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Social Discount Rate Dilemmas in Benefit-Cost Analysis

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The choice of discount rate for benefit-cost analysis has been an unresolved dilemma for decades. Discounting at the Social Time Preference Rate will understate capital costs, while discounting at the Social Opportunity Cost Rate may understate future benefits. This article describes a two-rate discounting method that uses the Social Opportunity Cost Rate only to compute capital costs but not to discount future benefits and uses the Social Time Preference Rate to discount future benefits but not to compute capital costs. This removes the need to choose between the two rates, as both are used to arrive at correct results. The feasibility hurdle rate is the Social Opportunity Cost Rate, the Social Time Preference Rate plays no role in this determination. Other questions addressed are the following. The question whether the understatement of capital costs resulting from conventionally discounting at the Social Time Preference Rate can be compensated by generic Shadow Price of Capital or Marginal Cost of Funds adjustments is answered in the negative. The Ramsey equation is a logical tautology that cannot predict Social Time Preference Rates. Weitzman's Declining Discount Rate proposal is based on a fallacy, so there is no reason to consider declining Social Opportunity Cost Rates. However, the declining STPRs that have been proposed can be handled by two-rate discounting. By providing answers to the stated dilemmas or questions, the paper offers a guide to benefit-cost analysts.

Keywords: *Social discount rate; STP discounting; SOC discounting; Two-rate discounting; Shadow Price of Capital; Marginal Cost of Funds. Ramsey Rule. Declining Discount Rates.*

Introduction

The discount rate plays a very important role in benefit-cost analysis (BCA) because it is the hurdle rate of return of project feasibility. It is called the social discount rate because BCA measures the effects of investment decisions on social welfare. If the capital markets in which interest rates are established were undistorted, the equilibrium interest rate would be the natural choice of Social Discount Rate (SDR). However, as capital markets are distorted by taxes, the markets determine two rates: one that borrowers pay and another that lenders receive after payment of taxes. Baumol (1968) observed that efficient resource allocation requires the social discount rate to be the rate paid by borrowers. Yet it is largely the net rate received by savers that determines social time preference between present and future consumption, because that is the rate by reference to which most people adjust their intertemporal consumption path. Setting the SDR to borrowers' interest rate would optimize resource allocation, while setting it to the savers' would be coherent with social preferences. Baumol (1968) concluded "We see now that no optimal rate

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exists. The rate that satisfies the one requirement cannot possibly meet the conditions of the other."

Arrow *et al* (1995) characterized the choices as either the prescriptive (or normative) approach – "How (ethically) should impacts on future generations be valued?" – or alternatively the descriptive or positive approach, which focuses on efficiency, that is, on "trade-offs across time." Following the former, nowadays called the Social Time Preference (STP) approach, means discounting by the Social Time Preference Rate (STPR), while following the latter, nowadays called the Social Opportunity Cost (SOC) approach, means discounting by the Social Opportunity Cost Rate (SOCR).

Choosing a discounting approach is not the only dilemma faced, however. There is also the question of what the rate should be within the chosen category. With constant discount rates, benefits in the distant future become insignificant sooner or later, the sooner the higher the rate. Chichilnisky (1997) thought that a constant discount rate embodies the "tyranny of the present over the future." Ben Groom et al, (2005) stated "The deleterious effects of exponential discounting ensure that projects that benefit generations in the far distant future at the cost of those in the present are less likely to be seen as efficient, even if the benefits are substantial in future value terms." According to Weitzman (1998) "Few are the economists who have not sensed in their heart of hearts that something is amiss about treating a distant future event as just another term to be discounted away ..." This observation moved him to propose a method of defining discount rates that decline in time. Groom et al (2022) report on proposals that elaborate on the Ramsey rule, which is frequently used to propose STPR values, and would result in declining discount rates.

It is clear that STP discounting leads to underestimating capital costs given that STPR<SOCR. Marglin (1963) proposed to remedy this through a shadow price of capital (SPC) adjustment. The OECD CBA Manual (2018:221) states that this seldom happens: "Using the Shadow Price of Capital approach (SPC) is advisable when using the [STPR], so that the opportunity cost of public capital can be reflected in the NPV calculation. This rarely happens in practice due to onerous informational requirements." Groom *et al* (2022) reported that an alternative remedy to overcome the undervaluation of capital costs could be "to evaluate the welfare cost of the marginal taxation required to fund public investment via the taxes raised and the distortions it introduces." This is called the adjustment for the marginal cost of public funds (MCF).

This paper proposes a discounting method that resolves the discounting approach dilemma and addresses the other issues mentioned above. Section 2 explains the two-rate discounting procedure that uses both the SOCR and the STPR. Section 3 questions the adequacy of the SPC adjustment. Section 4 answers the same question concerning the MCF adjustment. Section 5 reviews the nature of the SDRs. Section 6 reviews Weitzman's Declining Discount Rates (DDR). Section 7 offers a guide for the appropriate selection of discount rates. Section 8 presents conclusions.

Two-rate Discounting

To solve the discounting approach dilemma this paper explains how the two functions of discounting (imputing capital costs and discounting future benefits) can be performed in distinct steps. This was first proposed by Szekeres (2022) and refined in Szekeres (2026). In the first step, the SOCR is used to calculate the opportunity costs of capital (the interest cost that projects incur until they finish amortizing the capital they use) but not for discounting subsequent future net benefits. In the second step net benefits after capital costs are discounted using the STPR. The proposed two-rate discounting method correctly values both capital costs and future net benefits. The dilemma faced hitherto was: should one discount at the SOCR and accept the understatement of benefits in the distant future, or discount at the STPR and accept that capital costs will be understated. With two-rate discounting, this choice does not need to be made, as both rates are used in the role for which they are intended and neither error is committed. This largely solves the dilemma, because using two rate discounting there should be unanimous agreement on project feasibility, as no one wants welfare destroying projects. The choice of STPR, however, reflects social objectives, so no unanimous agreement about its value is expected. But unambiguous guidance can be given for the choice between alternative types of STPR.

We call the primary flow of an investment project the set of net yearly values that results from subtracting investment and operating costs from benefits. This is the flow that is normally discounted to obtain NPVs. The interest costs of the funding needed to cover the negative values that are typical of the early periods of projects are not included in primary flows because the process of discounting imputes them implicitly, so including them explicitly would result in double counting. However, the point of two-rate discounting is precisely to make these interest costs explicit. How this is done will be shown with the help of Figures 1-3.

Figure 1 shows the primary net flow of a hypothetical project. This cash flow is negative in years 0-2 and positive thereafter.

Figure 1. *Primary Net Flow of a Project*

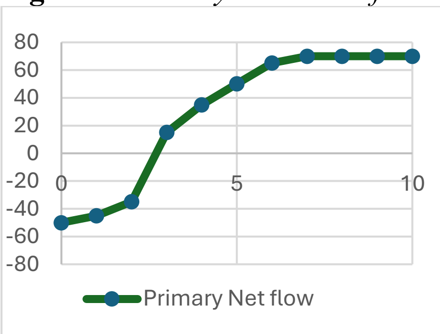


Figure 2 shows the funding flow of the project, which is derived from the primary flow and the interest rate that measures the opportunity cost of capital. Capital is needed to fund the negative cash flow of Years 0-2. While the primary flow is negative, the funding flow will be positive and equal to the negative of the primary flow. Total capital employed will grow faster than the accumulation of disbursements, however, because interest on the outstanding balances is capitalized. Once the primary flow turns positive, in Year 3 of this sample project, the funding flow turns negative, indicating that repayment of capital has begun. As all positive net benefits are devoted to repaying the outstanding capital stock and accrued interests, the funding flow is again the negative of the primary flow. The outstanding balance declines more slowly than the accumulation of repayments because fresh interest is capitalized as long as the outstanding balance is positive. The funding flow becomes 0 after all capital and accrued interest have been amortized.

Figure 1. *Funding Flow derived from the Primary Flow*

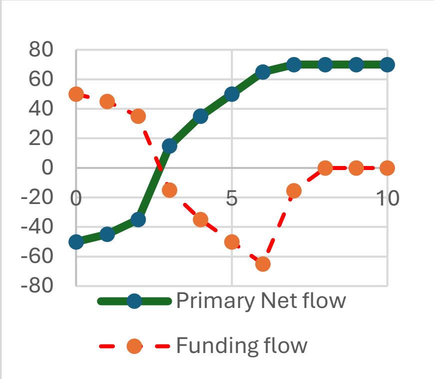
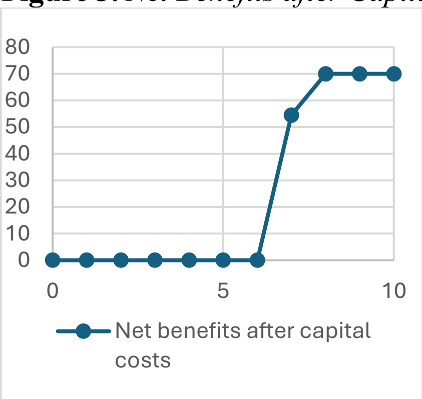


Figure 3 Shows net benefits after capital costs, which are obtained by adding the two flows shown in Figure 2. As the funding flow is always the negative of the primary flow, the net result is always zero until capital is fully amortized. Notice that this flow is still undiscounted. The SOCR used to compute the opportunity cost of capital is not used for discounting. It is just used for calculating the compound interest accrued on the capital employed. The net flow of Figure 3 is what needs to be discounted to obtain the NPV of the project. This is done using the second rate specified in two-rate discounting, which in our case is the STPR. There is no double counting of interest costs, despite interest costs having been explicitly included in the previous step because capital costs are absent from this flow. Only future net benefits after all cost are discounted by the STPR.

Figure 3. *Net Benefits after Capital Costs*



The two-rate NPV can be computed directly from the project's conventional net flow by compounding it forward at the SOCR while the cumulative compounded value is negative, thereafter doing so at the STPR until reaching the time horizon. Discounting the net future value (NFV) so obtained at the STPR yields the two-rate NPV. A project will have a positive welfare impact if, as in Figure 3, there are positive undiscounted net benefits after capital costs. Then their NFV will be positive, or, equivalently, the NPV of this NFV, discounted at the STPR, will be positive.

This makes it clear that the SOCR is the hurdle feasibility rate, for it is by reference to the SOCR that we must determine if net benefits exist. This will be evident even before discounting. The rate at which we discount any such benefits to obtain an NPV is immaterial to the feasibility determination.

Two-rate discounting accurately splits conventional single rate discounting into its two components: imputation of capital costs and intertemporal weighing. We can ascertain that this is true by computing any two-rate NPV with SOCR = STPR = *r*. The result will be the same as obtained by conventionally discounting the primary flow using rate *r*. This is necessarily so because two-rate discounting works by compounding the primary cashflow at different rates to obtain a NFV and then discounts that NFV at one of the rates. If r is used for compounding the primary flow to obtain the NFV and *r* is used for discounting the NFV, the result will be the same as discounting the primary flow at *r*. The congruence between NPV and NFV ensures this.

Formally, the recursive process described above operates as follows (Szekeres, 2026). C_i is the cumulative compound value of the project primary net flow for period *i*:

$$Ci = Ci\text{-}1\ (1{+}ri) + bi \qquad (1)$$

where *i* is the time-period index ranging between 0 and time horizon *t*.

b_i = primary net flow for period *i*
r_i = SOCR if $C_{i-1} < 0$
r_i = STPR if $C_{i-1} \geq 0$
$r_0 = 0$
$C_{-1} = 0$

The FV of the project is *Ct*, which is obtained by recursively applying expression (1) from years 1 to *t*. A project will have a positive welfare impact if its FV is positive, or, equivalently, if the PV of this FV is positive. This method works well for projects that require capital more than once, such as those needing periodic large reinvestments that make their primary flows turn negative repeatedly. Two rate discounting can be used whenever NPVs need to be computed. A practical application of two-rate discounting is shown in the Appendix.

Shadow Price of Capital

To correct for the understatement of capital costs under STP discounting, Marglin (1963) proposed an adjustment by an SPC factor that would multiply the investment costs of projects. To illustrate this, he assumed a project with infinite life that has capital costs *K* and yearly net operational benefits *b* accruing in perpetuity. In computing the present value of this project, Marglin's key insight was to "replace the money cost" *K* of the project by its future yearly opportunity cost $K \times SOCR$. The project's NPV therefore is:

$$NPV = \frac{b}{STPR} - \frac{K \times SOCR}{STPR} \tag{2}$$

This expression states the assumption that committing capital K forever to the project incurs an opportunity cost of $K \times SOCR$ every year, forever. Incidentally, this shows that both the STPR and the SOCR are needed to calculate the NPV of the project. Neither rate can do it by itself.

We can interpret the ratio *SOCR/STPR* as the SPC adjustment factor. After having applied it by multiplication of *K*, it appears that the STPR can compute a correct NPV by itself, because it is the sole rate used for discounting benefits. This ignores the fact, however, that the condition for *NPV* being positive is:

$$\frac{b}{STPR} > \frac{K \times SOCR}{STPR} \tag{3}$$

which reduces to $b > K \times SOCR$. This means that the feasibility hurdle rate of return for projects is the SOCR. The STPR is irrelevant to this question. This is logical, as projects that fail to cover the opportunity costs of the capital they use are welfare destroying, a fact that is unaffected by the value of STPR.

Reformulating expression (2) as follows

$$NPV = \frac{b - K \times SOCR}{STPR} \tag{4}$$

we see that Marglin's NPV is in principle the same as two-rate discounting (net benefits after capital costs computed using the SOCR are discounted by the STPR) but for the fact that Marglin assumed that capital would be committed forever to the project, while two-rate discounting assumes that capital is amortized as fast as possible. A simple numerical example shows the difference in outcomes. If in expression (2) we assume that SOCR= 5%, STPR= 2%, K =1 and b = 0.07, then NPV = (0.07/0.02) – (0.05/0.02) = 1. However, using a standard continuous compounding amortization formula, we can determine that K can be amortized in 26.64 years. This leaves an after capital-costs net flow of zero during the first 26.64 years and of \$0.07 thereafter. Discounting this after capital costs net benefit flow, NPV = (0.07/0.02) / exp(26.64 × 0.02) = 2.05, much higher than the result obtained from expression (2).

Expression (4) is excellent for illustrating the basic principle of two-rate discounting, but the infinitely long commitment of capital implicit in it is not realistic. Cline (1992) recognized this and proposed an SPC adjustment that assumes even capital repayment over the life of the project, as quoted in the OECD CBA Manual (2018:207). While that is far more realistic, it is still not what corresponds to conventional discounting, which assumes that project benefits are first devoted to capital amortization, that is, that amortization is as fast as possible.

Many authors, e.g. Newell *et al* (2023), kept Marglin's infinite time horizon assumption, however, and tried to define a SPC correction with economy wide applicability by tracing the future welfare impact of $1 not devoted to investment. This procedure is extremely complex and error prone[1] and regrettably it is not useful in BCA, because it ignores the actual capital usage of specific projects. The implicit eternal capital commitment assumption means that there is no need to repay capital, a fatal flaw.

For the above simple numerical example, with the perpetual capital commitment assumption, SPC = *SOCR/STPR* = (0.05/0.02) = 2.5. However, when Marglin's conceptual definition of SPC is adapted to the actual capital usage of this project, the SPC is computed as the present value of the fastest possible capital amortization, discounted at the STPR, as shown in Szekeres (2026):

$$SPC = \int_0^{26.64} 0.07\, e^{-0.02\, t}\, dt = 1.45 \quad (5)$$

Conceptually, the SPC is the present value of the opportunity cost of capital, discounted at the STPR. Using this SPC value, we have NPV = 0.07/0.02 – 1.45 = 2.05. This is the same result as was obtained above by two-rate discounting. Therefore, the correct SPC factor for any specific project can always be derived from the result of the two-rate NPV calculation.

Total capital cost is composed of the investment amount that needs to be amortized and of the interest earned by capital while committed to the project. The opportunity cost of capital (the interest) is not only a function of the investment made and the SOCR that defines the applicable interest rate, but also of the timing of the benefits that can amortize capital costs. This is the reason why no generic factor that multiplies one of the components of total capital costs (investments) can hope to measure correctly the entirety of capital costs. A correct SPC value can always be derived for any project from the NPV computed by two-rate discounting, but it would only be valid for that single project, so it is not worth calculating.

Marginal Cost of Public Funds

According to Groom *et al* (2022) some analysts believe that the Marginal Cost of Public Funds (MCF) can perform the corrective adjustment that STP discounting

[1]Newell et al (2023): "The SPC is defined as the change in immediate consumption equivalent to the present value of the stream of consumption losses associated with the immediate displacement of $1 of capital, discounted at the consumption discount rate. Computing this requires considering the effect that such an immediate displacement of capital would have on consumption over time, given savings rates, investment returns, and depreciation." "The degree of the displacement's persistence is determined by broad economic equilibrium dynamics, including depreciation and savings, suggesting that the SPC should be guided by macro-derived models of savings and investment."

needs to overcome its understatement of capital costs. The MCF measures the costs of raising tax revenues, including the welfare effects of the distortion that taxing causes. For a survey of the literature on this subject see Massiani and Picco (2013).

Harberger (2007) explained how the MCF adjustment should be applied in BCA. "The cleanest, most straightforward way to take tax financing and the excess burden associated with it into account is to apply an extra charge or benefit of λ to each and every cash outflow or cash inflow from and to the public treasury, over the life of the project." In other words, whenever appropriate, costs and benefits of projects should be multiplied by (1+ λ). An example of when the costs of a project should be so adjusted is when its benefits are not collected in fees and its costs are tax funded. An example of when benefits should be so adjusted is when the project generates revenues for the public treasury.

The MCF adjustment alters the project flow, which still needs to be discounted to compute an NPV. The MCF cannot measure the cost of capital because capital is a stock, the cost of which can only be measured by an interest rate, such as the SOCR, and not by a factor, such as (1+ λ). Modifying the flow that gives rise to a stock says nothing about the cost of using that stock. In conclusion, the MCF adjustment cannot correct the underestimation of capital costs implicit in conventional STP discounting.

Social Discount Rates

A General Equilibrium Capital Market Model

Szekeres (2022) built a computable capital-market general equilibrium model that will illustrate some of the concepts discussed in this paper. It will be briefly described in this section. It is a two-period, agent-based model, with 58 agents. All have constant elasticity of substitution utility functions with coefficients of risk aversion equal to 1. Half of them have Year 0 income that follows the distribution of Figure 4 and Year 1 income that is 2% higher than that of Year 0. The other half have the same Year 0 income distribution, but Year 1 income that is 2% lower than that of Year 0. The agents optimize their consumption path by borrowing, lending, or investing in an equity with stochastic yield (an even chance of a 40% gain or a 20% loss, with an expected yield of 10%), the supply of which is infinitely elastic. Given the time profile of their incomes, half of the agents are inclined to borrow, and the other half are inclined to lend. Figure 5 describes the market for loans; Figure 6 describes investment behavior. We can see by comparing Figures 5 and 6 that at higher interest rate savers lend more and invest less. The model finds the equilibrium market rate of interest (3.28%) at which lending equals borrowing ($27,200) and establishes the amount invested in equities ($45,988). There is a 40% tax to be paid on interest income and dividends.

Figure 4. *Income Distribution of Agents*

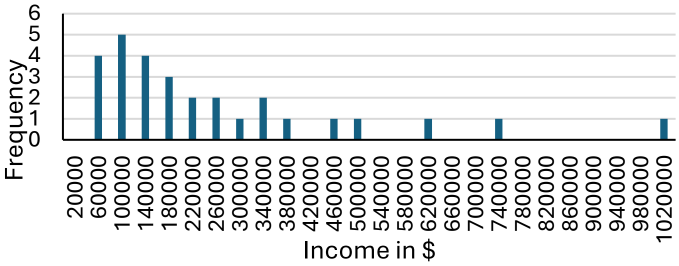


Figure 5. *Lending and borrowing as a Function of Interest Rate*

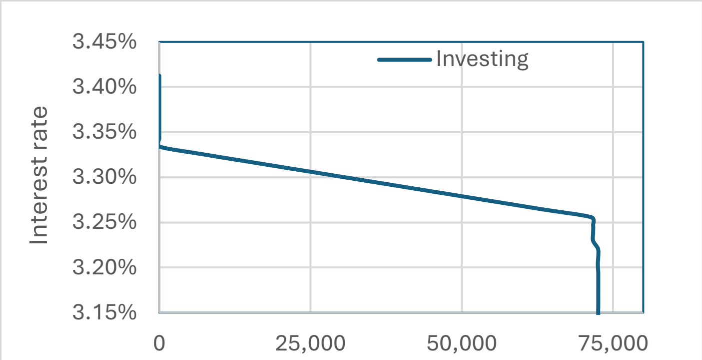
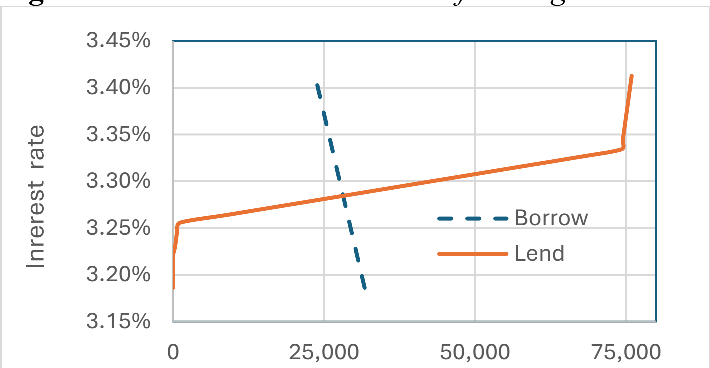


Figure 6. *Investment as a Function of lending Market Interest Rate*



The model calculates endogenously both the STPR and the SOCR. It can simulate the effect of financing a public sector project through bonds that agents subscribe to. The net benefits of the project are distributed equally among all agents (or if negative, collected as a lump sum tax).

The model compared the NPV of several sample projects calculated using the two-rate discounting method described in Section 2 to the aggregated amount that agents would be willing to pay collectively for the projects' future net benefits. The model records the level of expected utility attained by each agent after consumption path optimization for both the with-project and the without-project situations. After this, taking the without-project expected utilities as a benchmark, the model calculates how much additional Year 0 income would make each agent's expected utility equal to that of the with-project situation. The aggregate welfare improvement attributable to the project is the aggregation of these compensating variations. For all tested sample projects, the aggregate welfare gain (or loss) coincided with the NPV's computed using the two-rate NPV calculation method. This congruence is the consequence of discounting with the endogenous STPR, the calculation of which is explained in the next section.

The Social Time Preference Rate

Harberger (1972) assumed that "the 'social rate of time preference' refers to an appropriately weighted average of the different marginal rates of time preference applicable to the individuals who compose the society." The model described in section 5.1 above computes the STPR by finding the compensating variation payment that must be given to agents in Year 0 so that their utilities are the same as when they receive a very small payment in Year 1, after agents have optimized their consumption path. From the aggregation of these values the STPR can be computed. This rate turns out to equal the weighted average of the rates implicit in the agents' MRS between consumption in Year 0 and Year 1. The value obtained was 2.62%, which is a weighted average of 3.28% (the market interest rate for borrowers) and 1,97% (the market interest rate net of taxes for lenders). It is interesting to note that this is so irrespective of the investment in equities that agents may also have made.

The STPR so calculated, which corresponds exactly to Harberger's (1972) cited description, could be called *the revealed preference* STPR and is the only one that will compute NPVs that are congruent with the social valuation of future benefits. It is because this is the rate used for discounting net benefits in the model that the computed NPVs agree with the aggregated equivalent compensating variation of agents. Any other STPR value would either underestimate or overestimate the present value of future benefits from the point of view of aggregate welfare as measured by the affected people's revealed preferences.

This is not how Arrow *et al* (1995) defined the STP approach, however. Rather, their stance was based on normative or ethical considerations. Perhaps all definitions that depart from Harberger's (1972) description should be called normative, because they replace the revealed preferences of individuals by the preference of those empowered to know better, be they politicians, social planners, or anyone else with the authority to decide. Two surveys of the field of social discounting provide a comprehensive overview of possible normative STPRs: Greaves (2017) and Groom *et al* (2022).

The Ramsey Rule is one method of defining the STPR that is often used according to both surveys. Groom *et al* (2022) characterizes the simple Ramsey Rule as a "workhorse" model for social discounting. Greaves (2017) states that "The standard approach to determining the discount rate is via *the Ramsey equation*," which takes the following form[2]:

$$r = \delta + \eta g \tag{6}$$

[2]See Greaves (2017) for a derivation.

where *r* is the market interest rate, *δ* is the pure rate of time preference, *η* is the constant elasticity of marginal utility of consumption, which can also be interpreted as the coefficient of risk aversion, and *g* is the growth of consumption.

Economists have used this expression to estimate the STPR. Greaves (2017) reports that Stern assumed $\delta = 0.1\%$, $\eta = 1$ and $g = 1.3\%$ to arrive at $r = 1.4\%$, whereas Nordhaus (2019) states that "I adopt a time discount rate of 1.5 percent per year with a consumption elasticity of 2. These yield an equilibrium real interest rate of 5.5 percent per year with the consumption growth that is projected over the next century by the DICE-2007 model."

The Ramsey formula may have become the "workhorse" model for social discounting, but it is problematic because it is a circular definition. Its derivation is based on the premise that consumption path optimization has taken place, *r* is therefore already known, and *g* is a consequence of this optimization, not an exogenous variable. The Ramsey formula is a logical tautology that is true for any point along a saver's indifference curve, provided it is tangent to an exogenously determined *r*, for only then will expression (6) be true. Consequently, the Ramsey formula is not a behavioral function that can be used to determine an unknown *r*.

Figure 7 depicts the aggregated indifference curve of all 58 agents in the capital market model. In Year 0 the aggregate income of all agents is $15m. Ramsey equation (6) is true of all points of this indifference curve.

Figure 7. *Indifference Cure of the Entire Market*

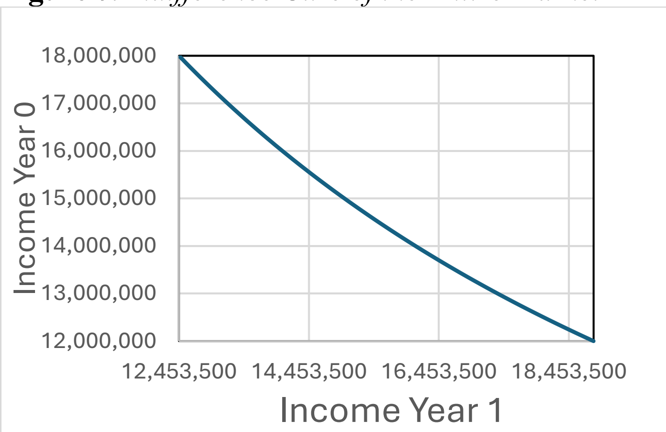


Table 1 shows selected values from the indifference curve of Figure 7. For example, if $r = 15.31\%$ then *g* will become 13.31 %. This interest rate induces the representative agent to save to increase future consumption. If r=2%, there will be no consumption reallocation between periods, so $g = 0$. If $r = -9.1\%$ the representative agent will find it expeditious to borrow and reduce future consumption so $g = -11.1\%$. It is not *r* that is the dependent variable, it is *g*. It is clearly fallacious to pick a growth rate of consumption exogenously and calculate a market interest rate from it.

Table 1. *Selected Incomes, g and r*

Year 0	Year 1	*g*	*r*
17 100 000	13 123 457	-23.25%	-21.25%
15 900 000	14 134 479	-11.10%	-9.10%
15 000 000	15 000 000	0.00%	2.00%
14 100 000	15 977 216	13.31%	15.31%
13 050 000	17 289 464	32.49%	34.49%

The closest we could come to use economic growth as an exogenous variable would be to change Year 1 income. Increasing Year 1 incomes of all agents in the model by 0.5% only resulted in a growth of consumption of 0.03% because consumption smoothing by agents caused them to also increase their Year 0 consumption. The actual value of *g* turned out to be much lower than that of the income growth that induced it.

A moment of reflection should suffice to realize that no expression containing data only about one side of the market, the supply of savings, carries enough information to determine the market interest rate (or set of distorted market rates). For that, information about the demand for loanable funds is also necessary, not least of which is the rate of technological progress that drives the productivity of investments.

The Ramsey rule, therefore, cannot be used to estimate real-world STPRs, however it might serve to formulate normative STPRs.

The Social Opportunity Cost Rate

Harberger (1972), in writing on measuring the social opportunity cost of public funds, defines what we call the SOCR as the weighted average of the net-of-tax yield of savings and the gross-of-tax cost of borrowing paid by investors, with the weights being proportional to their respective elasticities of supply and demand. The welfare cost of funding a public sector project through borrowing can be calculated as a weighted average of the two cited rates and constitutes the SOCR. Burgess and Zerbe (2011) expanded Harberger's formulation by adding foreign borrowing as a third source of funds, thereby adding a third rate and a third weight to the formula.

Table 2. *SOCR and Effective Interest Cost*

OPPORTUNITY COST		
Forgone equity profit		$94.11
Interest opportunity cost		$1.21
Compensating variation		$0.11
Total welfare cost		**$95.43**
	SOCR	**9.54%**
COST OF FINANCE		
Interest payment		$32.84
Taxes forgone		$62.52
Total cost		**$95.37**
	Effective interest rate	**9.54%**

The model described in previous section adds further elements to the Harberger formulation to arrive at the endogenous SOCR. As in the model some agents borrow for consumption smoothing, funding a public sector project also displaces some consumer borrowing, the opportunity cost of which is the market interest rate (3.28%), which is lower than the opportunity cost of reducing investment in higher yield equities (10%). This recognizes that the capital market is segmented. Public fundraising in the bond market affects investment in the stock market. Further, a compensating variation value is computed that leaves all agents with the same expected utility that they would have had absent public sector borrowing.

Table 2 shows the consequences of the public sector borrowing $1,000 in the model. A decline of $941.1 in equity investments results in an opportunity cost of $94.11. A decline in private borrowing of $37 has an opportunity cost of $1.21. The computed compensating variation of $0.11 is small, which interestingly shows that agents are compensated nearly in full for the consequences of public borrowing by adapting their consumption path to the new market realities. Because in the model the supply of equities (which is demand for funds for investment purposes) is infinitely elastic, the weighted average SOCR (9.54%) is pulled strongly upwards.

The model corroborates a statement by Burgess and Zerbe (2013): “the [SOCR] criterion measures the impact of the project on the government’s budget...” In Table 2 the sum of the effects that define the SOCR nearly exactly corresponds to the sum of two financial items: the interest paid by the public sector on the amount it borrowed ($32.84), and the amount of taxes forgone because of its intervention in the capital market ($62.54), making its effective interest rate equal to the SOCR. Thus, the result of a welfare impact calculation equals the computed out-of-pocket expense of the public sector, making the opportunity costs of capital very tangible. Harberger (2007) explains and illustrates this phenomenon with a numerical example.

Declining Discount Rates

The perception that discounting shortchanges future generations has led to the search for discounting methods that avoid the fast decline in time of the present value of future benefits that exponential discounting entails. A number of rationales have been proposed that result in declining discount rates (DDR). Because two-rate discounting uses two rates, we address this issue separately for the STPR and the SOCR.

For STP discounting, Groom *et al* (2022) report extensively on ways in which a social planner might arrive at DDRs by considering factors such as risk aversion, prudence, intergenerational inequality aversion, etc. All of the proposed methods are conceptual, rather than based on empirical fact, and fall into our normative STPR category. For this reason, these methods will not be discussed further. As will be seen in the next section, BCA analysts may choose to use a normative STPR as the second rate to be used in two-rate discounting, following the policy guidance to which they may be subject.

Concerning SOC discounting, Martin L. Weitzman (1998) claimed that if future interest rates are uncertain and autocorrelated, then risk neutral certainty equivalent interest rates decline over time. As market interest rates are the foundation of any estimation of the SOCR, Weitzman effectively argued for declining SOCRs. Weitzman (2001) defined the expected value of the discount factor for a future benefit of $1 at time *t* as follows[3]:

$$A(t) \stackrel{\text{def}}{=} \sum p_i e^{-r_i t} \tag{7}$$

where *t* is a future time period, r_i is the interest rate random variable and p_i are the probabilities of the r_i. Expression (7) is simply probability-weights the deterministic discount factor e^{-rt}. This is why it was unquestioningly accepted by nearly all. The certainty equivalent discount rate that can be derived from (7) is the following:

$$r^w(t) = -\left(1/_t\right)\ ln\left(\sum_i p_i e^{-r_i t}\right) \tag{8}$$

which is a declining function of time. In the above, r^w is Weitzman's certainty equivalent rate assuming risk neutrality. We can see that *A(t)* is a declining function of *t* because the higher the *t*, the lower the present values of $1 that are computed at higher interest rates. This pushes the value of *A(t)* downwards in the weighted average computation, and likewise the value of r^w. At the limit, as *t* grows, r^w will converge to the lowest possible r_i. This is how we get DDRs.

Gollier (2004) pointed out, thereby launching the Weitzman-Gollier puzzle, that if the same procedure is applied to future value factors, to compute the expected future value (EFV) of $1, the opposite conclusion follows.

$$EFV(t) \stackrel{\text{def}}{=} \sum p_i e^{r_i t} \tag{9}$$

$$r^*(t) = \left(1/_t\right)\ ln\left(\sum_i p_i e^{r_i t}\right) \tag{10}$$

The certainty equivalent rate r* is an increasing function of time by the same logic. Future values computed at lower interest rates will be lower than those computed at higher rates, pushing the weighted average upwards, taking *r** with it. At the limit *r** tends towards the highest possible r_i.

A vast literature emerged to deal with the Weitzman-Gollier puzzle, with diverse theories and explanations to bridge the disturbing gap between alternative conclusions. Eventually Gollier ended up endorsing DDRs and many jurisdictions adopted the policy of using DDRs for very long-lived projects. Groom and Hepburn (2017) observed: "The successful deployment and dissemination of DDRs suggests that, for better or worse, academic economists can enslave practical men with economic ideas."

For worse, as it turns out, for Weitzman's DDR derives from a fallacy.

Weitzman's *A(t)*, expression (7), will not compute the correct expected present value (EPV) of a future $1 available at time t. The correct EPV of a future $1 can

[3]In this expression Weitzman's continuous probability density function of interest rates was converted to a finite set of interest rates $\{r_i\}$ with probability $\{p_i\}$

be derived directly from the textbook definition of present value. It is the amount that will compound to the expected future value (EFV) of $1 at the going (stochastic) market rate *r*. If *EFV(t) =1,* then

$$EPV(t)\ \sum p_i e^{r_i t} \overset{\text{def}}{=} 1 \tag{11}$$

It is a fact of mathematics that the correct EPV derived from (11) is the following:

$$EPV(t) = \frac{1}{\sum p_i e^{r_i t}} \tag{12}$$

The correct *EPV(t),* expression (12), is different from Weitzman's *A(t),* as defined in (7):

$$EPV(t) \overset{\text{def}}{=} \frac{1}{\sum p_i e^{r_i t}} \neq \sum p_i e^{-r_i t} \overset{\text{def}}{=} \text{A(t)} \tag{13}$$

The trap into which Weitzman fell, along with everyone else (this author included, for a very long time) was not realizing that Weitzman's *A(t)*, which is the expected value of the inverses of the compound factors, is not equal to the inverse of the expected compound factor (which gives the correct EPV). Asserting that the expected value of a function (the inverse) is the function (the inverse) of the expected value of its argument is a fallacy. The difference between the two can be calculated from the following well-known statistical relationship:

$$E[XY] = E[X]E[Y] + cov(X,Y) \tag{14}$$

where *E[·]* is the expectation operator and *X, Y* are random variables. Let us define random variable *X* to stand for e^{rt} and random variable *Y* to stand for $1/e^{rt}$, where *r* is the random interest rate. Given that *E[XY] = 1*, because *Y* is the reciprocal of *X,* we can rearrange (13) as follows:

$$E[Y] = \frac{1 - cov(X,Y)}{E[X]} \tag{15}$$

which means that

$$\sum p_i e^{-r_i t} = \frac{1 - cov(\, e^{rt},\ e^{-rt})}{\sum p_i e^{r_i t}} \tag{16}$$

We can see from (16) that Weitzman's PV calculation (the left-hand side of the above expression) is only correct if the correlation of interest rates is zero[4], in which

[4]The covariance in (15) is between compound and discount factors at time *t*. If interest rates are uncorrelated, then the certainty equivalent r* and r^w will be the same, and the covariance between the discount and compound factors will be zero.

case the right-hand side becomes the correct EPV of \$1 at time *t*. This is the true explanation of the Weitzman-Gollier puzzle.

Correcting Weitzman's error results in increasing certainty equivalent discount rates, rather than declining ones, if interest rates are autocorrelated. See Szekeres (2020) for an empirical demonstration using Weitzman's own data. There is no reason to believe, therefore, that SOCRs would be declining and tending to the lowest possible value through time.

This does not mean, of course, that one should ignore the possibility of the SOCR changing over time. The relevance of this question depends on the benefits of the project, however. For economically feasible projects, capital might not be needed for all that long, and there is seldom much uncertainty about the cost of the capital that is about to be committed to a project. If IRR>SOCR, then the initial capital investment will be paid off before the end of the life of the project. If this happens in reasonable time, comparable to the maturities that market players typically handle, then the SOCR derived from their actions is a good guide that does not need revising[5]. If a project takes a very long time to break even, then obviously explicit or implicit refinancing of the initially committed capital stock will be necessary, and a forecast of the SOCR for the appropriate time period will be required. Computing capital costs with variable SOCRs is trivial. Projects that never break even (IRR<SOCR), however, should not be undertaken to begin with.

We conclude that DDRs do not apply to the first step of two-rate discounting, in which the SOCR is used to calculate the capital costs of projects. In the second step, that of net benefits discounting, however, two-rate discounting can handle any normative STPR that the analyst chooses to use, be it a declining DDR schedule, or even STPR = 0, which would reflect complete equality between generations.

The Choice of Discount Rates

The objective of BCA is to determine whether project benefits exceed project costs. The first step of two rate discounting makes the capital costs of projects explicit and adds them to projects' primary flows. This leaves after capital costs project flows with initial zero values followed by positive ones, for projects that have benefits in excess of all costs (see Figure 3), or alternatively, all zeroes until the project time horizon and a negative residual value (the unamortized balance of capital costs). In the latter case project benefits are insufficient to cover project costs. Because capital costs are computed using the SOCR, there will be no disagreement about project feasibility provided there is agreement on the SOCR used. Two rate discounting will therefore help BCA fulfill its objective.

The preceding shows that the SOCR is the feasibility hurdle rate and that there is no choice but to use it for the purpose of feasibility determination. The STPR plays no role in establishing project feasibility.

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[5]For reasonable SOCR values projects that are unable to covert their capital costs in a few decades at most are likely to be unfeasible. The uncertainty about future SOCR over that time frame is manageable.

For feasible projects, however, the choice of the rate at which to discount positive net benefits remains open. The choice to be made depends on the objective of the BCA being conducted:

1. If the objective is to measure the welfare impact of projects as valued by the beneficiaries themselves, then what we have called *the revealed preference* STPR in Section 5.2 should be used to discount benefits. This is what would be congruent with a BCA that defines consumption as the numeraire.
2. If the objective is to maximize the benefits attainable from a public sector investment budget, then the SOCR should be used instead because that will result in the highest possible volume of future benefits obtainable from a given budget. This would mean using two-rate discounting with the same rate in both roles, which would be unnecessary, because doing so is equivalent to single-rate discounting at the SOCR. This is what would be congruent with a BCA that uses the cost of public funds as the numeraire.
3. Finally, if the objective is to fulfill social preferences as expressed by a normative STPR, then that rate should be chosen for the second step in two-rate discounting.

Consequently, knowing the objective of the BCA being performed, the choice of rates to be used in two-rate discounting will be unambiguous, which completely resolves the choice of discount rate dilemma.

Conclusions

As conventional discounting both attributes capital costs and discounts future benefits at the same rate, having to choose between undervaluing capital costs and undervaluing future benefits posed the discounting dilemma. While in theory the SPC correction could correct for the undervaluation of capital costs, this is rarely undertaken in practice or very likely yields wrong results when it is. Two-rate discounting eliminates the need to choose between alternative rates. It uses the SOCR to measure capital costs fully and accurately and the STPR to value future benefits according to the desired social time preferences.

What two-rate discounting does is to make project capital costs explicit. Capital is thus recognized as just another project input that needs to be valued at its shadow price, the SOCR. Failure to do so invalidates the BCA results, just like undervaluing any other cost would. This makes it clear that the SOCR is the project feasibility hurdle rate, because projects that are unable to cover their capital costs do not generate net benefits, just welfare losses, measured by the unamortized capital left at the time horizon of the analysis.

As the full accounting of all projects cost is essential to obtaining correct BCA results, there should be universal agreement on project feasibility. Feasible projects will generate benefits after their full payback period, which need to be discounted to obtain a project NPV. The SOCR plays no role in discounting benefits. For that a STPR must be chosen that suits the objective of the analysis being undertaken: it could be a revealed preference one, in congruence with the willingness to pay principle of BCA; it could be the SOCR, to maximize the impact of an investment budget; or it might be an ethically mandated normative one, perhaps even be zero. Two rate discounting eliminates the discount rate dilemma: the SOCR *must* be used to compute capital costs and the choice of STPR derives directly from the objective of the analysis.

The use of the SPC correction is unnecessary, because two-rate discounting is more accurate and simpler to use. Common SPC estimation methods have onerous data requirements and because they are generally based on tracing the alternative fate of a hypothetical $1 not devoted to a project over an infinite time horizon they disregard the capital needs of specific projects. Consequently, such methods are inaccurate. A SPC value can be derived from any project's primary flow and the SOCR, but it would be project specific and would lead to the same result as two-rate discounting, which is easier to apply.

The MCF measures the cost of raising public revenues, which constitute a flow. For this reason, the MCF cannot be used to measure capital costs. Capital used in a project is a stock, and its yearly costs are a function of the volume of the committed stock and the SOCR, which is a rate.

The Ramsey rule is a logical tautology that is true for any point along the representative consumer's indifference curve. The growth of consumption is a consequence of consumption path optimization given a known market interest rate, not an independent variable that determines the interest rate. Therefore, the Ramsey formula cannot be used to estimate real-world STPRs, however it might serve to formulate normative STPRs.

There is no valid reason for assuming declining SOCR rates, as suggested by Weitzman's theory of DDRs, which is based on a fallacy. However, the relevant literature has formulated declining normative STPRs. Two-rate discounting can accommodate their use, should that be the objective of the BCA being performed.

Two-rate discounting easily and accurately measures the NPV of projects in a manner that is consistent with the values of the SOCR and the STPR, a set of social parameters that can be chosen unambiguously if the objectives of the BCA are clear. Obtaining correct BCA results through this method will help avoid undertaking projects that diminish future generations' welfare. As Becker, Murphy and Topel (2011) stated: "Future generations would not thank us for investing in a low-return project."

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Appendix[6]

In this Appendix the two-rate discounting method is illustrated with data from a real-life project. The data come from a feasibility study performed by Information for Investment Decisions, Inc., of Washington, DC, in August 1979. The project was the development of the Cerro Colorado copper mine. The plan was to exploit copper deposits found in the Chiriquí province of Panama that were estimated to contain 1.4 billion tons of ore with a tenor of 0.785%. The project included the following components: mine development, ore concentration plan, smelter and sulfuric acid plant, in addition to infrastructure such as roads, a seaport and a water collection dam. Total capital requirements were estimated to reach nearly $2 billion. The study was conducted using Monte Carlo simulation to quantify uncertainty about the yield of the mine, investment and operating costs and the forecast of copper prices. Values of the primary project flow shown in this Appendix are expected values obtained from the Monte Carlo simulations undertaken. An Excel file with the data is available for downloading at https://doi.org/10.3886/E244982V1

Table 1. *(values in $ million)*

Years	Primary Flow	Opening Balance	Oppty. Cost	Delta Capital Stock	Closing Balance	Financing Flow	Net Benefits after CC		Two-rate shortcut method
1	-183.0	0.0	0.0	183.0	183.0	183.0	0.0		-183.0
2	-327.9	183.0	14.6	327.9	525.5	327.9	0.0		-525.5
3	-447.8	525.5	42.0	447.8	1 015.4	447.8	0.0		-1 015.4
4	-301.5	1 015.4	81.2	301.5	1 398.1	301.5	0.0		-1 398.1
5	-87.5	1 398.1	111.8	87.5	1 597.4	87.5	0.0		-1 597.4
6	54.7	1 597.4	127.8	-54.7	1 670.5	-54.7	0.0		-1 670.5
7	47.1	1 670.5	133.6	-47.1	1 757.1	-47.1	0.0		-1 757.1
8	108.1	1 757.1	140.6	-108.1	1 789.5	-108.1	0.0		-1 789.5
9	174.6	1 789.5	143.2	-174.6	1 758.1	-174.6	0.0		-1 758.1
10	352.0	1 758.1	140.6	-352.0	1 546.7	-352.0	0.0		-1 546.7
11	354.5	1 546.7	123.7	-354.5	1 316.0	-354.5	0.0		-1 316.0
12	165.4	1 316.0	105.3	-165.4	1 255.8	-165.4	0.0		-1 255.8
13	74.9	1 255.8	100.5	-74.9	1 281.4	-74.9	0.0		-1 281.4
14	81.5	1 281.4	102.5	-81.5	1 302.5	-81.5	0.0		-1 302.5
15	125.0	1 302.5	104.2	-125.0	1 281.7	-125.0	0.0		-1 281.7
16	318.9	1 281.7	102.5	-318.9	1 065.3	-318.9	0.0		-1 065.3
17	279.0	1 065.3	85.2	-279.0	871.5	-279.0	0.0		-871.5
18	309.0	871.5	69.7	-309.0	632.2	-309.0	0.0		-632.2
19	306.7	632.2	50.6	-306.7	376.1	-306.7	0.0		-376.1
20	226.4	376.1	30.1	-226.4	179.8	-226.4	0.0		-179.8
21	219.5	179.8	14.4	-194.2	0.0	-194.2	25.3		25.3
22	240.1	0.0	0.0	0.0	0.0	0.0	240.1		265.9
23	272.9	0.0	0.0	0.0	0.0	0.0	272.9		544.1
24	276.5	0.0	0.0	0.0	0.0	0.0	276.5		831.5
25	284.1	0.0	0.0	0.0	0.0	0.0	284.1		1 132.2
26	281.2	0.0	0.0	0.0	0.0	0.0	281.2		1 436.1
27	294.0	0.0	0.0	0.0	0.0	0.0	294.0		1 758.8
28	295.7	0.0	0.0	0.0	0.0	0.0	295.7		2 089.7
29	452.4	0.0	0.0	0.0	0.0	0.0	452.4	**FV**	2 583.9
IRR	10.3%					8.0%			
NPV @SOCR	365.5					0.0	365.5		
NPV Two-rate	1 484.1					0.0	1 484.1		1 484.1
NPV @STPR	2 615.7					-1 131.5	1 484.1		

[6]This Appendix is quoted from Szekeres (2026)

The second column of Table 1 shows the primary flow of the project given in millions of dollars for years 1-29. The primary flow is negative for the first five years, sums that need to be financed. The following four columns describe the financing mechanism. The first column of this block is the Opening Balance of capital employed at the beginning of the period in question. The next column shows the Opportunity Cost of the outstanding capital balance. It is calculated using SOCR = 8%. The following column, labeled Delta Capital Stock, shows the change in the capital stock that occurs in the given year. For Year 1, since the primary flow is –$183m, the capital stock increment is a positive value of this magnitude. The following column contains the Closing Balance for the year, which is simply the sum of the values of the three preceding columns in the same row.

From here on we proceed recursively. The Opening Balance of Year 2 is the Closing Balance of Year 1. This balance incurs Opportunity Costs of $14.6m and the negative primary flow of Year 2 is also added to the capital stock, leading to a Closing Balance of $525.5m for Year 2. This procedure is repeated for the following years.

Notice that in Year 6 the Primary Flow becomes positive, and we can see in the Delta Capital Stock column that this positive flow of $54,7m is entirely devoted to amortizing the capital stock, which is reduced by this amount. However, the Closing Balance of Year 6 does not diminish, because the Opportunity Cost of the Opening Balance is higher than the positive net flow of this year. Looking at the Closing Balance column, we can see that its values keep growing because of the accruing Opportunity Costs of capital. This process continues until Year 21, in which capital is finally fully amortized. This is the full payback period. From here onwards there are no further capital costs.

The column labeled Financing Flow contains financing proceeds and the amortization of the capital stock. Notice that its values are identical to those of the Delta Capital Stock column. By adding this flow to the primary flow, we obtain Net Benefits after Capital Costs, shown in the next column. We can see that there are no net benefits until capital is fully amortized in Year 21. That year the amortization of capital is concluded, and the small residual of $25.3m that remains after final amortization becomes a net benefit. Thereafter net benefits coincide with the values of the primary flow.

We now proceed to discuss the financial performance indicators that can be derived from these flows. The IRR of the primary flow is 10.3%. The IRR of the financing flow is 8%. This is so by definition, given that the interest rate is the SOCR = 8%.

The NPV of the primary flow discounted at the SOCR is $365.5m. The NPV of the financing flow at the same rate is 0. Again, this is so by definition, given that the SOCR was used to compute the opportunity costs of capital. We can see that the NPV of the primary flow is the NPV of Net Benefits after Capital Costs less the NPV of the Financing Flow. In keeping with the interpretation of NPV, this is the PV of benefits after all costs, including those of capital.

The two-rate NPV of the project is obtained by discounting net benefits after capital costs at the STPR, which we have assumed to equal 2%. The NPV has a value of $1,484.1m.

The conventional NPV of the primary flow discounted at the STPR is \$2,615.7m. It is interesting to see what this amount consists of. Part of it is the same \$1,484.1m PV of net benefits that we computed for the two-rate NPV, minus the NPV of the Financing Flow, discounted at the STPR, which is – \$1,131.5m. This value in positive terms is by how much the cost of capital has been underestimated by STP discounting, and it is therefore the amount by which unadjusted conventional STPR overestimates the welfare impact of the project.

The objective of two-rate discounting is to avoid this overestimation, while keeping the PV of future benefits the same as with STP discounting.

The last column of Table 1 shows how the shortcut method used to compute a two-rate NPV works. As described in the main text, this column contains the compounded accumulation of the primary net flow values using the SOCR as the compounding rate while the accumulated total is negative and using the STPR thereafter. Notice that this flow is identical to the values of the Closing Balance column in Table 1 during the capital amortization period. These balances diminish until we reach the full payback period. Thereafter, the primary net flow values are compounded forward using the STPR until teaching the time horizon in Year 29. The last value in this series is the FV of the project, \$2,589m in this case. Discounting this value to Year 1 using the STPR we obtain the same two-rate NPV of \$1,484.1m that we obtained before, much more laboriously.

The example presented is that of a feasible project, because its IRR>SOCR. Should it be that IRR<SOCR, the project would be unfeasible, the full payback period would not precede the time horizon, Delta Capital Stock would never cease to be negative, the Closing Balance would keep growing, the Financing Flow would not cease to be negative, and there would be no net benefits. Using the shortcut method the values compounded forward would remain negative and grow in absolute value. The FV would be the negative of the Closing Balance column[7].

Could a SPC correction remedy the understatement of the opportunity cost of capital implicit in conventional STP discounting? Yes, a project specific SPC factor can be found to accomplish this. Using the goal seek feature of Excel, we find that if the initial investments in the project, that is, the negative values of the primary flow, are multiplied by 1.87, then the NPV of the thus modified primary flow conventionally discounted at the STPR will yield the same result as that obtained by two-rate discounting.

Another project specific SPC calculation was mentioned in the main text, that which is based on the present value of the amortization payments. It cannot be expressed as a factor in this case, because investments are spread over 5 years, but the concept can be illustrated. The PV of capital amortization (negative values of the Financing Flow in Table 1) discounted at the STPR is – \$2,431.3m. If we modify the Primary Flow by putting this value in Year 1, set the values of Years 2-5 to zero and then conventionally compute the NPV of the modified flow at the STPR, we get a value of \$1,484.1m, the same as with two rate discounting.

But to make either project specific SPC correction we first had to compute a two-rate discounting NPV. Therefore, finding the SPC factor adds no new

[7]Readers who downloaded the Excel file can see an illustration of this by setting SOCR to a value higher that the IRR of the primary flow.

information and is unnecessary. It should be noted that no generic SPC factor will ever be appropriate except by pure chance. The following table will help explain why.

Table 2. *Composition of the Capital Costs of the Cerro Colorado Project*

Total investments	$1 347.7m
Opportunity cost	$1 824.3m
Total capital costs	$3 172.0m

The values in Table 2 are undiscounted. Total investments is the sum of the negative values of the primary flow during its first five years. This is the amount that the SPC factor multiplies. But initial investments are not the sole determinants of total capital costs. The opportunity cost is another important component that is a function of two other factors as well: the SOCR and the timing of project benefits. It is these, along with the total investments, that determine the time profile of the project's capital needs, and hence the derived opportunity cost of capital. No generic SPC correction, which necessarily ignores two key determinants of the opportunity cost component of total capital costs, can effect an accurate correction by multiplying one of the components s of total capital costs.

The MCF correction is not illustrated with this project, even though it could be. Data exist to show the project's incremental impact on public finances, in the form of increased tax revenues. The way to effect the correction would be to add the product of the incremental tax revenues by the appropriate factor λ as an extra benefit, which would result in changes in the project's primary flow. This type of adjustment has no conceptual bearing on the opportunity cost of capital.