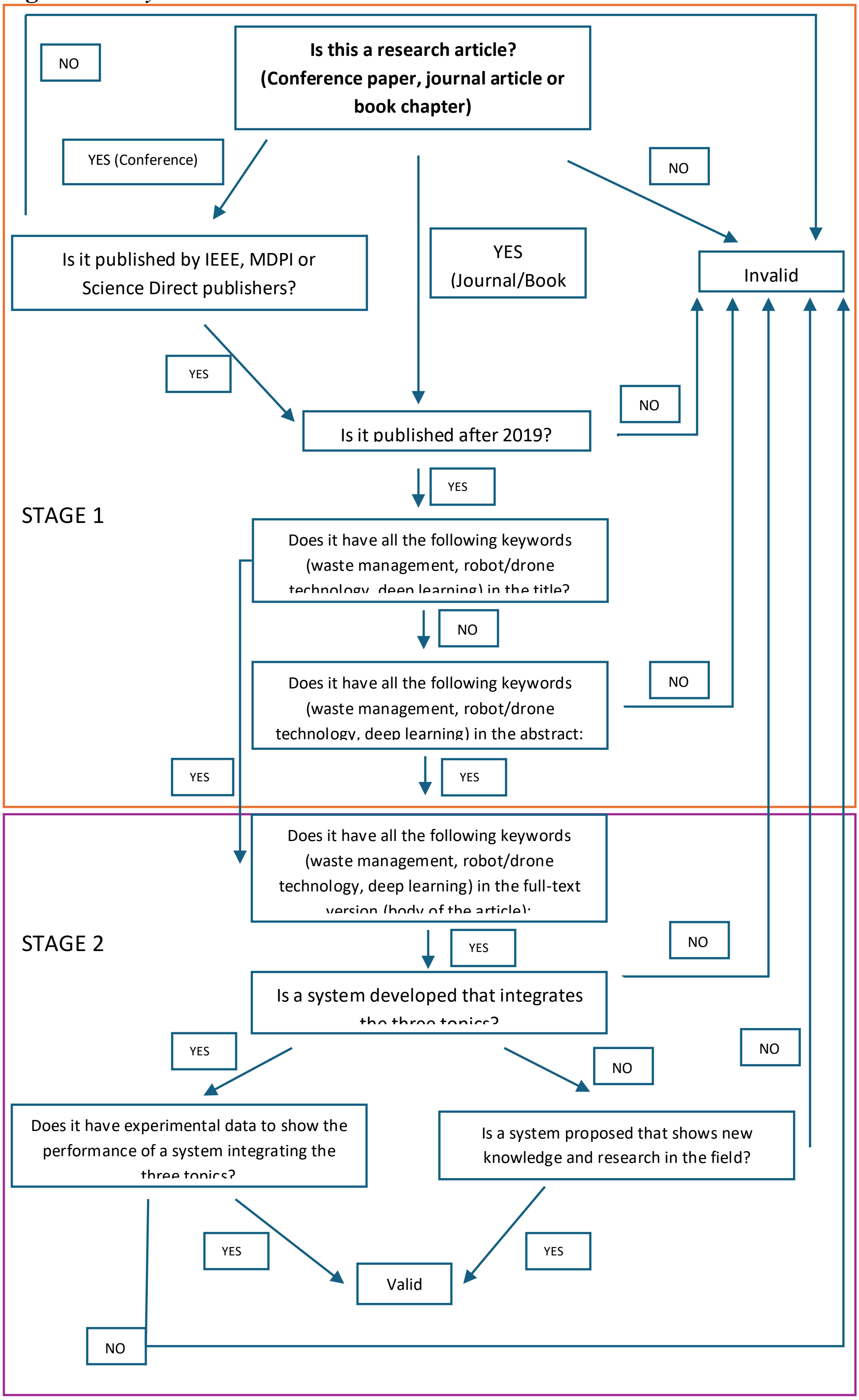
Information Source and Search Strategy

The database chosen for this research was IEEE Xplore. This database was selected as it covers engineering, computer science and technology research topics. It had an added advantage that these research articles were primarily published in accredited journals or conference proceedings that were internationally reviewed. This ensured that the research is peer-reviewed by experts in the engineering field.

To search for research articles, the database required terms, keywords and publication year. To ensure that all the databases were used effectively, the following information was used:

- Terms/Keywords: robotic and drone technology, waste management, deep learning.
- Publication year: from 2019 to 2024 (June).
- Type of publication: research articles (conference papers or journal articles) and book chapters (books). Conference papers will only be considered provided that they are published in conference proceedings by IEEE publishers. This will ensure the research is reviewed and accepted at an academic standard.

Figure 3. *Study Selection Process Flow Chart*



Study Selection Process

Once the search results were completed, the researcher began with a pilot test to ensure that the inclusion and exclusion criteria were applied correctly. The researcher choose a small sample set from each database and confirmed the eligibility. This was done by screening the research papers' titles and abstracts. Once completed, the researcher began stage one of the screening process. The researcher screened all the academic papers from IEEE and confirmed the eligibility using the same method. Once the eligibility was confirmed, the remaining papers were disregarded. Stage two of the screening process involved retrieving and evaluating full-text versions of the academic papers. The flow diagram in Figure 3 outlined both phases described.

After the screening process, all eligible academic documents were collated and were ready for data collection.

Data Extraction Process

The researcher developed a data extraction form. The data extraction form was created to ensure consistent data was extracted from the study during analysis. It's a critical step in the research, allowing the researcher to synthesise evidence and draw conclusions. The data extraction form required the following information:

1. General Information (study ID and year).
2. Study Character (study focus).
3. Methodology (technology used).
4. Results (key findings and statistical analysis).
5. Reviewer Information (Extracted by date and completion details). – It was ignored as one reviewer conducted the research over a period of 6 months.

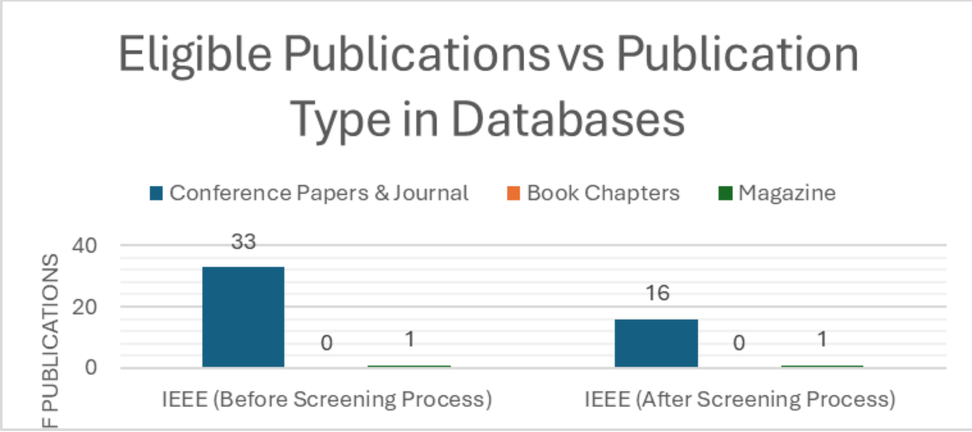
Data Synthesis Method

Since the focus was on understanding the state of drone and robotic technology research in waste management, a qualitative synthesis was chosen. This was sufficient as it provided a method for integrating and interpreting findings from various studies to draw a comprehensive conclusion on the state of research.

Results

The systematic review and data analysis results began with the study selection process. The results of this process are highlighted in Figure 4. The graph in Figure 4 shows the number of eligible publications versus the publication type. This was based on the eligibility criteria outlined in the study selection process.

Figure 4. *Number of Eligible Publications Versus the Publication Type per Database*



Once the study selection process was complete, the data extraction process began. The process was outlined in the data extraction section of the Methodology. This information was summarised and can be seen in Table 1. This highlights a few samples of research papers that underwent data extraction. This ensured that there was a fair process to ensure that sufficient data was collected for the research analysis. The full table can be seen in Appendix.

Table 1. *Data Extraction Form Summary (A Sample)*

Year	Study ID	Study Focus	Technology Used	Results
2021	(Rashida et al. 2022)	Implementation of Faster Region-Based Convolutional Neural Network for Waste Type Classification	Faster R-CNN (with ROI Pooling Layer); Data Augmentation; Pascal VOC (Format); ALEXNET	Precision - 95.87%; Recall - 90.1%; Average Detection Time - 0.119s, Overall Accuracy - 91.2
2023	(Joseph et al. 2023)	Use of deep learning algorithm and robotic technology, known as EIRT, to detect, classify and sort waste.	EfficientNET-D2 architecture, e-waste hunting module, robotic arm module, waste classification module, sorting module, pressure testing module, TrashBox dataset, Waste Pictures, eWaste.	Accuracy classification - 82.32%, recall - 72.92% to 97.12%, precision of model – 76.21% to 91.14%. F1 score - 78.24% to 90.97%.
2023	(Jalali et al. 2023)	Practical Implementation of Real-Time Waste Detection and Recycling based on Deep Learning for Delta Parallel Robot	3-DOF Delta Robot, MultiStage Detection (ResNet50), One-Stage Detection (YOLOv5), TrashNET.	ResNet50 Accuracy - 99.31%. Capable of classifying over five images per second. YOLOv5x achieved mAP (@IoU = 0.75) of 97.4% at 33 FPS.
2021	(Assis et al. 2021)	Garbage Collecting Robot Using YOLOv3 Deep Learning Model	One-Stage Detection (YOLOv3), MultiStage Detection (CNN), Raspberry Pi 3B+, Arduino Uno R3, Ultrasonic sensor.	YOLOv3 Accuracy, Precision, Recall & F1 Score - 0.9341, 0.9722, 0.84, 0.9012. CNN Accuracy, Precision, Recall & F1 Score - 0.849, 0.85, 0.8491, 0.8512.
2023	(Thanh et al. 2023)	Designing of A Plastic Garbage Robot With Vision-Based Deep Learning Applications	3 DOF Robot Arm (SCARA), Pneumatic end-defectors, cameras, One-Stage Detection (YOLOv5), Polygon Method using RoboFlow (Labelling Method), Bounding Box using RoboFlow (Labelling Method), Data Augmentation	YOLOv5 (Polygon Method): Precision – 92.1%, recall - 87.3%, Success Rate – 91.5%

2023	(Samsudeen et al. 2023)	Sky Sweeper: A Drone Surveillance Model Using YOLOV8 and Jetson Nano for Plastic Waste Monitoring System	Nvidia Jetson Nano Kit, CSPDarkNet53, YOLOv8 (Object Detection Algorithm)	Precision (98.6%); Recall (96.13%); Success Rate (99.3%)

Based on the data extracted from IEEE, the research has shown significant advancements in the application of technology in waste management through the use of deep learning algorithms and robotic systems. One of the notable systems by (Badoi & Lie 2022), utilising Faster R-CNN with ROI Pooling, Data Augmentation, Pascal VOC Format, and ALEXNET, achieving a precision of 95.87%, recall of 90.1%, and overall accuracy of 91.2%. The efficient average detection time of 0.119 seconds highlights the potential for real-time applications.

The EfficientNET-D2 architecture used by (Joseph et al. 2023) integrates e-waste hunting and robotic modules, displaying an accuracy classification of 82.32%, with precision ranging from 76.21% to 91.14% and F1 scores between 78.24% and 90.97%. This showcased the robustness of integrated systems of robotic technology and deepening learning through single detection for e-waste management.

The research conducted by (Jalali et al. 2023) on real-time waste detection using a 3-DOF Delta Robot with ResNet50 or YOLOv5 achieved a remarkable accuracy of 99.31% or mAP of 97.4% at 33 FPS. This study provided a comparison between multi-stage detection networks versus single-stage detection networks. While the multi-stage detection network (ResNet50) achieved greater accuracy, the single-stage detection architecture achieved the results in real-time environments. This highlighted the improved efficacy of waste management systems using deep learning models as it enhanced detection speeds and accuracy.

In terms of drone technology and deep learning, (Samsudeen et al. 2023) used YOLOv8 and Jetson Nano for drone surveillance exhibited precision of 98.6% and a success rate of 99.3%, highlighting the effectiveness of drones in environmental monitoring.

These studies collectively demonstrate that the integration of advanced deep learning models with robotic systems significantly enhances the efficiency and accuracy of waste management processes. Technologies like YOLO, Faster R-CNN, and EfficientNET, combined with real-time data processing capabilities, present promising solutions for automating waste segregation and recycling, potentially leading to substantial environmental benefits.

Discussion

The primary objective of this research was to evaluate the effectiveness of applying robotic and drone technology in waste management, with a specific focus on image recognition and deep learning for waste identification, categorisation, and sorting. This study aligned with the overarching goal of improving waste management practices and advancing sustainable solutions in response to the increasing global waste production.

In terms of assessing the precision of robotic and drone systems in identifying various waste types, research demonstrated significant advancements in the accuracy of waste identification using image recognition and deep learning algorithms. For instance, the study by (Rashida et al. 2022) using Faster R-CNN achieved a precision of 95.87% and an overall accuracy of 91.2%. This high accuracy indicates that these technologies can effectively distinguish between different waste materials, which is crucial for efficient waste sorting and recycling processes. This highlights the effectiveness of Faster R-CNN networks with data augmentation and AlexNet. (Samsudeen et al. 2023) showcased the use of YOLOv8 and Jetson Nano for drone surveillance, exhibited precision of 98.6% and a success rate of 99.3%. While this research highlighted significant results, little application work was conducted when verifying the results.

The efficiency of robotic and drone technologies in categorising waste was substantially more when the integration of advanced neural networks and robotic/ drone were developed. Studies such as Joseph et al. (2023), Jalali et al. (2023) and Thanh et al. (2023) using robotic/drone systems coupled with deep learning technology to achieve a significant success rate (which can be seen in the results above). These success rates highlight the potential of deep learning technologies to enhance the waste sorting process, reducing manual labour and increasing overall efficiency.

In order to determine the overall impact of robotic and drone technologies on waste management practices, the application of these technologies have demonstrated potential in reducing human exposure to hazardous waste, improving the speed of waste processing, and enhancing the precision of waste data collection. This is present in all the research that was investigated and analysed. These findings indicate that such technologies can significantly enhance the effectiveness and safety of waste management practices.

Conclusion

The findings of this systematic review and meta-analysis highlighted the transformative potential of robotic and drone technologies in waste management. The results aligned with the research objectives, demonstrating advanced image recognition and deep learning algorithms that can enhance the accuracy, efficiency, and overall impact of waste management processes.

Effective waste management increasingly relies on innovative technologies that can handle the growing volume and complexity of waste. The studies reviewed provide compelling evidence that integrating robotic and drone technologies with deep learning algorithms offers a viable solution for modern waste management challenges. These technologies enhance the precision of waste identification and categorisation while contributing to safer and more efficient waste management practices.

Future research should address the identified gaps, such as improving the scalability of these technologies and integrating them seamlessly into existing waste management frameworks. Further advancements in deep learning models and robotic systems will be crucial for developing more robust and adaptable waste management solutions. Additionally, exploring the application of these technologies in diverse environmental contexts can provide deeper insights into their potential global impact.

Overall, this research highlights the success of robotic/drone systems coupled with deep learning for sustainable waste management. Which is essential for addressing the environmental challenges posed by increasing waste production.

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Appendix

Year	Study ID	Study Focus	Technology Used	Results
2021	(Badoi & Lie 2022)	Automatic Waste Segregation System	Faster R-CNN (with ROI Pooling Layer); Data Augmentation; Pascal VOC (Format); ALEXNET	Precision - 95.87%; Recall - 90.1%; Average Detection Time - 0.119s, Overall Accuracy - 91.2
2023	(Joseph et al. 2023)	Use of deep learning algorithm and robotic technology, known as EIRT, to detect, classify and sort waste.	EfficientNET-D2 architecture, e-waste hunting module, robotic arm module, waste classification module, sorting module, pressure testing module, TrashBox dataset, Waste Pictures, eWaste	Accuracy classification - 82.32%, recall - 72.92% to 97.12%, precision of model – 76.21 % to 91.14 %. F1 score - 78.24% to 90.97%.
2023	(Jalali et al. 2023)	Practical Implementation of Real-Time Waste Detection and Recycling based on Deep Learning for Delta Parallel Robot	3-DOF Delta Robot, MultiStage Detection (ResNet50), One-Stage Detection (YOLOv5), TrashNET.	ResNet50 Accuracy - 99.31%. Capable of classifying over five images per second. YOLOv5x achieved mAP (@IoU = 0.75) of 97.4% at 33 FPS.
2021	(Rumahorbo et al. 2021)	Development of Robot to Clean Garbage in River Streams with Deep Learning	Autonomous River Robot, VGG16 Network	Accuracy – 72%
2021	(Kumar et al. 2021)	Dronezilla	Image Stitching, Drone, Firebase, Raspberry Pi, YOLO (One-Stage Detection), Tensor Flow , Datasets - Github & Kaggle, OPENCV & SIFT (Image processing & feature extraction method), Drone.	Detects waste. No parametric results.
2021	(Assis et al. 2021)	Garbage Collecting Robot Using YOLOv3 Deep Learning Model	One-Stage Detection (YOLOv3), MultiStage Detection (CNN), Raspberry Pi 3B+, Arduino Uno R3, Ultrasonic sensor.	YOLOv3 Accuracy, Precision, Recall & F1 Score - 0.9341, 0.9722, 0.84, 0.9012. CNN Accuracy, Precision, Recall & F1 Score - 0.849, 0.85, 0.8491, 0.8512.
2022	(Rashida et al. 2022)	Implementation of Faster Region-Based Convolution Neural Network for Waste Type Classification	Robotic arm, raspberry pi, ultrasonic sensors, infrared sensors, SSD-Mobile Netv2 module and Garbage Classification dataset	Average accuracy detection of 81% with paper – 68%, plastic – 84%, glass – 85%, and metal 87%
2021	(Koskinopoulou et al. 2021)	Robotic Waste Sorting Technology	ABB IRB360 Delta Robot, ZED camera, Vacuum gripper module, Mask-regional CNN, Region proposal network, TrashNet & Taco dataset, Data Augmentation	RCNN (with Data Augmentation) Precision – 87.4%, recall – 90.1% & F1 scores – 88.7%.
2023	(Thanh et al. 2023)	Designing of A Plastic Garbage Robot With Vision-Based Deep Learning Applications	3 DOF Robot Arm (SCARA), Pneumatic end-defectors, cameras, One-Stage Detection (YOLOv5), Polygon Method using RoboFlow(Labelling Method), Bounding Box using RoboFlow (Labelling Method), Data Augmentation	YOLOv5 (Polygon Method): Precision – 92.1%, recall - 87.3%, Success Rate – 91.5%
2023	(Samsudeen et al. 2023)	Sky Sweeper: A Drone Surveillance Model Using YOLOV8 and Jetson Nano for Plastic Waste Monitoring System	Nvidia Jetson Nano Kit, CSPDarkNet53, YOLOv8 (Object Detection Algorithm)	Precision (98.6%); Recall (96.13%); Success Rate (99.3%)
2022	(Johnson et al. 2022)	Application of AI and Machine Vision to improve battery detection and recovery in E-Waste Management	RoboCRM, DenseNet121, Fast R-CNN	No Results

2022	(Tornero et al. 2022)	Detection and Location of Domestic Waste for Planning Its Collection Using an Autonomous Robot	Autonomous Robot (Blue), NVIDIA DGX-A100 GPU, an RGBD Camera (Intel Realsense d435i with LIDAR) and various YOLO ((YOLOv3, YOLOv3-SPP, YOLOv5s, YOLOv51, YOLOv51 Augmented))	Precision – 0.991, recall – 0.992 and Rate of Detection – 36 ms
2022	(Bobulski & Kubanek 2021)	Vehicle For Plastic Garbage Gathering	Custom-built Plastic Sorting Robot, Rasberry Pi3, CNN (with Relu)	Accuracy – 87 % with Learning Rate (0.01), Max epoch (30), Regularization (0.1)
2019	(Hossain et al. 2019)	Autonomous Trash Collector Based on Object Detection Using Deep Neural Network	Customised Robot Vehicle, Raspberry Pi 3 & camera module, CNN (with Relu, Maxpool & Softmax functions), Arduino Mega, Ultrasonic sensor (HC-SR04)	Accuracy (training) – 0.98, accuracy (testing) – 0.96, Loss (training) – 0.389, Loss (testing) – 0.392.
2020	(Liu et al. 2021)	Garbage Collection and Sorting with a Mobile Manipulator using Deep Learning and Whole-Body Control	GarbageNet (Recognition & Localisation), Taco dataset (with augmentation), ROS, Grasp Pose Detection (GPD) system, Robotnik SUMMIT-XL Steel mobile platform, Franka Emika Panda robot, Whole Body Impedance Controller, RGB-D RealSense camera	GarbageNet: Mean average precision – 40.43 (with IoU of 0.75) Capture rate – 30 fps GPD system: Success rate – 90%
2022	(Akram et al. 2022)	Automatic Robotic Action Towards Plastic Collection	MultiStage Detection (Mask R-CNN), ResNet101, COCO (pre-trained), Region Proposal Network (RPN), ROI pooling, Arduino Uno R3, VGG Annotator, Data Augmentation,	Prediction Confidence (0.98 – 0.95), speed of detection (Moderate to high)
2021	(Zhou et al. 2021)	Towards the Urban Future: A Novel Trash Segregation Algorithm Based on Improved YOLOV4	Single Stage Detection (YOLOv4), CSPDarkNet53, Ghost module, DToU NMS, Spatial pyramid pooling module, Path aggregation network.	Accuracy (YOLOV4) - 94.83% and 91.25%.

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Tropical Cryptography – The State of the Art and Future Prospects

*By Mariana Durcheva**

The first paper introducing the use of tropical semirings in public key cryptography was published in 2013. Since then, numerous tropical schemes have been proposed, incorporating various idempotent semirings. The most commonly used idempotent semirings for cryptographic purposes are the min-plus and max-plus semirings, with some schemes also utilizing the max-time and min-time semirings. The main mathematical problems that ensure the security of these schemes include the tropical discrete logarithm problem, the tropical semigroup action problem, the tropical semidirect product problem; the problem of solving two-sided linear systems in tropical semirings; tropical polynomial factorization and finding greatest common divisor problems, and the tropical matrix power function problem. Unfortunately, most of the proposed tropical schemes have been successfully attacked, raising concerns about the future of tropical cryptography. In this work, we review existing tropical schemes, discuss the complexity of the underlying problems, examine current attacks on these schemes, and explore future prospects and directions for cryptography based on tropical semirings.

Keywords: *semirings, tropical semirings, tropical cryptography, tropical problems, crypto attacks*

Introduction

Semirings were introduced by Vandiver (1934), but they remained unnoticed by mathematicians for a long time. According to Vandiver, a semiring is a nonempty set R, equipped with two operations: addition (+) and multiplication ($\cdot$), satisfying the following conditions: $(R, +)$ forms a commutative semigroup, $(R, \cdot)$ forms a semigroup, and multiplication distributes over addition.

Semirings find applications in diverse fields, including automata theory, formal languages, finite state machines, recognizable languages, speech recognition, and image compression. Formal systems used for studying the correctness and effectiveness of computer programs, such as dynamic algebras, Hoare algebras, and Kleene algebras, are closely related to various semirings. Idempotent semirings bridge algebraic formalism and dynamic and temporal logic, enabling the modelling of properties of computer programs and transition systems. Additionally, semirings play a crucial role in large-scale scientific applications, including high-dimensional data and graph analytics for linear algebra computations.

Tropical algebra was first proposed by Cuninghame-Green (1979), while the term *idempotent analysis* was proposed by Maslov (1986). The name *tropical* was coined in honor of Brazilian mathematician Imre Simon, whose work (Simon

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1988) laid important foundations for tropical algebra. In tropical algebra, tropical addition means taking the minimum or maximum of two numbers, while tropical multiplication corresponds to usual addition. The concepts of tropical algebra were further developed through the framework of semirings, leading to the formal definition of the algebraic structure of tropical semirings. This theory introduced a novel approach by replacing the field of real numbers, $\mathbb{R}$, with the semifield $\mathbb{R}_{max}$. One of its key advantages is the ability to transform many problems that are nonlinear over real numbers into linear problems via this new arithmetic. In essence, these problems become linear over the appropriately chosen semiring.

Since 1995, the research field of tropical algebra has experienced remarkable growth, driven by numerous findings and diverse applications in areas such as control theory and optimization, phylogenetics, the modelling of cellular protein production, and railway scheduling. Tropical algebra has become a significant subject of study in algebraic theory. A major research focus in tropical algebra is the investigation of the semigroup of tropical matrices, which includes exploring the structure of multiplicative semigroups, Green's relations, regularity, and abundance. In recent years, the study of the multiplicative semigroup of tropical matrices has gained prominence as a hot research topic. Scholars are increasingly leveraging tropical algebra for cutting-edge research in areas such as noise removal and optimal control. Moreover, researchers are exploring additional methods of tropical algebra to address practical problems, including the variational iteration method and the homotopy perturbation method.

Max-algebra has found significant applications in various fields, including nonlinear partial differential equations and various optimization theories. The min-algebra, which is isomorphic to max-algebra, is utilized in areas such as statistics, computational biology, and net calculus for fading channels. Brunsch et al. (2012) extended the use of idempotent semirings to interval analysis, proposing conditions for the existence of a non-linear projector in the solution set of certain inequalities. Trendafilov & Tzvetkov (2021) introduced the concept of derivations in a product of additively idempotent semirings, highlighting their utility in diverse areas such as switching circuits, automata theory, and decision theory. Krivulin in several papers (Krivulin 2020, Krivulin & Gubanov 2021) proposed different applications of idempotent semirings, including solving location problems with Chebyshev and rectilinear distances in multidimensional space and in the two-dimensional plane. These semirings were also used to address optimal project scheduling problems in project management, with the goal of minimizing deviations in starting times of tasks. Idempotent semirings have additionally been applied in genetic algorithms for solving resource-constrained project scheduling problems (Bulavchuk & Semenova 2023) and for modelling coordination schemes and weighted connectors in component-based systems architectures (Fountoukidou & Pittou 2022).

On the one hand, information security is crucial in today's interconnected world, where data is a key asset. It protects organizations, individuals, and nations from a wide array of risks, ranging from financial loss and reputational damage to personal privacy violations and national security threats. As cyber threats continue to evolve, the importance of maintaining strong and adaptive information security cannot be overstated, including establishing information security policy (Nagata 2023).

On the other hand, cryptography is vital to information security, and modern cryptographic methods are used to secure communication, protect stored data, and ensure secure authentication processes.

Before 1976, secret key cryptography was the standard approach to secure communication over open channels. However, Diffie and Hellman (1976) introduced a groundbreaking concept: public key cryptography, which revolutionized the field and opened new direction for research, including methods for interaction between logic gates, blockchain and key generation (Medelin 2022). Numerous applications rely on public-key cryptosystems and protocols, including SSH (Ylonen & Lonvick 2006), OpenPGP (Callas et al. 1998) and SSL/TLS (Dierks and Allen 1999).

Maze et al. (2007) proposed one of the first cryptosystems based on semirings. However, this cryptosystem was successfully attacked by Steinwandt et al. (2011).

Grigoriev and Shpilrain (2013) were the first to demonstrate the use of tropical semirings in public key cryptography. A discussion on the potential of semirings in cryptography can be found in Durcheva (2020). Tropical and supertropical semiring-based protocols have emerged as a powerful solution for modern IoT ecosystems, offering remarkable robustness and efficiency (Durcheva & Danilchenko 2024, Ponmaheshkumar & Perumal 2024, Jackson & Perumal 2024). In these environments, devices often operate with limited resources while requiring both rapid and secure communication. Additionally, such schemes integrate seamlessly into blockchain systems, enhancing the security of trade operations and digital contracts while mitigating common vulnerabilities in distributed networks.

In this paper, we provide a review of tropical schemes, discuss the complexity of the underlying problems, examine existing attacks on these schemes, and explore the future prospects of cryptography based on tropical semirings.

The paper is organized as follows: Section 2 presents relevant preliminaries, Section 3 introduces the state of the art, Section 4 discusses the future of tropical cryptography and Section 5 offers concluding remarks.

Preliminaries

A *semiring*, as defined in Vandiver (1934), consists of a nonempty set R, equipped with two binary operations: addition (+) and multiplication ($\cdot$). These operations satisfy the following requirements:

- $(R, +)$ is a commutative semigroup, i.e. $a + b = b + a$ and $a + (b + c) = (a + b) + c$ for all $a, b, c \in R$;
- $(R, \cdot)$ is a semigroup, i.e. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in R$;
- the distributive laws hold: $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$ for all $a, b, c \in R$.

It is noteworthy to mention that the definition provided by Golan (1999) assumes the presence of a zero and a one in the semiring.

Let $R = (R, +, \cdot)$ be a semiring. If there exists a neutral element 0 in the semigroup $(R, +)$ that satisfies the condition $0 + a = a + 0$ for all $a \in R$, this element is called a *zero*. If there exists a neutral element 1 of the semigroup $(R, \cdot)$, meaning $a \cdot 1 = 1 \cdot a = a$ for all $a \in R$, this element is called a *one*. A semiring R is called *commutative* if the semigroup $(R, \cdot)$ is commutative.

Furthermore, when the commutative semigroup $(R, +)$ is an abelian group, the semiring is referred to as a *ring*; when the multiplicative semigroup $(R, \cdot)$ is a group, then a semigroup is called a *semifield*.

An element a of the semiring $(R, +, \cdot)$ is called *additively idempotent* if $a + a = a$. If every element of the semiring is additively idempotent, then R is referred to as an *additively idempotent semiring*. A semiring is termed *idempotent* if it is an additively idempotent.

We will highlight several important idempotent semirings:

$\mathbb{R}_{max,min} = (\mathbb{R} \cup \{-\infty, +\infty\}, \max, \min)$ is an idempotent semiring, which is not a semifield. In this semiring, $\mathbf{0} = -\infty, \mathbf{1} = +\infty$. The inverse element with respect to the operation *min* does not exist, and the maximal element is $+\infty$.

The following semirings are often utilized as building blocks for different protocols:

$$\mathbb{R}_{max,+} = (\mathbb{R} \cup \{-\infty\}, \max, +); \ \mathbb{R}_{min,+} = (\mathbb{R} \cup \{+\infty\}, \min, +);$$
$$\mathbb{R}_{max,\times} = (\mathbb{R}_+ \cup \{0\}, \max, \times); \ \mathbb{R}_{min,\times} = (\mathbb{R}_+ \cup \{+\infty\}, \min, \times),$$

when $\mathbb{R}$ is the field of real numbers and $\mathbb{R}_+ = \{x \in \mathbb{R} | x > 0\}$. The semirings $\mathbb{R}_{max,+}$, $\mathbb{R}_{min,+}$, $\mathbb{R}_{max,\times}$, $\mathbb{R}_{min,\times}$ will be referred to as *tropical semirings*.

Over idempotent (tropical) semirings can be considered matrices and polynomials in a usual way. Let $(M, \oplus, \otimes)$ be an idempotent semifield. For some random matrices:

$$A = (a_{ij}) \in M^{n \times n}, \qquad B = (b_{ij}) \in M^{n \times n},$$

the addition and multiplication of matrices, as well as the multiplication of a matrix by a scalar $\alpha \in M$ are defined as follows:

$$(A \oplus B)_{ij} = a_{ij} \oplus b_{ij}, \qquad (A \otimes B)_{ij} = \oplus_{k=1}^{n} a_{ik} \otimes b_{kj},$$

$$(\alpha \otimes A)_{ij} = \alpha \otimes a_{ij}.$$

The set $M^{n\times n}$ of $n \times n$ matrices with entries from M equipped with matrix addition, matrix multiplication, and scalar multiplication is an idempotent algebra. In this algebra, there is a null matrix O (where all of entries are 0) and a unit matrix I (where diagonal entries are 1 and all other entries are 0).

Tropical polynomials are defined in the conventional manner as follows:

$$P(x) = \bigoplus_{i=0}^{n} a_i \otimes x^{\otimes i},$$

where $x^{\otimes i} = \underbrace{x \otimes x \otimes \ldots \otimes x}_{i \text{ times}}$ and $a_i \in M$ for $i = 0, 1, \cdots, n$.

It follows from the properties of operations in tropical semirings that:

$$P(x) \otimes Q(x) = Q(x) \otimes P(x).$$

For further information on semirings, including matrices and polynomials over semirings, we recommend the monograph by Golan (1999).

The State of the Art

Key Problems Ensuring the Security of Tropical Protocols and Corresponding Attacks

In this section, we discuss the main hard problems that underpin the security of tropical protocols, the schemes that utilize these problems, and the attacks against these schemes. We note that all coefficients of tropical polynomials, as well as the entries of tropical matrices are from some tropical semiring T, which can be specified if needed for a protocol.

- **The Tropical Discrete Logarithm Problem (TDLP)**

We begin by recalling the classical Discrete Logarithm Problem (DLP). Given a cyclic group G with a generator g and an element h in G, the DLP involves finding an integer x such that: $g^x = h$.

Let A, B, C be suitable tropical matrices, and m be a positive integer. The TDLP is defined as follows: Given the matrices A, B, C, find m from the equation:

$$C = A \otimes B^{\otimes m} \text{ (or } C = B^{\otimes m} \otimes A). \quad (1)$$

Protocols, such as the one proposed by Muanalifah et al. (2023), rely on this problem for their security. The discrete logarithm problem in a tropical semiring can be completely solved in certain cases (specifically, on a group and a module). In other cases, a 100% success rate can be achieved through alternative methods (Muanalifah & Sergeev 2022).

Most often, this problem needs to be solved in its two-sided form. Given matrices *A, B, C* and *D*, the goal is to find the positive integers m and n from the equation:

$$C = A^{\otimes m} \otimes D \otimes B^{\otimes n}. \tag{2}$$

- **The Tropical Semigroup Action Problem (TSAP)**

We first recall the *Semigroup Action Problem* (SAP) as defined in (Maze et al. 2007).

An *action* of a finite set S on a semigroup $(G, \cdot)$ is a map $\varphi: G \times S \to S$ such that:

$$\varphi(g \cdot h, s) = \varphi\big(g, \varphi(h, s)\big). \tag{3}$$

The *Semigroup Action Problem* is then formulated as follows: Given a semigroup G acting on a finite set S through the (left) action φ, and elements $s \in S, t \in \varphi(g, s)$, find an element $g \in G$ such that $\varphi(g, s) = t$.

Tropical Semigroup Action (τ). A semigroup G is said to act on a tropical semiring T:

$\tau: G \times T \to T$

if the following conditions hold:

(1) For any element a of the tropical semiring T and an element g of the semigroup G, there exists an element a^g that belongs to the tropical semiring T.

(2) For any elements a, b of the tropical semiring T and element g, h of the semigroup G, the following properties are satisfied:

$$(a \otimes b)^g = a^g \otimes b^g, \qquad a^{gh} = (a^g)^h .$$

Tropical Semigroup Action Problem. Given a semigroup G acting on a tropical semiring T through the action (τ) and elements $a \in T, b \in \tau(g, a)$, the problem is to find an element g of the semigroup G such that $\tau(g, a) = b$.

It is claimed in Goel et al. (2020) that the Semigroup Action Problem can be considered a generalization of the Discrete Logarithm Problem. Huang et al. (2024b) argue that most cryptosystems based on the SAP, DLP, and matrix-action problem on a group and on a module cannot resist quantum cryptanalysis. However, cryptographic systems based on the SAP on a semi-module are likely more resilient against quantum attacks.

Additionally, a protocol based on the bi-semigroup action problem (BSAP) is proposed by Trendafilov & Durcheva (2010).

- **The Tropical Semidirect Product Problem (TSPP)**

We first recall the *Semidirect Product Problem* (SPP) in arbitrary semigroups (Shpilrain 2018): Given a semigroup G and elements $(g, h, h^{-m}(hg)^m, h^{-n}(hg)^n)$ in G, the goal is to find $h^{-(m+n)}(hg)^{m+n}$, where $m, n \in \mathbb{N}$.

Tropical Semidirect Product. For a semigroup G and a tropical semiring $\mathbb{R}_{max}$ ($\mathbb{R}_{min}$), the semidirect product is defined by the following condition:

$$(a, g)(b, h) = (a^h \otimes b, g \otimes h) \tag{4}$$

for any elements a, b in the tropical semiring and elements g, h in the semigroup G.

We formulate the *Tropical Semidirect Product Problem* (TSPP) in the matrix semiring, since it has been used in this setting (Grigoriev & Shpilrain 2019): Given the tropical matrix semiring $(M_n(\mathbb{Z}), \oplus, \otimes)$, two matrices, M and H, chosen from $M_n(\mathbb{Z})$, and knowing the first components A, B of the expressions $(M, H)^m = (A, H^m)$, and $(M, H)^n = (B, H^n)$, the problem is to determine $(M, H)^{m+n}$.

A scheme whose security relies on this problem is presented by Grigoriev & Shpilrain (2019). This scheme has been attacked in several works, including Jackson & Perumal (2023), Rudy & Monico (2021), Isaac & Kahrobaei (2021), and Muanalifah & Sergeev (2022).

- **The problem of solving a two-sided linear system in tropical semiring**

Let A, B, C, D, X, Y, K be suitable tropical matrices (matrices over tropical semiring). Solving a two-sided tropical linear system involves finding the unknown matrices X and Y from the given matrices A, B, C in the following system:

$$\begin{cases} A \otimes X = X \otimes A, \\ Y \otimes B = B \otimes Y, \\ X \otimes Y = C. \end{cases} \tag{5}$$

Solving this system is equivalent to solving the two-sided tropical equation**:** given the matrices K, C, D, find the unknown matrices X and Y from the equation:

$$X \otimes D \otimes Y = K, \tag{6}$$

with the condition that

$$X \otimes Y = C.$$

Several protocols, including Grigoriev & Shpilrain (2013), Huang et al. (2022), Amutha & Perumal (2023), Durcheva (2013), Durcheva & Trendafilov (2012), Durcheva (2014a), Durcheva (2014b), Durcheva & Rachev (2015), Durcheva & Ivanova (2018), Ahmed et al. (2022), rely on this problem or on similar problems for their security.

The problem represented by equation (6) turns out to be polynomially equivalent to the problem of determining the winner in *mean payoff games*

(Grigoriev 2013). This problem is known to belong to the intersection of NP and coNP complexity classes.

One of the most important attacks (Isaac & Kahrobaei 2021) on the protocols whose security is based on the difficulty of solving of system (5) (or eq. (6)) in tropical semirings ($\mathbb{R}_{min,+}$ and $\mathbb{R}_{max,+}$) exploits the property known as *almost linear periodicity* of tropical matrices.

A sequence of matrices $A^n, n \in \mathbb{N}$ over tropical semirings ($\mathbb{R}_{min,+}$ and $\mathbb{R}_{max,+}$) is called *almost linear periodic* if there exists a period *p*, a linear factor a, and a defect d such that for all $n > d$, the following holds:

$$A_{ij}^{n+p} = a + A_{ij}^{n}.$$

For the semirings $\mathbb{R}_{min,\times}$ and $\mathbb{R}_{max,\times}$, the sequence of matrices exhibits almost linear periodicity if:

$$A_{ij}^{n+p} = a \cdot A_{ij}^{n}\ .$$

Based on this property, the corresponding two-sided system, similar to system (5) in these semirings, was successfully attacked in Buchinskiy et al. (2023b). Further improvements to these types of attacks have also been proposed in the review paper (Ahmed et al. 2023).

Another attack on system (5), proposed by Kotov and Ushakov (2018), employes the idea of representing the unknown matrices X and Y in the following form:

$$\bigoplus_{i=0}^{u} x_i \otimes A^{\otimes i},\ \ \bigoplus_{j=0}^{u} y_j \otimes B^{\otimes i},$$

where the coefficients $x_i, y_j \in \mathbb{Z}$ are unknown and u is the upper bound for degrees of polynomials in the protocol.

The idea presented by Kotov and Ushakov is generalized in Alhussaini et al. (2023), where finding the smallest cover is suggested instead of enumerating all possible covers. Such kind of attacks have been applied to several protocols based on modified circulants, like those suggested in Ponmaheshkumar & Perumal (2023), Huang et al. (2022) and Amutha & Perumal (2023).

To avoid linear periodicity, in Ahmed et al. (2022) a modified protocol based on pairs is proposed. Several attacks to this protocol are proposed in Alhussaini et al. (2024), where the ultimate periodicity of those columns and rows of tropical matrix powers whose indices correspond to the critical nodes is exploited.

Another approach for avoiding linear periodicity involves the use of a commutative subset of duo circulant matrices, as stated in Amutha & Perumal (2024).

- **Factorization of tropical polynomials. Finding the GCD of two tropical polynomials.**

The problem of the *factorization of a tropical polynomial* is as follows:

Let $r(x), p(x), q(x)$ be one-variable tropical polynomials: $r(x), p(x), q(x)$. For a given polynomial $r(x)$, find $p(x)$ and $q(x)$ from the equation: $r(x) = p(x) \otimes q(x)$.

The problem of *finding a Great Common Divisor* (*GCD*) *of tropical polynomials* is as follows: Given the tropical polynomials Pol₁ = $(p_1(x) \otimes q_1(x))$ and Pol₂ = $\left(p_2(x) \otimes q_2(x)\right)$, find a common divisor of Pol₁ and Pol₂.

The problem of factoring polynomials over a tropical semiring, under several conditions, is NP-complete (Th 7, Kim & Roush 2005). Additionally, the GCD of polynomials over a tropical semiring is not in general unique.

A cryptographic scheme based on this problem was introduced by Chen et al. (2023). This protocol was subsequently attacked in Panny (2023) and Brown & Monico (2023). To overcome these attacks, Géraud-Stewart et al. (2023) suggested the use of the Fiat-Shamir Transform (FST) claiming that this approach could also be applied to other cryptosystems.

- **Tropical Matrix Power Function problem**

The concept of a *Matrix Power Function* (*MPF*) (Sakalauskas et al. 2017) is defined as follows: Given a matrix $Q = (q_{ij})_{n \times n}$, raising it to the power of a matrix $Y = (y_{ij})_{n \times n}$ from the right produces a resulting matrix $P = (p_{ij})_{n \times n}$ (denoted $P = Q^Y$), where $p_{ij} = \prod_{k=1}^{n} q_{ik}{}^{y_{kj}}$. Similarly, raising Q to the power of a matrix $X = (x_{ij})_{n \times n}$, from the left gives a matrix $S = (s_{ij})_{n \times n}$ (denoted $S = {}^{X}Q$), where $s_{ij} = \prod_{k=1}^{n} q_{kj}{}^{x_{ik}}$.

We first recall the *Matrix Power Function Problem* (*MPFP*) problem as defined by Sakalauskas et al. (2017): Let A, B, C, D, X, Y, Q be suitable matrices, and matrices A, B, C, D, Q be given. The MPF problem (MPF) involves finding unknown matrices *X* and *Y* from the system:

$$\begin{cases} {}^{X}Q^{Y} = E, \\ X^{-1}AX = C, \\ Y^{-1}BY = D. \end{cases} \tag{7}$$

Sakalauskas & Mihalkovich (2018) proved that the MPF problem, under certain constrains, is NP-Complete.

In tropical algebra, this problem is known as the *Tropical Matrix Power Function* (TMPF) problem and it can be defined as follows: Given a tropical matrix *K*, and tropical circulant matrices Q_1 and Q_2, find the unknown tropical matrices *X* and Y from the equation:

$${}^{X}Q_1 \otimes {}^{Y}Q_2 \otimes M = K \text{ (or } Q_1{}^{X} \otimes Q_2{}^{Y} \otimes M = K\text{)}. \tag{8}$$

The security of the first stage of the protocol presented by Durcheva (2022) relies on the difficulty of solving the Tropical Matrix Power Function Problem. Some attacks to this protocol are suggested in Jiang et al. (2023) and Buchinskiy et al. (2023a).

Additional analysis of protocols based on the *MPF* problem, including the *TMPF* problem is provided in Mihalkovich et al. (2024). The discussion presented in this Section is summarized in Table 1.

Table 1. *Schemes and the Attacks on them*

Scheme	Semiring	Hard problem	Attack	Method used in the attack
Grigoriev & Shpilrain (2013)	Min-plus	Solving two-sided linear system in a tropical semiring	Kotov& Ushakov (2018) Alhussaini et al. (2023)	1) Generating a set containing all the minimal covers. 2) Applying a heuristic algorithm. Improving the attacks of (KU,2018) by suggesting the identification of the smallest cover instead of enumerating all possible covers.
Durcheva (2013 Durcheva & Trendafilov (2012)	Max-plus	Solving two-sided linear system in a tropical semiring	Ahmed et al. (2023)	Utilizing the linear periodicity property of powers of tropical matrices.
Durcheva (2014a) Durcheva (2014b) Durcheva & Rachev (2015) Durcheva & Ivanova (2018)	Dual semirings: max-plus/ min-plus; max-time/ min-time	Solving a two-sided linear system comprising dual idempotent semirings	Ahmed et al. (2023) Buchinskiy et al. (2023a)	Utilizing the linear periodicity property of powers of tropical matrices in semirings (max+, min+). For max-time/min-time matrices, the relationship between corresponding elements of successive powers is not linear. Utilizing the behavior of sequences of powers of matrices over max-times and min-times algebras.
Grigoriev & Shpilrain (2019)	Min-plus with adjoint multipli-cation	The Tropical Semidirect Product problem	Rudy & Monico (2021) Isaac & Kahrobaei (2021) Muanalifah & Sergeev (2022) Jackson & Perumal (2023)	Utilizing the linearity in the order of the sequence of the employing matrices. Exploiting the “almost linear periodic property” exhibited by the sequence of matrices in use. Solving the TDLP in certain cases and reducing it to the optimal paths problem in others. Utilizing the fact that the difference between the secret key and the product of the shared matrices is independent of the users' choice of public parameters.

Huang et al. (2022) Amutha& Perumal (2023)	Min-plus	Solving the two-sided tropical circular matrix action problem	Buchinskiy et al. (2023b) Alhussaini et al. (2023)	Using some heuristic tricks to reduce checking a small portion of the covers. Using the isomorphism between max-times and max-plus semirings, while improving the attacks of (KU, 2018).
Ponmahesh-kumar & Perumal (2023)	Max-time	Solving a max-product non-linear system	Alhussaini et al. (2023)	Heuristic implementation of the (KU, 2018) attack utilizing a cover.
Ahmed et al. (2022)	Max-plus	Solving tropical two-sided non-linear equation with 6 unknown variables	Alhussaini et al. (2024)	Exploiting the ultimate periodicity of the columns and rows of matrix powers whose indices correspond to the critical nodes.
Durcheva (2022) 2- phase key exchange protocol	Min-plus; Max-plus	Solving MPF problem in tropical semiring	Jiang et al. (2023) attacked 1st phase Buchinskiy et al. (2023a) attacked 2nd phase	Solving the linear equations on the tropical semiring. Computing the defect and period of a public matrix M.
Chen et al. (2023)	Min-plus	Factoring one-variable tropical polynomial	Panny (2023) Brown & Monico (2023)	1) Searching for small modifications to the coefficients of the resultant matrices. 2) Instead of full factorization, finding only small-degree divisors. Swapping certain symbols involved in computing the signature; factoring the hash; factoring the public key; re-hashing an honest signature.

Future prospects

It is well-known that finding cryptographic primitives to serve as building blocks for cryptosystems that can resist quantum computer attacks is highly challenging. The problem of developing a secure key-exchange scheme over an insecure (public) channel remains unsolved. Taking into account the vulnerabilities of key-exchange protocols based on tropical algebra, we pose the question: Could tropical (or, more broadly, idempotent) cryptography provide a solution to this issue?

Looking ahead, we contemplate the future of cryptography based on idempotent algebra and envision several potential directions.

Exploring Different Hard Problems/Different Cryptographic Primitives within Tropical Semirings

Examples of such protocols include:

- The protocols proposed by Huang et al. (2024a) and Huang & Li (2022), which are based on the problem of finding the multiple exponentiations of tropical matrices (METM).

The authors of these protocols assert that the *METM* problem can be transformed into a constructive membership problem of a semigroup in polynomial time, which is a hard problem resisting quantum computer attack, as demonstrated in (Childs & Ivanyos 2014). In a recent paper (Huang et al. 2024b), it is stated that it has not yet been proven that the *METM* problem can be reduced to the Hidden Subgroup Problem, which is vulnerable to the quantum cryptanalysis.

- The protocol suggested by Durcheva & Danilchenko (2024), which utilized tropical block matrices as building blocks.

The security of this protocol relies on the matrix decomposition problem based on block matrices. Although a recent attack on this protocol in certain cases has been discussed in (Otero Sánchez et al. 2024), we believe that employing tropical block matrices as a cryptographic primitive offers a promising approach.

Building Alternative Types of Protocols, which Combines Various Techniques

Examples of such protocols include:

- An encryption protocol based on max-plus (MP) wavelet transforms.

MP-Wavelets are computationally simple and efficient, as noted by the authors of the protocol presented in (Subiono et al. 2022). The advantage of using them for building an encryption scheme is that the number of possible decryption keys is very large, even if the plaintext is short.

- Protocols based on T-norms.

The authors of several Stickel's like protocols based on max-min (max-T) semirings (Alhussaini & Sergeev 2024) argue that the advantage of max-min protocols over tropical ones is that the max-min protocols demonstrate greater resilience against the two-sided discrete logarithm attacks. Existing attacks such as the Kotov-Ushakov method (Kotov & Ushakov 2018) have been shown to be not very successful.

Utilizing Enriched Idempotent Semirings, such as Complete Idempotent Semirings (Quantales)

The connection between quantales and tropical mathematics is discussed by Fujii (2019). While no cryptosystems have been built yet based on this algebraic structure, we believe it represents a promising direction for future development.

Conclusions

We have been conducting research on semirings since 2010, with a specific focus on idempotent semirings and cryptography based on them since 2013. Over this period, dozens of tropical protocols have been developed. However, a significant portion of these proposed protocols has been exposed to various types of attacks, revealing vulnerabilities in key exchange protocols based on tropical algebra. Additionally, the advent of quantum computing has posed significant threats to classical public-key cryptosystems, raising the legitimate question of whether tropical cryptography has a viable future. Post-quantum cryptography relies on NP-hard problems that are considered harder than those used in classical cryptography and are specifically designed to resist quantum attacks. According to NIST's Post-Quantum Cryptography Project (https://www.nist.gov/pqcrypto), the main categories of post-quantum cryptography currently include *Hash-Based Cryptography*; *Lattice-Based Cryptography; Code-Based Cryptography*; *Multivariate-Based Cryptography,* and *Isogeny-Based Cryptography*. Although tropical-based cryptography has not yet been included in this list, certain problems in different semirings can be considered computationally hard, as no known quantum algorithm has been proven to solve them efficiently. For example, the security of certain cryptosystems, such as the one proposed by Huang et al. (2024c), which utilizes the digital semiring, is based on the 3-satisfiability problem. This problem can be polynomially reduced to solving systems of quadratic polynomial equations over the semiring, a task that is recognized as NP-hard.

In this paper, we provided a review of the current state of tropical cryptography and shared our insights on its future directions. We envision several approaches for its development, including exploring different hard problems or different cryptographic primitives within tropical semirings; developing alternative types of protocols that combine various techniques; and utilizing enriched idempotent semirings, such as complete idempotent semirings (quantales).

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Exploring Architectural Design 3D Reconstruction Approaches through Deep Learning Methods: A Comprehensive Survey

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In recent years, substantial advancements in computer vision have been propelled by the advent of deep learning techniques. The transformative impact of deep learning is evident across various computer vision applications, spanning object detection, image classification, and semantic segmentation. Within the dynamic realm of architectural parametric design, a pivotal force shaping contemporary and future architectural pursuits, designers frequently encounter scenarios necessitating the reshaping, augmentation, or modification of building components and facades. This demand arises from diverse factors, including the integration of human-interactive facades, the implementation of sustainable facade systems, and more. In such instances, the foundational structure must possess malleability, enabling openness to edits. However, the grand scale and intricate details inherent in architectural designs pose a challenge, as a densely detailed mesh comprising millions of elements becomes increasingly unwieldy and challenging to modify for subsequent stages. This challenge underscores the critical need to generate objects capable of seamless evolution in tandem with evolving design requirements. This paper, will focus on categorizing 3D deep learning approaches into Learning-based 3D reconstruction in architecture and delving into their methods.

Keywords: *Algorithm Development, Computational Design, Learning-based Generative Design, Deep Learning, 3D Reconstruction*

Introduction

Architecture is intensively relying on the concept, philosophy, and perception of the environment and the human behavior in the three-dimensional space and over the time. The new realm of the digital fabrication and rapid manufacturing is the field that requires an automated approach to accurately help architects during the design and manufacturing process. Besides the significant transformative impact of the deep learning in object detection, classification, and segmentation, its semantic inference capability makes it more powerful and suitable to analyze and interpret the data. Deep learning in three dimensions has the potential to transform the field of architecture by introducing novel approaches to construction, maintenance, and design. This entails the design and enhancement of neural network architectures algorithms that possess the ability to analyze and extract significant insights from a wide range of three-dimensional data formats, such as point clouds, meshes, volumetric data, and depth maps. 3D deep learning has the potential to greatly augment Building Information

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Modeling (BIM) in the discipline of architecture through the automation of model development and refinement, resulting in improved accuracy and efficiency. By facilitating the generation of numerous design options that are optimized for aesthetics, structural integrity, and sustainability, it facilitates generative design and optimization. In addition, it facilitates the development of dynamic and intricate building facades and forms, thereby augmenting both functionality and aesthetic appeal. The convergence of deep learning and interactive augmented reality (AR) enables the creation of immersive design experiences by facilitating the visualization and interaction with designs in real-time. Moreover, the utilization of 3D deep learning is of major significance in the digital reconstruction and preservation of historical structures within the 3D dense reconstruction, thereby contributing to the maintenance of cultural heritage. By harnessing these capabilities, architects have the potential to attain elevated levels of ingenuity, accuracy, and productivity, ultimately revolutionizing the architectural domain. The integration of 3D deep learning into digital fabrication, robotic fabrication, and 3D printing for large-scale buildings is revolutionizing the construction industry. These technologies enable greater precision, efficiency, and innovation in building processes. Applying deep learning to the digital fabrication process prepares a context to streamline the generative design to the fabrication step by optimizing material and improving the structure, ensuring the fabricated components pass the required specifications and also monitoring the production line for defect detection and for adaptive path planning in robotics fabrication and assemblies. However, our paper would be the initial one that concentrates on 3D reconstruction methods for architecture, there are other papers that explain the method, such as (Lu et al. 2005, Choy et al. 2016, Seitz et al. 2006, B. Zhao et al. 2018, Samavati & Soryani 2023, Pan et al. 2019, Parket al. 2019).

A significant contribution of this work, compared to existing literature, can be summarized as follows:

1. To the best of our knowledge, this is one of the first comprehensive survey papers covering 3D reconstruction in architecture.
2. The paper addresses both classical and deep learning-based methodologies, providing a broader perspective on 3D reconstruction capabilities.
3. Unlike existing reviews, we specifically focus on 3D reconstruction and its applications in architecture.
4. The paper aims to include the most recent and robust techniques in this field, highlighting their relevance in accurate industry workflows and research.

This paper is structured as follows: Section *ii* provides an overview of 3D reconstruction and its applications in architectural design and digital fabrication. Section *iii* delves into classical 3D re-construction methods, while Section *iv* explores deep learning-based 3D reconstruction techniques. Section *v* presents a discussion, highlighting the technology's impact on architectural design. Finally, Section *vi* offers the conclusion.

Background

3D Reconstruction

Extending deep convolutional networks, the cornerstone of image-based techniques, to the third spatial dimension presents a number of challenges. Because of the higher data dimensionality, these networks have a tendency to multiply in terms of both time and space complexity. Conventional dense surface representations, like quad or triangular meshes, pose more challenges to the training process. These challenges arise from the randomly generated topology and the fluctuating vertex count of these models. These problems have severely limited the deep learning algorithms' efficacy, flexibility, and accuracy. Applying these algorithms to 3D data input analysis or generating object segmentations in 3D space particularly highlights these limitations. Because 3D data is inherently complicated, more advanced models and computing power are required, which might hinder training and reduce algorithmic performance. The inconsistent mesh topologies make it more difficult to create reliable and effective deep learning frameworks. Since every 3D model can have a different structure, algorithms must be able to adjust to a variety of vertex combinations and geometries. This heterogeneity necessitates the use of more sophisticated methods for data augmentation and preprocessing, in addition to increasing the computational load. There are different methods to construct a 3D mesh from an image, such as:

- *Classical 3D Reconstruction:* Traditional reconstruction methods usually adopt discrete representations, such as voxel grids, 3D point clouds, and depth maps. The depth map representation is flexible and appropriate for parallel computation, so it is the most widely used representation (Long et al. 2022). Therefore, traditional techniques often provide noisy and partial reconstructions because they are susceptible to reflections, picture noise, and variable illuminations.
- *Deep Learning-based 3D Reconstruction:* These methods typically represent the surfaces to be reconstructed as Signed Distance Function (SDF) and utilize surface rendering or volume rendering to render the pixel colors of the input images for optimizing the SDF fields (Long et al. 2022).

3D Reconstruction in Architecture

In terms of conceptualizing, visualizing, and realizing forms and structures, 3D reconstruction can have an impact on digital production and architectural design. Architects and designers can produce extremely realistic and detailed models of buildings and other structures by utilizing sophisticated computational tools. This improves planning, analysis, and communication during the design process. 3D reconstruction in architecture makes it possible to create detailed digital representations of real locations.

- *Conceptualization and Visualization:* Using accurate 3D models that offer a realistic image of the proposed structure, architects can develop and revise design concepts. This facilitates the visualization of the design's spatial relationships, proportions, and aesthetics, improving stakeholders' comprehension and assessment of the undertaking.
- *Detailed Element Design:* 3D models allow for accurate planning of structural elements, materials, and construction methods. These models can be used to simulate and optimize various design aspects, such as load distribution, energy efficiency, and acoustics, leading to more informed decision-making.
- *Collaboration and Virtual Interaction:* The use of 3D reconstruction enhances communication among architects, engineers, and clients. Web-based interactive technologies, virtual reality, and augmented reality can be used to instantly exchange and analyze interactive 3D models, guaranteeing that everyone involved has a consistent and clear knowledge of the design concept.

Classical 3D Reconstruction

Structure from Motion (SfM): Snavely et al. (2006) presented a method using camera pose (location, orientation, and field of view) and sparse 3D scene information to create new interfaces for browsing large collections of photographs. Knowing the camera pose enables placing the images into a common 3D coordinate system and allows the user to virtually explore the scene by moving in 3D space from one image to another using our photo explorer. We use morphing techniques to provide smooth transitions between photos. In addition, the re-constructed features provide a sparse but effective 3D visualization of the scene that serves as a backdrop for the photographs themselves. Table. 2 shows popular SfM packages and software.

Multi-View Stereo: Seitz et al. (2006) categorized multi-view stereo algorithms into four classes. The first class operates by first computing a cost function on a 3D volume and then extracting a surface from this volume. A simple example of this approach is the voxel coloring algorithm and its variants, which make a single sweep through the volume, computing costs and reconstructing voxels with costs below a threshold in the same pass. The second class of techniques works by iteratively evolving a surface to decrease or minimize a cost function. This class includes methods based on voxels, level sets, and surface meshes. Space carving and its variants progressively remove inconsistent voxels from an initial volume. Other variants of this approach enable adding as well as deleting voxels to minimize an energy function. Level-set techniques minimize a set of partial differential equations defined on a volume. Like space carving methods, level-set methods typically start from a large initial volume and shrink inward; unlike most space carving methods, however, they can also locally expand if needed to minimize an energy function (Seitz et al. 2006). The third class is image-space methods that compute a set of depth maps. To ensure a single consistent 3D scene interpretation, these methods enforce consistency constraints between depth maps or merge the set of depth maps into a 3D scene as a post process. The final class consists of algorithms that first extract and match a set of feature points and then fit a surface to the reconstructed features (Seitz et al. 2006). The Multi-View Stereo (MVS) technique utilizes *Structure from Motion (SfM)* output to compute depth and normal data for each pixel within an image. The amalgamation of depth and normal maps from various images in a three-dimensional space subsequently yields a comprehensive point cloud representation of the scene. Table. 1 compares SfM and MVs.

Table 1. *Structure from Motion (SfM) vs. Multi-View Stereo (MVS)*

SfM	MVS
Generally employs a collection of photos not arranged in a particular sequence, captured from diverse views and time instances, and frequently exhibiting variations in focal lengths and resolutions.	It often involves a collection of calibrated photos captured from predetermined angles with constant camera settings.
In the context of camera calibration, SfM is a technique used to estimate the calibration parameters, such as the focal length, camera rotation, and camera translation, as an integral component of the overall process.	It often necessitates the use of pre-calibrated cameras that possess predefined characteristics.
Utilizes the process of feature matching between pictures to ascertain the relative camera postures and the scene's sparse three-dimensional (3D) structure.	It encompasses several techniques, including direct methods that do not need explicit feature matching and methods that include feature matching.
Initially generates a sparsely reconstructed scene by detecting a restricted set of points. Dense reconstruction can be accomplished as the following procedure by utilizing depth map estimation techniques.	Frequently pursues the objective of achieving dense reconstruction in a direct manner, wherein the complete visible surface of the 3D object or scene is reconstructed.
The technique of inferring the 3-dimensional structure of a scene and the camera's motion is commonly referred to as SfM.	The primary objective of multi-view reconstruction is to reconstruct the three-dimensional structure by utilizing numerous views without explicitly considering the camera's motion.
It is often employed in several domains, including photogrammetry, robotics, and computer vision jobs that utilize unordered pictures captured by handheld cameras or drones.	It is a widely employed technique in several fields, including computer graphics, medical imaging, and industrial applications, mainly when calibrated cameras are accessible.

Simultaneous Localization & Mapping: Simultaneous Localization and Mapping (SLAM) is a technique employed to get information on the surrounding environment by utilizing various sensors such as cameras, LIDAR (Light Detection and Ranging), and IMUs (Inertial Measurement Units). The system concurrently calculates the estimates: *Localization,* refers to determining the location and orientation, also known as the pose, of a robot or device within its surrounding environment. *Mapping,* The process of producing a visual representation, known as a map, that depicts the spatial arrangement of the surrounding physical or conceptual world. SLAM aims to resolve the inherent challenge of the chicken-and-egg dilemma. In order to construct a map, the system needs knowledge of its stance. However, to estimate its pose, the system relies on the existence of a map. The SLAM algorithm effectively addresses this issue through its iterative process of updating both the map and pose estimations in response to incoming input. SLAM methodologies can be categorized into *feature-based, grid-based,* and *topological techniques*. The *feature-based* technique uses recognizable elements like corners for mapping and localization. *Grid-based* techniques discretize the environment into grids, including occupancy information, and use this information to estimate the map and pose. *Topological* techniques represent the environment as a graph. The SLAM system consists of three components: *data association, state estimation, map update,* and *loop closure*. Data association matches observable features in the environment with map features. State estimation infers the robot's current state, including posture and map, based on sensor data. Map update integrates recent data, and loop closure identifies robot return instances for improved accuracy and consistency. There are three popular types of SLAM algorithms *Filter-based SLAM, Graph-based SLAM,* and *Visual SLAM (VSLAM)*. Table. 2 shows popular SLAM packages and software.

Reconstructing with Voxels Using Occupancy Grid: This representation discretizes the three-dimensional space into a grid of uniform cubic parts. Each part has two states: empty or occupied. Due to memory limitations, the grid resolution is often limited to 10243 and lower. Despite this limitation, this type of representation easily fits into the deep learning frameworks. Therefore, models that infer 3D shapes using this representation are very easy to implement (Samavati & Soryani 2023). Choy et al. (2016) introduced a 3D Recurrent Reconstruction Neural Network (3D-R2N2). This network is designed to understand the 3D shape of objects by learning from a wide array of synthetic data and mapping images to their respective 3D structures. The network processes one or multiple images of an object from any angle and then generates a 3D occupancy grid, essentially creating a reconstructed version of the original object.

Table 2. *Popular SfM & SLAM Software Packages*

Package	Description	Method
COLMAP	A 3D reconstruction software that includes both sparse and dense matching modules, which includes Radial and Fish-eye camera models (Schonberger & Frahm, 2016).	SfM
OpenMVG	A well-known sparse reconstruction software that provides Radial, Brown, Fisheye, and Sphere, camera models. It can directly process spherical images (Moulon, Monasse, Per-rot, & Marlet, 2017).	SfM
OpenVSLAM	A visual SLAM system that supports various camera models, including perspective, fisheye, and spherical cameras (Sumikura, Shibuya, & Sakurada, 2022).	SLAM
Cubemap-SLAM	A visual SLAM system that converts fisheye images into perspective images and achieves image orientation based on the ORB-SLAM (Y. Wang et al., 2018).	SLAM

Space Carving: By framing shape reconstruction as a constraint satisfaction problem, Kutu-lakos and Seitz (Kutulakos & Seitz 2000) established an approach to reconstruct 3D scenes from photographs. In their paper, they demonstrated that any collection of photographs of an immutable scene defines a set of image constraints delighted by each scene that can be projected onto those images. They represented "the set of all 3D shapes that satisfy these constraints and use the underlying theory to design a practical reconstruction algorithm, called *Space Carving*, that applies to fully-general shapes and camera configurations" (Kutulakos & Seitz 2000). Space Carving (Kutulakos & Seitz 2000, Broadhurst et al. 2001, Slabaugh et al. 2004, Aykin & Negahdaripour 2016, R. Yang et al. 2003) or Volumetric Reconstruction techniques involve the removal of space from the volume based on the coherence of 2D images captured from multiple perspectives.

Depth Map Fusion: A number of techniques have been developed for reconstructing surfaces by integrating groups of aligned range images. A desirable set of properties for such algorithms includes incremental updating, representation of directional uncertainty, the ability to fill gaps in the reconstruction, and robustness in the presence of outliers (Curless & Levoy 1996). Using depth-sensing cameras to generate a set of depth maps (Merrell et al. 2007, Zach 2008, Zhu et al. 2010) combined to create a 3D model constitutes Depth Map Fusion. Generally speaking, depth map fusion represents a technique applied to computer vision to blend *(fuse)* several depth maps into an individual, precise, or comprehensive depth map. Depth maps are images in which each pixel comprises a value indicating the distance from the object at that image location to the camera.

Light Field Methods: Light field (Gershun 1939, Levoy & Hanrahan 1996, W.C. Chen et al. 2002) imaging has emerged as a technology allowing us to capture richer visual information from our world. As opposed to traditional photography, which captures a 2D projection of the light in the scene integrating the angular domain, light fields collect radiance from rays in all directions, demultiplexing the angular information lost in conventional photography. On the one hand, this higher dimensional representation of visual data offers powerful capabilities for scene understanding and substantially improves the performance of traditional computer vision problems such as depth sensing, post-capture refocusing, segmentation, video stabilization, material classification, etc. (G. Wu et al. 2017).

Poisson Surface Reconstruction: (Kazhdan et al. 2006, Bolitho 2009, Estellers et al. 2015) Poisson surface reconstruction algorithm is an implicit function method. It combines global and local fitting schemes. Yet, the basis functions are associated with the ambient space rather than the data points, are locally supported, and have a simple hierarchical structure that results in a sparse, well-conditioned system. But this algorithm reconstructs the isosurface using the Marching Cube method (Long et al. 2022, Newman & Yi 2006, Lopes & Brodlie 2003), which will generate a simple smooth surface and waste lots of time on the detecting of non-null cubes (X. Li et al. 2010).

Ball Pivoting Algorithm (BPA): (Bernardini et al. 1999, Digne 2014, Stelldinger 2008) is a method for reconstructing surfaces from unorganized point clouds. The BPA, initially proposed by Bernardini et al. (1999), has been broadly adopted because of its simplicity, effectiveness, and excellent outcome. According to (Bernardini et al. 1999), the method is conceptually simple. Starting with a seed triangle, it pivots a ball around each edge on the current mesh boundary until a new point is hit by the ball. The edge and point define a new triangle, which is added to the mesh, and the algorithm considers a new boundary edge for pivoting. The output mesh is a manifold subset of an alpha shape of the point set. Some of the nice properties of alpha-shapes can also be proved for our reconstruction.

Signed Distance Function (SDF): The Signed Distance Function (SDF) (Zhou, et al. 2016, Bylow et al. 2013, Osher et al. 2003, Zollhöfer et al. 2015) is explained, given its intersection with the categorization of 3D deep learning. In simultaneous localization and mapping, 3D reconstruction and surface tracing are con-ducted, which presents a continuous and differentiable surface representation that is advantageous for camera estimation poses and reconstructing the scene's geometry.

Determining image pixels' depth ranks among the foremost fundamental challenges of 3D re-construction. Deducing the spatial structure of an environment from two-dimensional images is the task of depth estimation. The objective of this task is to recover the crucial spatial data that was lost during the 3D-to-2D projection procedure during image capture. Classical 3D reconstruction approaches determine the depth of pixels using Triangulation and Epipolar Geometry. Nevertheless, these techniques require knowledge of internal and external parameters associated with the cameras, and the images must be captured with minor rotational and translational adjustments to the camera. Based on these algorithms, a variety of methodologies have been established for 3D re-construction algorithms. In addition to Poisson Surface Reconstruction, Ball Pivoting Algorithm, and Signed Distance Function methods, there are other algorithms such as Marching Cube and Truncated Signed Distance (TSDF).

Deep Learning-based 3D Reconstruction

Applying deep learning techniques to the processing and analysis of three-dimensional data is known as 3D deep learning. It involves the creation of neural network designs and algorithms that can comprehend and extract useful information from 3D data, including depth maps, 3D models, point clouds, and volumetric data. The efficient handling of 3D data by deep learning opens up a variety of applications in many industries. These domains cover a wide range of industries, such as architecture, computer-aided design (CAD), robotics, autonomous cars, augmented reality (AR), medical imaging, and game creation.

3D deep learning extends the utilization of deep learning algorithms to 3D data, such as point clouds (Achlioptas et al. 2018, L. Jiang et al. 2018, Fan et al. 2017, K. Li et al. 2018, Thomas et al. 2019, G. Yang et al. 2019), volumes (Brock et al. 2016, Choy et al. 2016, Gadelha et al. 2017 Jimenez Rezende et al. 2016, Riegler et al. 2017, Stutz & Geiger 2018), multi-view images (Xie et al. 2019, Rhodin et al. 2018, Xu et al. 2016, Tatarchenko et al. 2016, B. Zhao et al. 2018), meshes (Zhou & Tuzel 2018, Groueix et al. 2018, Kanazawa et al. 2018, Liao 2018, Pan et al. 2019), etc. as opposed to traditional deep learning, which primarily operates on 2D data. This shift enables machines to interpret and analyze a more comprehensive and detailed representation of physical entities, thereby producing more accurate results and decisions. As a consequence, it is increasingly being utilized in diverse areas such as autonomous vehicles, robotics, virtual reality, and healthcare (Z. Wu et al. 2015). The methodology of 3D deep learning encompasses a multitude of strategies, each serving a different function and possess-ing certain advantages and disadvantages. The main methods include volumetric, multi-view, point cloud-based, and graph-based.

One pioneering 3D deep learning technique involves the volumetric analysis of 3D data. These methods convert 3D data into a volumetric grid and then apply traditional 3D convolutions to capture spatial correlations. An example is VoxNet, a 3D Convolutional Neural Network (3D-CNN) that operates on voxel grids to recognize and classify 3D shapes (Maturana & Scherer 2015). 3D-CNNs are a direct extension of traditional 2D-CNNs to the 3D domain, enabling the processing of 3D data directly (Ji et al. 2012). Various voxel-based approaches have been developed following this work, such as PointNetVLAD by Zu et al. (Uy & Lee 2018), which aggregated local features for global scene representation using an encoding module. Nevertheless, volumetric methods are typically computationally intensive because of the substantial dimensionality of 3D data. Researchers have proposed multi-view methods as a solution to the computational difficulties of volumetric methods. MVCNNs process 3D data by projecting it onto multiple 2D views and then analyze these 2D views using 2D-CNNs (Su 2015). However, this method may fail to capture certain spatial relationships as some information is lost during the projection process. The PointNet architecture developed by Qi et al. (2017) has become among the most widely recognized within this category. It explicitly applies neural networks to point clouds, considering the input an unsorted collection of points. Later architectures, including PointNet++ (Qi et al. 2017) and DGCNN (Y. Wang et al. 2019), enhanced the authentic PointNet by more accurately preserving the point cloud's local structures. PointNet operates directly on 3D point cloud data, which are unordered sets of points in a 3D coordinate system (Qi et al. 2017). Other notable point-based methods include PointConv by Wu et al. (2019), which introduced learnable convolutional operators on point clouds. Graph-based strategies encode 3D data as graphs, manipulating the raw connectivity data within 3D shapes. The Graph CNN (GCNN) by Bronstein et al. (2017), which extrapolates the convolution layers to non-Euclidean spaces, is a seminal work in this field. These strategies are able to represent complex spatial relationships; however, graph creation from point clouds can be complex. GNNs are designed to process data structured as graphs, a very generic form of data, which allows them to handle a variety of 3D data formats (Bronstein et al. 2017). Graph Attention Networks (GAT) by Veličković et al. (2017) utilized attention mechanisms to weigh the importance of neighboring points during information propagation.

3D Geometry is a subfield of mathematics concerned with studying three-dimensional shapes and figures. Numerous shapes are involved, including cubes, pyramids, spheres, and cylinders, and are widely utilized in computer graphics, simulations, and game design due to the advancement of computer technology. As Kato et al. (2020) stated, "Most 3D estimation methods rely on supervised training regimes and costly annotations, which makes the collection of all properties of 3D observations challenging. Hence, there have been recent efforts towards leveraging easier-to-obtain 2D information and differing levels of supervision for 3D scene understanding. One of the approaches is integrating graphical rendering processes into neural network pipelines. This allows transforming and incorporating 3D estimates into 2D image-level evidence." Rendering in computer graphics is the process of generating images of 3D scenes defined by geometry, materials, scene lights, and camera properties. Rendering is a complex process, and its differentiation is not uniquely defined, which prevents straightforward integration into neural networks (Kato et al. 2020). The technique of Differentiable Rendering (DR) (Loper & Black 2014, Loubet et al. 2019, J. Chen et al. 2019) bridges the divide between 3D graphics and artificial intelligence. It is a process by which the procedure for rendering becomes differentiable, i.e., output gradients can be computed concerning their input which enables backpropagation of gradients, a technique essential for training neural networks.

Differentiable Rendering (DR) constitutes a family of techniques that tackle such integration for end-to-end optimization (Kotary et al. 2021, Ballé et al. 2016, Sitzmann et al. 2018) by obtaining useful gradients of the rendering process. By differentiating the rendering, DR bridges the gap between 2D and 3D processing methods, allowing neural networks to optimize 3D entities while operating on 2D projections. Previously, rendering was a process that generated a 2D image from 3D shapes, textures, illumination, and perspective. This function is typically not differentiable everywhere, making it challenging to use conventional optimization (Reddy et al. 2017, Banos et al. 2011) methods or develop deep learning models that entail a rendering phase. DR algorithms attempt to achieve the rendering operating as differentiable as feasible, allowing gradients to be backpropagated over it with various applications, including training deep neural networks that generate 3D shapes and adjusting the hyper-parameters of a 3D model to suit an image. Differentiable rendering algorithms can be divided primarily into four main categories based on (Kato et al. 2020). Here, we discuss differentiable rendering in detail:

Mesh-based Methods

Analytical Derivative: According to (Kato et al. 2020), "a mesh represents a 3D shape as a set of vertices and the surfaces that connect them." It is frequently utilized, particularly in computer graphics, due to its ability to represent sophisticated three-dimensional shapes compactly. Given the rendering data as input sources and pixel information, estimating the color consists of two steps. First, defining a triangle around the pixels and, second, calculating the pixel color based on the hues of the vertices from the allocated triangle.

Approximated Gradient: In optimization methods, an approach known as approximated gradient applies to the estimation rather than the accurate calculation of a function's gradient. This method is frequently employed when computing the gradient would be expensive or complex. The gradient may be estimated by making specific assumptions or applying numerical techniques, which enables the use of gradient-based optimization techniques. As Kato et al. (2020) stated, "gradients should be objective-aware for better optimization. Since the objective of OpenDR is not to provide accurate gradients but to provide useful gradients for optimization, a loss-aware gradient flow is required for this purpose."

Approximated Rendering: (Reeves & Blau 1985, Lighthill 1949, Loper & Black 2014) In-stead of approximating the backward pass, other methods approximate the rasterization (or the forward pass) of the rendering in order to be able to compute useful gradients (Kato et al. 2020). Creating a picture from a 3D model is known as approximate rendering, focusing more on speed and efficiency than precise accuracy. These methods can quickly create visuals that are visually near to a correct depiction by making approximations and simplifications. This method is frequently applied in scenarios where rendering in real-time is necessary, as in simulations or video games.

Global Illumination: (H. Jiang et al. 2021, Y. Li et al. 2022, Jensen 1996, Sillion et al. 1991, Landis 2002) is an essential notion in 3D reconstruction that describes how light is reflected and dispersed throughout a scene. Global lighting considers the indirect impacts of light because it bounces and scatters across a scene, in contrast to local or direct illumination, which solely analyzes the direct interaction across lighting sources and surfaces. A reconstructed picture may become much more realistic with accurate modeling of global lighting. Still, it also becomes more complicated since it must replicate all the ways light interacts with the environment's surfaces.

Voxel-based Methods

In three-dimensional space, a voxel, a phrase derived from *volume* and *pixel,* denotes a value on a regular grid. It is a pixel's two-dimensional counterpart in three dimensions. Volumetric in-formation is determined in a Cartesian coordinate system and is contained in each voxel. A voxel carries information, including density, color, or material attributes, based on the context, much like a pixel has a particular color associated with it. Before delving into the specifics, let's clarify two key concepts:

Occupancy Grid: This is a representation of space in the form of a grid, where each cell denotes its occupancy status—either occupied or free. Typically, cells can have binary values, indicating whether they are occupied or not. Alternatively, they can hold a probability value that signifies the chances of the cell being occupied

Deep Learning-based Signed Distance Function (SDF): This describes space as a continuous scalar field in which every point is assigned a value corresponding to the shortest distance to the closest surface. These values are positive when the point lies outside the surface and negative when it's inside the surface.

It is common practice to encode occupancy (Snow et al. 2018) in voxel occupancy, the scene is represented as a set of 3-dimensional voxels, and the task is to label the individual voxels as filled or empty (Snow et al. 2000). Voxel information using a binary value and transparency using a non-binary one. Shapes can be represented as the shortest path from the center of each voxel to the surface of the object rather than by storing occupancy. The shortest distance between a point and a surface is returned by a computer graphics algorithm called a Signed Distance algorithm (SDF) (Bylow et al. 2013, Osher et al. 2003, Zollhöfer et al. 2015). This distance is positive if the point is outside the surface and negative if it is within, as indicated by the word "signed" in the function's name. SDFs, as implicit surface reconstruction methods, are used extensively in generating and manipulating intricate geometric objects. SDFs have the ability to specify a form in space with just one function, which makes them an effective tool for developing procedural content. SDFs may be used to generate complicated forms with less computational expense than conventional polygon-based models, which is especially beneficial in real-time 3D rendering and graphics.

Point Cloud-based Methods

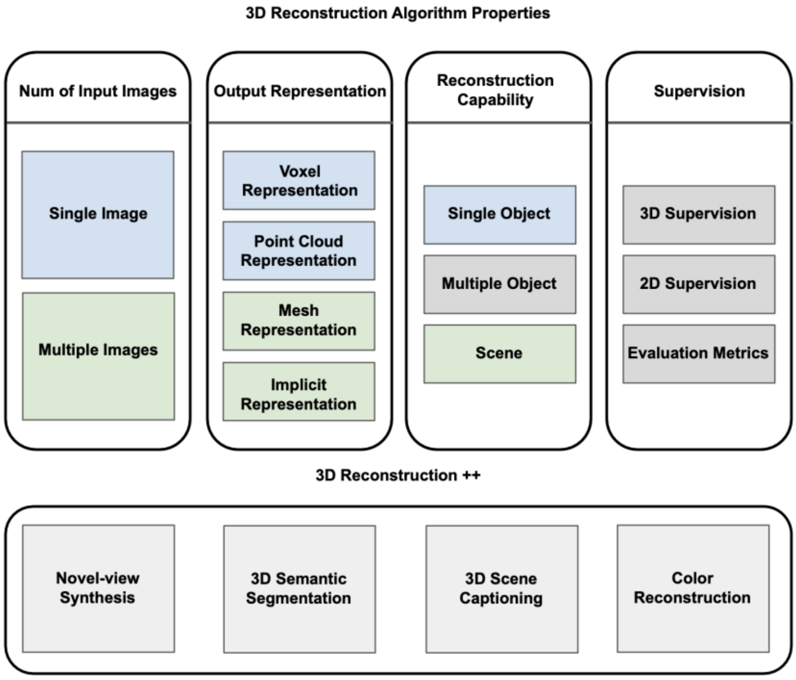
A point cloud is a group of data points arranged in three dimensions. Each cloud point cor-responds to a location where a surface was identified. A considerable amount of points are often measured on the surfaces of nearby objects using 3D scanners or photogrammetry software, which produces a file with these 3D coordinates. There are three primary processes for rendering a point cloud. Initially, matrix multiplication is used to determine a 3D point's screen space coordinates, much like when rendering a mesh. This makes this step differentiable. Secondly, each 3D point's impact on the color of the target pixel is calculated. To get the final color value for each pixel, the points are finally combined based on their influences and z-values. Several techniques have been put forth to successfully handle this last stage of aggregation.

Implicit Methods

(Niemeyer et al. 2020, Liu et al. 2020, Cole 2021) Using functions that implicitly determine a form, implicit representations in differentiable rendering approaches indicate a technique for modeling and rendering 3D shapes. In contrast to traditional 3D modeling, which employs explicit representations such as meshes or point clouds to specify the vertices and edges of a shape, implicit 3D modeling uses a function to specify whether a particular point in 3D space is located within or outside the shape.

Unlike voxel-based methods, in implicit representations, memory usage is constant with respect to spatial resolution. Therefore, a surface can be reconstructed at infinite resolution without excessive memory footprint. In implicit representations, memory utilization is constant with respect to spatial resolution, unlike voxel-based techniques. This allows for the recreation of a surface at an infinite resolution and with a minimal memory footprint.

Figure 1. *3D Reconstruction Methods' Properties Overview*



3D Reconstruction in Architectural Design Discussion

Deep Learning-based 3D Reconstruction Model Selection Criteria

Architectural forms can be found in a wide range of styles, from the elaborate intricacies of Gothic designs to the sleek minimalism of Modern architecture and even experimenting with avant-garde shapes. The extreme range of architectural entities in terms of scale and nature is a significant difficulty in this field. The difficulty of creating a complete dataset encompassing all these styles stems from the scattered and frequently insufficient data for individual architectural disciplines.

Because of these limitations, choosing a model that can produce reliable findings without excessively depending on large datasets was essential. Architectural entities are highly detailed stylistically, and their actual forms vary widely. They can be as large as skyscrapers or lightweight structures that resemble shelters and are sometimes referred to as non-watertight structures. Because of this variety, a method that is adaptable enough to allow reconstructions regardless of the structural complexity of the form is required.

Based on (Samavati & Soryani 2023), "as voxel grids are discrete, the output resolution must be kept at low levels due to resource constraints. Thus, the reconstructed shapes are low-detailed, and the reconstruction accuracy is low. Additionally, this type of representation cannot be directly used in some applications, such as game and movie industries, as it requires a different algorithm to obtain more flexible and natural outputs." Therefore, a middle-tier method that can enhance voxel outputs and give them a more flexible and realistic appearance is required. Adding to these difficulties is that voxels (3D-R2N2 (Choy et al. 2016), (Xie et al. 2019)) naturally yield results with lower resolution, primarily because of memory usage limitations. There is, however, a bright side. Based on the observations made by Kato (2020), it is clear that voxels can be effectively converted into useful informational resources by using specific methods. Using the Signed Distance Function (SDF) (Takikawa et al. 2021, Y. Jiang 2020) is one technique that expands the possible uses of voxel-based reconstructions; however, SDF is useful when used for watertight shapes.

Point Cloud-based approach does a more accurate job with a better resolution. According to (Samavati & Soryani 2023), "A point cloud is simply a set of unordered 3D points in a coordinate system. Compared to ordered structures like meshes that store combinational connectivity patterns, predicting 3D shapes in point-cloud representation is less challenging." It may use different types of networks, mainly the Encoder-Decoder model (Zhang 2020). However, there are advances to the primary models using Multilayer Perceptron (MLP), especially for the shape completion (Qi et al. 2017). Point Cloud-based methods mainly rely on AE models, and they require training steps in which the decoder knowledge is confined to the 3D data shapes and categories used in the training phase.

Typically, conventional signal representations exhibit discreteness. For example, when rep-resenting 3D shapes, they are commonly specified as grids consisting of voxels, point clouds, or meshes. Conventional signal representations employ a discrete methodology. Consider 3D shapes as an illustrative example, which are frequently depicted through the utilization of voxel grids, point clouds, or mesh structures. In contrast, implicit neural representations adopt another approach. The researchers define a signal as a continuous function that establishes a connection between the domain of the signal and the corresponding value or entity associated with that coordinate. This value could represent either an occupancy probability or an SDF (Park et al. 2019) (also see Table. 3) value. Instead of employing explicit representations, implicit neural techniques approximate the continuous function by utilizing an MLP network. MLP possesses the capability to replicate any function to a notable extent accurately. One notable advantage of these strategies is their high level of memory efficiency. This methodology enables the neural network to accurately infer complex 3D shapes at different resolutions. One prominent limitation of the SDF models is "limited to objects with closed surfaces since they adopt Signed Distance Function (SDF) as a surface representation which requires the target shape to be divided into inside and outside", (Long et al. 2022). Additionally, methods such as NeuralUDF (Long et al. 2022) (Unsigned

Distance Function) try to rectify this issue; a comprehensive training dataset is inevitable.

In the past, SfM, multi-view stereo, and depth-map fusion-based methods were widely used for active 3D acquisition tasks. However, as alluded to above, these classical approaches have limitations in the dense 3D reconstruction of an object (Lee et al. 2022). Additionally, classical methods like multi-view have been integrated with deep learning for better performance and results, such as the work by Niemeyer et al. (2020). According to (Lee et al. 2022) "NeRF learns an implicit function to represent complex 3D shapes such that the novel poses of the object can be synthesized. The implicit function is learned by an MLP (Samavati & Soryani 2023). NeRF is a simple yet effective volume rendering approach. It represents the continuous static scene as 5D neural radiance fields, parameterized by multi-layer perceptron (MLP). It demonstrated that regressing density and light fields via an MLP could yield photo-realistic rendering. Due to its remarkable ability to capture complex geometric details, it gathered significant interest from the community (Deng 2022). Although NeRF requires more views than other models and the training will be expensive; however, according to (Deng et al. 2022) "Neural rendering with implicit representations has become a widely-used technique for solving many vision and graphics tasks ranging from view synthesis to re-lighting, to pose and shape estimation, to 3D-aware image synthesis and editing, to modeling dynamic scenes. On the other hand, NeRF is built upon perspectives of particular objects rather than relying on extensive collections of diverse data. This methodology places greater emphasis on the visual representation of the object as opposed to the quantity or diversity of data points, which can be adjustable for a variety of architectural styles. The seminal work of Neural Radiance Fields (NeRF) demonstrated impressive view synthesis results by using implicit functions to encode volumetric density and color observations.'' Overall, it is important to consider that every solution has its drawbacks. However, we pursued a solution offering greater efficiency, enhanced flexibility, and reliance on cases. Table. 3 compares the SDF-based methods with NeRF architecture.

Table 3. *Comparison between SDF and NeRF*

SDF	NeRF
Is a mathematical representation that depicts a three-dimensional shape by assigning a scalar value to every point in space, representing the shortest distance from that point to the shape's surface. The scalar's sign indicates whether the point is located inside *(negative)* or outside *(positive)* the geometric shape.	Employs a neural network architecture to effectively model and describe a continuous volumetric scene function. The process involves mapping a three-dimensional coordinate and a viewing direction to determine the radiance value and volume density.
Is employed in form representation, reconstruction, and generation.	Is designed to generate original perspectives of a particular scene based on a collection of two-dimensional photographs captured from predetermined vantage points.
Suitable for shape completion, shape interpolation, 3D modeling, and more.	Novel view synthesis, 3D scene representation from 2D images, AR/VR.
Focusing on representing the shape or boundary of objects.	Focusing on representing the entire volumetric scene, including color, lighting, and density.
Using 3D data (like point clouds or meshes) for training.	Using 2D images of a scene for training.
	Computationally intensive (particularly during training).

With the introduction of NeRF (Mildenhall et al. 2021), a completely new direction for novel-view synthesis has been established. Given a sparse set of images capturing an object of interest from different viewpoints, NeRF learns an implicit function to represent complex 3D shapes such that the novel poses of the object can be synthesized (Samavati & Soryani 2023). Meanwhile, NeRF was established on scene training, using sparse views of an object. As stated by Samavati et al. (2023), "the training and inference times of the original NeRF are high; therefore, instead of representing the entire scene as a single implicit field, Neural Sparse Voxel Fields (NSVF) (Liu et al. 2020) organize the scene as a sparse voxel octree and bound a set of implicit functions to its voxels. This speeds up rendering by a factor of ten. While NeRF only operates on static scenes, there are some other methods proposed to handle dynamic ones." This gives the opportunity to employ architectural-related works as it is independent of a massive, diverse training dataset. Figure. 1 shows the more advanced 3D reconstruction methods that NeRF could be categorized under *Novel-View Synthesis* class.

3D Reconstruction in Architecture vs. Other Fields

3D Reconstruction is a commonly utilized technique for simplifying the process of object observation from images and generating 3D meshes. By employing this approach, any person can obtain a three-dimensional object without requiring sophisticated technologies or 3D scanners. Its reliability and precision have been established through extensive use to the extent that it has been integrated into various sectors such as medicine, robotics, and autonomous vehicles. However, the rationale for this transformation from 2D to 3D varies among academic fields. In computer vision, research on structure from motion (SFM) and multi-view stereo (MVS) has produced many compelling results, in particular accurate camera tracking and sparse reconstructions, and increasingly reconstruction of dense surfaces. However, much of this work was not motivated by real-time applications (Newcombe et al. 2011). Here, we will discuss the application of the 3D Reconstruction in robotics, medicine, and architecture.

In order to perform real-time data processing duties such as localization and grasping, robots must have an exhaustive understanding of their immediate environs. "Real-time 3D semantic re-construction is required in many robotics applications, such as autonomous navigation or grasping and manipulation. While a variety of well-known methods can reconstruct accurate 3D maps at real-time frame rates, the resulting point clouds contain no semantic-level understanding of the observed scenes. Hence, the problem of 3D semantic reconstruction is attracting increasing attention in the robotics research community. Recent methods not only generate a 3D point cloud map but also simultaneously assign a semantic label to each point in the cloud'', (C. Zhao et al. 2017). The utilization of semantic observation and reconstruction extends to numerous environmental attributes, including materials, due to its critical application in disciplines such as robotic grasping and rescue. According to Endres et al. (2013), "Manipulation robots, for example, require a detailed model of their workspace for collision-free motion planning, and aerial vehicles need detailed maps for localization and navigation.'' Moreover, in robotic systems, active vision involves adeptly adjusting the camera pose to gather the maximum amount of information from the scene. Existing methods that calculate the information yield for active robotic 3D Reconstruction focus on explicit 3D models like point clouds and voxels, using structure from motion (SfM). Using well-posed multi-view images, NeRF can provide an object's dense 3D reconstruction with favorable accuracy, overcoming the inherent limitations of SfM and depth fusion methods (Lee et al., 2022).

Medical images have been used in disease diagnosis and therapy since their discovery. Image processing techniques have been used to improve the quality of the images through tasks such as contrast enhancement and also in image analysis to aid in the interpretation of the images by clinicians (Ahishakiye et al. 2021). According to (Singh et al. 2020), 3D Deel Learning has multiple applications in 3D medical imaging such as *segmentation* (for brain lesions, skull stripping, etc.), *classification* (detecting dementia from MRI), *detection and localization* (detecting cerebral micro-hemorrhages in brain tissue), and *registration*. As Wang et al. (2021) explained in, "medical 3D reconstruction refers to computer processing of sampled single or sequence images from human organs by computerized tomography (CT), magnetic resonance imaging (MRI), and Ultrasonic imaging (UI). It includes data acquisition, pre-processing, point cloud splicing, and feature analysis. In medical images, 3D reconstruction of tissue sections, CT sections, and autoradiography sections have become an important part of biomedical research. Through the restoration of the 3D surface shape of the organ, the human-computer interaction can be further used to carry out various realistic operations on the reconstructed object, such as rotation, scaling, and cutting.'' Architecture spans a broad spectrum of applications, from the detailed conservation of historical sites to the dynamic realms of parametric and algorithmic designs. When focusing on historic preservation, there is a critical emphasis on achieving utmost accuracy, ensuring every intricate detail is retained. In this context, computer vision emerges as a powerful toolset, offering sophisticated techniques to aid in the preservation and restoration of historical treasures. Leveraging advanced algorithms and methodologies, such as object classification and feature detection, architects and preservationists can analyze and interpret collections of data for a specific element or site. Through object classification, computer vision algorithms can identify and categorize a variety of architectural elements and features present within historical structures, ranging from decorated facades to intricate interior details. By automatically recognizing and cataloging these components, preservation efforts can be streamlined, allowing for more efficient documentation and analysis of the site's architectural heritage. Moreover, feature detection techniques enable the identification of distinctive characteristics within historical imagery and architectural drawings. By establishing correspondences between these features across different data sources, computer vision facilitates the creation of accurate digital reconstructions and 3D models, enabling architects to visualize and analyze the site's evolution over time. By harnessing the capabilities of computer vision in historic preservation work, architects can enhance their ability to document, analyze, and protect our cultural heritage. Through the precise identification and preservation of architectural elements, these advanced technologies can ensure that the legacy of our past remains intact for generations to come. Computers and algorithms altered the architectural design in three major fields:

- *Parametric and algorithmic design*
- *Remodeling and renovating existing buildings*
- *Expanding the building by adding newly designed forms and more*

One of the most notable uses of 3D reconstruction is its ability to enable quick and precise re-modeling and building enlargement. This technology is especially useful for projects that include adding contemporary additions to old or existing buildings. As an example, Figure. 2 presents two well-known instances of building extensions in which traditional buildings have been modernized with computer-aided design. Because 3D reconstruction produces accurate digital models of the existing structures, it assists designers in such situations. Using these precise models as a starting point helps designers and architects combine new ideas with the existing structure in an easy-to-use manner. All aspects of the existing building, including its size, geometrical features, and structural details, are taken into account thanks to the high degree of detail obtained through 3D reconstruc-tion. 3D reconstruction makes the design process more effective and efficient. It makes it possible to analyze and visualize in great detail how new components will interact with the current structure, which can help spot any problems and improve integration.

Figure 2. *Royal Ontario Museum in Toronto, by Daniel Libeskind, Coop Himmelb(l)au's Rooftop Extension on Falkestrasse in Vienna*

Sources: weburbanist.com, dezeen.com

This capacity is especially helpful in maintaining the original building's historical significance and architectural integrity while adding cutting-edge, practical modern additions. Conversely, in parametric design, the requirements fluctuate based on the design's intent and underlying concept. In the dynamic world of parametric design, which is pivotal for contemporary and envisioned future architectural endeavors, designers often find themselves in situations demanding the reshaping, augmentation, or modification of building components or facades. This could be for integrating human-interactive facades, sustainable facade systems, and more. In such scenarios, the foundational structure needs to be shapeable and open to edits. With the grand scale and intricate details associated with architectural designs, a densely detailed mesh comprising millions of elements becomes less manageable, if possible, to modify for subsequent stages. This issue underscores the importance of generating objects that can seamlessly evolve in pair with design requirements.

3D Reconstruction Effects in Architectural Education & Professional Field

Creativity in architectural design compositions can be viewed as an emergence of new forms and shapes or relationships between forms and shapes from which new concepts can be discovered (Reffat 2003). Architects and designers are thus interested in situations where distinctions matter. For this purpose, architects may use or modify existing forms as a basis for their designs. As Leimer et al. (2017) stated, "with online repositories for 3D models like 3D Warehouse becoming more prevalent and growing ever larger, new possibilities have opened up for both experienced and inexperienced users alike. These large collections of shapes can provide inspiration for designers or make it possible to synthesize new shapes by combining different parts from already existing shapes, which can be both easy to learn and a fast way of creating new shapes." However, architects are inspired by a specific image and incorporate the concept into their design to produce a more inventive structure.

Current methodologies enable arduous manual form modification through the utilization of documents or images. Architects often draw inspiration from specific visual depictions or conceptual frameworks, which they then incorporate into their design paradigms to produce more innovative architectural structures. Conventional approaches have predominantly enabled the labor-intensive alteration of forms, frequently relying on manual modifications informed by visual imagery or documentary evidence. On the contrary, the emergence and widespread adoption of Machine Learning technologies have caused a profound transformation in the obligations and functionalities of architects. According to (Penney & Chen 2019), "traditionally, the relationship between computer architecture and ML has been relatively imbalanced, focusing on architectural optimizations to accelerate ML algorithms. In fact, the recent resurgence in AI research is, at least partly, attributed to improved processing capabilities". These emergent technologies facilitate rapid creativity and innovative solutions and accelerate the overall design lifecycle. Moreover, they introduce a forward-thinking approach that enables immediate alterations and the retracing of design decisions, thereby increasing efficiency and adaptability within the architectural process.

An outstanding attribute of these computational methods is their ability to algorithmically and iteratively manipulate geometry. Vale et al. (2003) stated, "engineering design is increasingly being influenced by the proliferation of computer tools in the design environment. CAD systems that previously supported human efforts through design representation and visualization are being supplemented with optimization algorithms that automate various routine design tasks." Instead of utilizing static, immutable designs, students are now capable of creating dynamic, adaptable models that can develop in reaction to various inputs or criteria. The utilization of structure from surface techniques empowers the fabrication of intricate and organic forms, whereas cladding techniques facilitate the seamless incorporation of recurring and modular design components. In addition, deformation permits the twisting, distorting, and transformation of simple shapes, which produces novel and complex design outcomes. As a result, designers may be required to interact with an existing form in numerous scenarios of the design computation process in order to create or redesign structures. A foundational framework is necessary for this to be modifiable throughout the design process. Professional practice, conversely, pertains to punctuality, expenditures, and precision. In circumstances where a redesign aims to modify, enlarge, or renovate the preexisting structure or to incorporate emerging technologies (such as interactive and smart facades, interiors, and so forth), the architect must possess the existing structure, which may be adapted to accommodate the new design. "The main reasons are that architecture design involves complex products, consisting of many distinct parts, and creating and editing these 3D models in 3D space is difficult. Hence, architectural design continues to be done on 2D drawings. However, for various construction applications, such as quantity surveying, inventory, construction, and visualization, it is necessary to convert the widely used 2D architectural drawings to accurate 3D models", according to (Lu et al. 2005). The requirement for a foundational framework that considers labor and time generates a need for the automation of laborious and time-consuming duties using artificial intelligence and geometric algorithms. In conjunction with scholarly and professional developments in deep learning, the algorithm expands the limits of computational and geometric methodologies within architectural research.

Virtual Reality Integration with 3D Reconstruction in Architecture

The research method could be applied in developing accurate, optimized 3D mesh structures that integrate into VR applications, ensuring a seamless and realistic presentation of architectural designs. These immersive experiences transcend traditional static presentations, enabling users to explore, interact, and modify architectural elements in real time. This process offers a human-centric interactive experience in the VR setting, allowing users to seamlessly transition from a 2D representation to an editable and amendable 3D object within the virtual environment. This level of interactivity empowers users to manipulate architectural elements, experiment with different design configurations, and acquire practical insights into the design's spatial relationships and aesthetic qualities. These VR architectural presentations also serve as helpful tools for architects and designers, improving the design process by providing a platform for collaborative exploration and iterative refinement. Design decisions can be made in real time, informed by user feedback from virtual environment experiences.

Furthermore, the adaptability of this approach allows for the integration of additional features and functionalities, such as real-time lighting simulations, material selection, and environmental modeling. This augments the influential experience and gives architects and designers a comprehensive understanding of how their designs will perform in different lighting conditions and contexts. Finally, the integration of the NeRF and 3D optimization methods into VR architectural presentations represents a paradigm shift in architectural visualization, offering outstanding interactivity and versatility. By adopting this innovative approach, architects and designers can collaborate more effectively, make informed design decisions, and ultimately create spaces that resonate with users on a profoundly immersive level.

Evolution of the Computer Vision in Architectural Design

Computer vision plays a multifaceted role in architectural design, influencing different stages from initial conception to final presentation. Beginning with the design strategy phase, computer vision tools facilitate the transition from early sketches to precise lines, enabling architects to trans-late their creative vision into tangible plans more efficiently and accurately. One of the significant contributions of computer vision lies in 3D Reconstruction, which assists in creating detailed 3D models of architectural structures from images or point clouds. This technology allows architects to visualize their designs in a realistic manner and make informed decisions about spatial arrangements and aesthetics. Moreover, object detection and classification, particularly within the context of Building Information Modeling (BIM), revolutionize the design process by automating the identification and categorization of architectural elements and regions. By leveraging these capabilities, both students and professional architects can explore multiple design alternatives efficiently, saving valuable time and resources that would otherwise be spent on manual labor. Furthermore, computer vision enhances the rendering and post-processing phases of architectural design by stream-lining the visualization process and improving the quality of final presentations. The accuracy and efficiency of design workflows are significantly enhanced through auto-detection and auto-classification functionalities, leading to more precise and visually compelling architectural representations. Therefore, the integration of computer vision technologies into architectural design practices not only empowers designers to unleash their creativity but also facilitates the generation of innovative solutions, streamlines workflow procedures, and eventually enhances the quality of architectural outcomes.

Conclusions

The integration of architectural principles with inspirations from philosophy, nature, and digital technology has ushered in a revolutionary period of architectural creativity characterized by cutting-edge innovation, precision, and ecological sustainability. In the future, continuous learning, adaptability, and cross-disciplinary collaboration will be essential to harnessing the full potential of these methodologies, anticipating a future in which architecture parallels our evolving global environment with an emphasis on sustainability. Deciphering three-dimensional data presents both opportunities and complexities within the 3D deep learning domain. The transition from 2D methodologies to 3D deep learning techniques, advantageous across multiple industries, exemplifies the field's dynamic evolution. This paper has explored various 3D reconstruction strategies, each offering distinct advantages for extracting depth from 2D images. With the rise of learning-driven 3D representations and the bridging potential of differentiated visualization, the future of this field appears bright. Given the complexity and immense potential of 3D data analysis, ongoing research, particularly in the field of architecture, is essential. As our journey to comprehend the 3D domain progresses, it becomes clear that the horizons for innovation and understanding are limitless. This promises a transformative future in architectural interactions with the world of three-dimensional deep learning.

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