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Articles

ATHENS INSTITUTE FOR EDUCATION AND RESEARCH
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The current issue is the fourth of the eighth volume of the *Athens Journal of Education (AJE), published by the Education Unit of ATINER.*

Gregory T. Papanikos
President
ATINER

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16-19 May 2022, Athens, Greece

The Education Unit of ATINER organizes its **24th Annual International Conference on Education, 16-19 May 2022, Athens, Greece** sponsored by the Athens Journal of Education. The aim of the conference is to bring together scholars and students of education and other related disciplines. You may participate as **stream leader**, presenter of one paper, chair a session or observer. Papers (in English) from all areas of education are welcome. Please submit a proposal using the form available (https://www.atiner.gr/2022/FORM-EDU.doc).

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Important Dates

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Athens Journal of Education - Volume 8, Issue 4, November 2021 – Pages 349-360

Can Multiple-Choice Questions Replace Constructed Response Test as an Exam Form in Business Courses? Evidence from a Business School

By Leiv Opstad[*]

The discussion of whether multiple-choice questions can replace the traditional exam with essays and constructed questions in introductory courses has just started in Norway. There is not an easy answer. The findings depend on the pattern of the questions. Therefore, one must be careful in drawing conclusions. In this research, one will explore a selected business course where 30 percent of the test is comprised of multiple-choice items. There obviously are some similarities between the two test methods. Students who perform well on writing essays tend also to achieve good results when answering multiple- choice questions. The result reveals a gender gap where multiple-choice based exam seems to favor the male students. There are some challenges in how to measure the different dimensions of knowledge. This study confirms this. Hence, it is too early to conclude that a multiple-choice score is a good predictor of the outcome of an essay exam. This paper will provide a beneficial contribution to the debate in Norway, but it needs to be followed up with more research.

Keywords: multiple choice test, constructed response questions, business school, gender, regression model.

Introduction

There are some studies related to applying the multiple-choice format in exam tests. Due to the complexity of using multiple-choice, one cannot simply transfer results from other studies. Therefore, it is of interest to explore the experience from a Norwegian business school. One needs more research to get a more complete view of how different kinds of exams work (Krieg & Uyar, 2001).

Contrary to Northern America, in Norway, it is normal to have a traditional 4-hour written school exam in business courses. This constructed-response (CR) questions format places test-takers under pressure. Many want a system with multiple-choice (MC) questions such as is heavily used in the US, especially in large-scale testing (Livingston, 2009). Becker and Watts (2001) suggested that MC-test make up 45 percent of the exams in economics. Some advantages of the MC-based exam are low administrative costs, the test is easy to complete, and one avoids subjective assessments. It is indisputable how many scores each student has achieved (Walstad, 1998).

The Business School of the Norwegian University of Science and Technology (NTNU) has considered increasing the use of MC-based exams. The reasons are as follows. Firstly, there will be a common and identical exam at three geographically separate campuses from 2020 onwards. There are different instructors and the enrollment qualifications may vary depending on university grounds. Altogether, there might be from 500 to 800 students in the same compulsory course. With

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several examiners involved, there can be inequality in judging of the students' performance. Two students with the identical result do not necessarily get the same grade. Ensuring equal treatment of the students requires substantial resources. With MC-based questions, there will be no bias or different assessment among the business classes. Secondly, there are financial pressures to reduce the costs associated with the exam. Traditionally, there have been two evaluators to determine a student's letter grade. Today, it is a pressure to have only one examiner, even in courses with many students. A third objective is to reduce the numbers of students asking for validation of the given grade and the numbers of students who complain about the grade decision request a new exam commission. Undergraduates in Norway have strong rights. It is easy and cost-free to ask for justification of the letter grade and demand new evaluation. The proportion of students doing this has increased. In many courses, the amount has surpassed 30 percent. One can assume that the number of undergraduates doing this will increase after establishing a joint exam among the campuses/colleges. Such reviews take up a lot of time for the instructors and the administrative staff.

The purpose of this article is to see if the MC-based exam can substitute CR-based exams in business courses. This will be done by exploring data from the macroeconomic course where one used both CR-based and MC-based tests over the last 10 years. Experiences from this course will hopefully be relevant to other subjects. This paper does not investigate whether replacing of CR with MC will affect the way undergraduates' study and acquire knowledge. This study is limited to looking at if the choice of test method affects the grade level and the ranking of students and will be achieved by comparing data from 2012 to 2016.

An important question is whether one measures the same competencies by the two methods. To answer this, it may be useful to start with Bloom's taxonomy of educational objectives (Bloom, 1956):

1. Knowledge
2. Comprehension
3. Application
4. Analysis
5. Synthesis
6. Evaluation

Knowledge is about memorizing fact while *comprehension* requires that one understands the meaning of knowledge using one's own words. *Application* measures if the student understands how to use the knowledge. *Analysis* means the student uses the principles learned to derive new knowledge. *Synthesis* implies the student is able to solve problems by combining the given information. *Evaluation* requires that the student can discuss the usefulness of the knowledge and theories.

The key question is to what extent is one able to measure these six levels using an MC-based test or a CR-based assessment? It is easy to create MC-based questions from the two first level of Bloom's taxonomy (Hickson & Reed, 2011). According to Buckles and Siegfried (2016), Scully (2017), and Zheng, Lawthorn, Lumley, and Freeman (2008), it is considerably more difficult but still possible to design questions that also capture the two next dimensions. Aiken (1982) suggests also that it is challenging for one to produce an MC-based test that involves higher-order thinking. It is easier to test higher levels of learning by presenting CR-based questions (Hickson, Reed, & Sander, 2012). As a part of the assignment, it is important to promote reflective judgement, analysis, and evaluation (Dubas & Toledo, 2016; Dwyer, Hogan, & Stewart, 2014). One likely can achieve this better by applying constructed questions and essays.

If the questions are connected to the lower level of Bloom's taxonomy, one can get one result, but it may differ if one uses the higher level (Bacon, 2003). A lot of MC answers are based on partial understanding of knowledge and one might obtain the wrong expression of the students' academic abilities (Simkin & Kuechler, 2005).

Literature Review

The literature is mixed in the analysis of comparability of MC-based and CR-based tests. Important contributors to this discussion are Becker and Johnston (1999), Walstad (1998), and Walstad & Becker (1994).

An advantage of MC-based tests, compared to CR-based tests, is that it is easy for examiner to determine if the correct answer has been provided. On the other hand, they are difficult to construct. There can be a substantial variation in the quality of the questions; this affects the results. Bush (2001) showed how one could achieve high score just by guessing. If there is a question with five possible answers, one can expect 20 percent correctly answered. Assume that, of half the questions, the student finds that only two of the options could be true. This increases the likelihood that one can guess the right answer to 35 percent. This guessing can lead to a lack of validity (Mbonigaba & Oumar, 2017).

The literature shows that the effect of the various types of exam depends on the form of the questions. One can produce a MC version that is difficult or easy for students to answer. The MC-based test can be too simple and hence will not be a good indicator of students' understanding (Simkin & Kuechler, 2005). By changing the design and form of the MC questions, the students will perform differently (Chan & Kennedy, 2002).

Many studies suggest that MC and CR formats do not measure the same understanding or performance. One can miss some basic understanding by using an MC-based test. It is hard to achieve the advanced level of knowledge by using the MC format (Simkin & Kuechler, 2005). Mbonigaba and Oumar (2017), basing their investigation on Bloom's taxonomy, found that students have different scores using MC or CR, depending on the level. Therefore, the MC-test ranking is inconsistent with the level of cognitive ability. Using CR-based test develops better cognitive and writing skills (Welsh & Saunders, 1998). With essays and CR-questions, the students can write with their own words, analyze a topic, and demonstrate originality. Even though there are differences between CR and MC, there is a link between them. For instance, the reported correlation coefficient between MC and CR is quite high, but this relationship depends heavily on the chosen questions (Chan & Kennedy, 2002). Walstad and Becker (1994) suggest the correlation coefficient is 0.69 in microeconomics and 0.65 in macroeconomics. Some authors report a substantially lower value (Hickson & Reed, 2011). The score is higher in more quantitative courses (chemistry, calculus, etc.) and less in non-quantitative courses (history, language, etc.). They ask if it is of enough economic value to include an essay component, since it does not give much more information. The connection between two CR-based tests in within the same course is similar to those one observes between MC and CR. Therefore, Hickson, Reed, and Sander (2012) suggest that the consequences will be rather small by switching to MC-test only. Furthermore, the two highest levels of academic skill (Bloom's taxonomy) are the most appropriate for master's- and PhD-level students. Only to a small extent are the students tested on these topics in the introductory courses. In addition, many students are attending introductory courses. This argues for using MC questions for undergraduates. This might explain why multiple-choice and essay scores are substantially significant for explaining achievement on a less advanced level (Becker & Johnston, 1999). However, if one wants to measure sophisticated dimensions of knowledge, there is a substantial difference between MC and CR.

Many researchers indicate there might be a gender difference in the performance depending on the exam form. The CR-based test seems to favor females, while the opposite is true for MC-based tests (Becker & Johnston, 1999; Livingston & Rupp, 2004; Smith & Edwards, 2007). The reason why females get relatively lower scores on multiple choice questions can be related to other factors than academic understanding, such as differences in reasoning, socialization, and instructional practice (Walstad & Robson, 1997). Females might have advantages at writing essays. This can be another factor explaining why they are falling behind when evaluated by MC-based questions in economic courses (Becker & Johnston, 1999).

Data, Model and Findings

The Sample

The data were taken from the school's database and from the instructor's note related to the determination of the grades. The data derive from a period of 5 years (2012–2016). The students had the same instructor and identical textbook during this interval. The exam form was based on the same principle or every year in the chosen macroeconomic course. It is compulsory and runs the second year for undergraduate students. The paper exam (4 hours) consists of three parts:

1. Apply economic theory and analysis using a familiar mathematical model.
2. Answering current issues by applying the theory.
3. A conventional multiple-choice test with five-answer questions and no negative marking.

Numbers 1 and 2 are structured/essay questions (CR). The first assignment makes up for 50 percent, the second 20 percent and the last section, 30 percent. The students were informed of this. There were 32 questions in the MC-based test. As a result, the score was determined by subtracting "2" from the number of correct answers. Since the CR- and MC-based tests were taken simultaneously for the same group of students, this gives a good opportunity to compare the outcome. The assignments reflect different levels of Bloom's taxonomy. The first segment is somewhat predictable, so the students can learn the process in advance, but part of the questions are challenging and require good academically skill. Exercise 2 is demanding. Here, the students are inspired to apply the theory to more or less new and unknown issues. The CR-based questions (part 1 and 2) primarily test the application and analysis levels of skills, some fragment of Exercise 1 includes the Comprehension level, while the advanced Synthesis level might be touched in Exercise 2. On the MC-based test, there are questions from the whole textbook. A suggestion is that about one-third of the questions is testing the Knowledge level, another third the Comprehension level, and the final third cover the Application or Analysis level.

Table 1. The Data

	Min	**Max**	**Mean**	**St. Dev.**	**N**
MC-score (scale 0 to 100)	13.3	100	60.5	15.4	1190
CR-score (scale 0 to 100)	4.3	100	49.9	19.2	1190
Macroeconomics (grade)	0	5	3.10	1.19	1190
Gender (0:F, 1:M)	0	1	0.41	0.49	932
GPA	44.0	65.4	51.9	3.39	613
Microeconomics (grade)	0	5	3.24	1.20	887
Business mathematics (grade)	0	5	2.85	1.62	887
Management (grade)	0	5	2.88	1.26	927
Compulsory midterm MC-based test (macro-economics), 32 questions	8	31	19.6	4.3	911

Note: 0:F, 1:E, 2:D, 3:C, 4:B, 5:A.

Table 1 presents the data where gender, GPA (Grade Point Average) from high school and performance in some of the compulsory courses first year are included. For making the scores from MC and CR sections directly comparable, the scale is measured in percentages. Data are missing for some of the variables. The females were in the majority (57%). Using the same MC-method as for the final exam, the score of a compulsory midterm test was 55.

The Correlation between MC and CR

The correlation coefficients give an indicator to which degree multiple-choice questions can substitute constructed response questions (see Table 2).

Table 2a. Correlation Coefficients for the Whole

	MC	CR	MC-midterm
MC		0.62	0.43
CR	0.62		0.36
MC-midterm	0.43	0.36	

Table 2b. Correlation Coefficient between CR and MC for Each Year

	2012	2013	2014	2015	2016
Correlation coefficient	0.50	0.54	0.71	0.66	0.67

The values of all correlation coefficients were strongly significant at 1% level. The score is about 0.6 between MC and CR, but notice (Table 2b) that there is a considerable variation from year to year (0.50 in 2012 and 0.71 in 2014). Furthermore, notice that the link between the MC-based midterm test and final MC-based test is rather low. The correlation coefficient is 0.43.

The Gender Impacts

Table 3 shows the result of bilateral comparison of women and men.

Table 3. The Gender Effect - Independent Sample T-Test of Mean (Assuming Equal Variance)

	Females	Males	Diff.	St. Dev.	*T*-value	Sign. level
MC	52.7	64.7	-7.4	0.98	-7.5	0.000
CR	48.3	55.6	-7.3	1.22	-6.0	0.000

The male students perform better than the female students.

Regression Model

By using a linear regression model, one can find out more about the impact the independent variables simultaneously have on the chosen exam form. The explanatory variables were gender, GPA score, and performance in business mathematics, microeconomics and management. The GPA score measures academic ability. There was also a strong positive link between GPA scores and performance in macroeconomics (Jones, Kouliavtsev, & Ethridge Jr, 2013; Raimondo, Esposito, & Gershenberg, 1990). Mathematics competency is a key factor for success in economics (Ballard & Johnson, 2004; Opstad, 2018). It is an indicator of analytical and quantitative skills. Microeconomics and macroeconomics are closely linked together (Perumal, 2012). The data in this study reveal a correlation coefficient between performance in microeconomics and macroeconomics of 0.672. The achievement in microeconomics with a traditional 4-hour CR-based test can provide a picture of the students' skills in economics. The ability to write and present the content in a proficient manner has an impact on the students' performance. This attribute is important for the grade in a subject such as management. Therefore, this course is included as an independent variable.

The chosen linear regression production function for comparing MC- and CR-score is:

$$Yi = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_5 X_5 + \varepsilon$$

where: Yi: Score in macroeconomics using MC- or CR-test.

α_0: Constant
X_1: Gender (0: F, 1: M)
X_2: GPA score
X_3: Performance in Business mathematics (0: F, 1: E, 2: D, 3: C, 4: B, 5: A).
X_4: Performance in Microeconomics (0: F, 1: E, 2: D, 3: C, 4: B, 5: A)
X_5: Performance in an introduction course in management (0: F, 1: E, 2: D, 3: C, 4: B, 5: A)
ε: stochastic error

Due to missing data, the valid numbers in the regression were substantial smaller than the whole sample presented in Table 1. However, it looks like they are random, since they did not affect the mean values of the variables. Results are presented in Table 4.

Independent of chosen exam test, the GPA had no significant impact on the student's performance. The gender effect in favour of males was more powerful and stronger using the MC-based test. Business mathematics influenced the score, but the differences between the two assessment formats are minor. Performances in microeconomics and macroeconomics were tightly related to each other. However, the impact is higher for CR-based questions (The B-coefficient is 8.01 for the CR model and 5.66 for MC model). Unlike the other test method, there is a substantial significant link between performance in management and the CR-score.

Discussion

The findings in this research do not give an unambiguous answer to whether this is a good idea or not to replace CR-based questions with MC- based questions. There is a near relationship between the two alternative test methods. This is similar to results of prior studies. However, the correlation coefficient depends heavily on the design of the questions. This can explain why this rate varies between 0.5 and 0.7 (Table 2b). On the other hand, this is substantially higher than the link between the similar test methods (MC) used within the same course at different times during the semester (see Table 2a). This is a point that Walstad and Becker (1994) and Hickson, Reed, and Sander (2012) highlight. Even with an identical test method, there will be a considerable variation in the students' achievement. From this point of view, the correlation rate between MC and CR is rather high. Hence, one needs to be careful in drawing conclusions by just comparing MC- and CR-score. Regardless of the chosen identical method, the correlation coefficient will probably be substantial lower than 1.0 for having two or more assignments for the same target group.

Table 4. Finding, Regression Model

	MC model			**CR model**		
	Coefficient (B) (Unstan-dardised)	***T*-value**	**Sig. level**	**Coefficient (B) (Unstan-dardised)**	***T*-value**	**Sig. level**
Constant	10.9 (8.52)			6.6 (9.73)		
GPA	0.48 (0,11)	1.27	0.20	0.17 (0.19)	0.90	0.366
Gender	5.01 1.12)	2.89	0.004 (***)	2.49 (1.28)	194	0.053 (*)
Business Math	1.45 (0.42)	4.46	0.000 (***)	1.24 (0.48)	2.58	0.010 (**)
Microeconomics	5.66 (0.61)	3.44	0.000 (***)	8.01 (0.70)	11.40	0.000 (***)
Management	0.13 (0.46)	0.27	0.787	2.22 (0.53)	4.20	0.000 (***)
	Adj. R^2=0.376 N=473			Adj. R^2=0.442 N=473		

Note: Standard error in parenthesis, *, ** and *** denote significance at the 10%, 5%, and 1% level, respectively. All VIP (Variable Importance of Projection) values are between 1 and 2.

Similarities between MC and CR

The regression model shows many similarities between the two assignments formats (CR and MC). The input variables explain nearly the same proportion of the variance for dependent variable (the value of adjusted *R*-square). The *GPA* has no significant impact on the performance on neither MC-based exam nor CR-based exam. The influence of *business mathematics* is almost the same for the two discussed test methods. It is not surprising, since a substantial proportion of MC-based questions require good skills in calculation.

Differences between MC and CR

The regression model also documents differences between the two exam forms. Adjusting for other variables, it turns out that there is a significant *gender gap*. Men will gain if one decides to select MC questions as an exam design. This is similar to many prior studies. There is strong gender equality in Norway but still there are substantial gender differences. This is a paradox. We will try to find some explanations (Ahlstrom & Asarta, 2019). The females struggle with quantitative courses (Naqvi & Naqvi, 2017; Opstad, 2020). This might be a reason why females prefer non-quantitative majors. Men select more quantitative subjects like finance. This result might be related to that the women choose less advanced mathematics at high school (Opstad, 2018). Therefore, they have poorer attitudes towards mathematics (Opstad & Årethun, 2019). This can explain why females perform poorer than males in quantitative courses like macroeconomics. This study confirms this. However, this factor does not explain why males outperform females with MC- based test compared to CR-based test. Data are not available in this research to explain this difference. One reason might be that women in general are more risk-averse than men are (Pekkarinen, 2015). The time and competitive pressure are probably stronger with MC-questions compared to CR-questions. Therefore, the female undergraduates are less confident in answering multiple-choice questions. Hence, they perform relatively better in CR-based exam. Another explanation can be sociocultural differences (Johnson, Robson, & Taengnoi, 2014) and student-specific characteristics (Opstad & Fallan, 2010; Johnson, Robson, & Taengnoi, 2014). However, Riener & Wagner (2017) emphasize that the gender difference in MC-designed exam depends on the difficulty of the questions.

There is a considerable connection between the *performance in management* and for CR score, but this link does not seem to exist for the MC score. The explanation is probably that writing and presentation ability is catch up to a greater extent with CR questions than MC questions. This confirms results from some prior research. *Performance in microeconomics* is a good indicator for success in macroeconomics, but we notice the influence is stronger for CR-test. One reason may be that this exam method more closely captured the desired skills.

Another explanation might be that the exam in microeconomics is CR-based. May be the effect would have been differently if there was a MC-format exam in microeconomics.

Following Bloom's taxonomy, the two test alternatives will probably emphasize the distinct dimensions differently. This problem is not easy to solve. Some aspects are harder to measure using MC-test. This paper shows that there are some differences between CR- and MC-based tests. The question is whether the two tests complement or substitute for each other. Probably, more investigation is needed to make such a determination. To do so, one must follow two steps. Firstly, one must increase the percentage of MC questions of the final exam from, for instance, 30 to 50 percent of the final exam. Secondly, one should put more effort into designing the MC questions as well as increase the proportion of testing at the application/analysis level.

Limitations and Further Research

This study has compared the two exam formats with data from only one business school. Since the design of questions is critical for the result, one must be careful with respect to drawing general conclusions. More independent variables in the regression model would have been desirable, but available information from the school's database was limited. An important issue that is not touched in this paper is whether a change of exam format from CR to MC will alter the way students acquire their knowledge.

To further research, it would be interesting to find out more of why there is a gender difference between the two test methods. One should also investigate in how the test methods influence the learning style and the academic qualification of the undergraduates.

Conclusion

In Norway, it is normal to use structured/essay questions as an exam format. There is a limited tradition of using MC questions. Due to identical exams from several colleges, it is challenging to avoid judgment bias; it also incurs considerable administrative costs. Therefore, there is pressure to increase the use of MC-based tests, especially in courses with more than 500 students. The findings of this study suggest that those tests can partly replace each other and partly supplement each other. The result indicates that MC-based test favor male students. The decision maker must consider this. It is difficult, however, to measure some dimensions of understanding by using MC.

The discoveries derived from this research can give constructive input into the discussion about more MC-based tests in introductory courses at business schools in Norway. However, one should experiment more with the design of the questions. There are many arguments against excluding essay questions, and only use MC-based questions in the final exam. For the time being, it might be valuable to follow the advice of Becker and Johnston (1999), who suggest using both forms of testing.

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Mathematics Teachers' Pedagogical Content Knowledge Involving the Relationships between Perimeter and Area

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This study aims to investigate and compare mathematics teachers' knowledge of pedagogical content, knowledge of students' understanding and knowledge of instructional strategies, subcomponents of pedagogical content knowledge, student errors in the relationship between perimeter and areas in rectangles, squares, and parallelograms. 10 pre-secondary school in-service mathematics teachers and 10 prospective mathematics teachers participated in the study. The qualitative case research approach was used. To collect the data, an interview form consisting of four questions showing student errors related to perimeter and area was prepared. The participants were asked to comment on the questions given in the form, and their answers were recorded. Later, they were asked to write down their answers to these questions. According to the outcomes, there is a lack of knowledge of students' understanding and knowledge of instructional strategies, which are the subcomponents of the pedagogical content knowledge of prospective mathematics teachers. Moreover, prospective teachers are found to be incompetent as regards the necessary mathematical subject matter knowledge. To prevent difficulties, when the concepts of perimeter and area are taught, instead of giving formulae initially, concrete materials or real-life examples about these concepts should be provided.

Keywords: pedagogical content knowledge, perimeter, area, mathematics teachers, prospective mathematics teachers.

Introduction

The topics of perimeter and area are the basic subjects for the competence of elementary school teachers (Reinke, 1997). However, these are confusing topics as both involve measurement, their formulae are taught almost simultaneously to students, and memorizing these formulae may be confusing (Van de Walle, Karp, & Bay-Williams, 2014). This is confusing for both students and teachers. Some studies show that teachers often confuse the concepts of perimeter and area because they assume a constant relationship between area and perimeter (Baturo & Nason, 1996; De Sousa, Gusmão, Font, & Lando, 2020; Yeo, 2008). According to several studies, prospective teachers have also been found to be incompetent in comprehending concepts, their knowledge is based on rules and formulae, they have some difficulties in explaining what these formulae are for, and they focus on using formulae rather than activities designed to reinforce concepts (Baturo & Nason, 1996; Berenson et al., 1997; Livy, Muir, & Maher, 2012; Menon, 1998; Reinke, 1997; Runnalls & Hong, 2019). According to Zacharos (2006), using formulae first while measuring area leads to misconceptions about area measurement and makes it difficult to interpret the physical meaning of the numerical representation of the area. According to Baturo and Nason (1996),

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many prospective teachers are devoid of concrete measurement experiences such as covering the surface area with measurement units, and they think of area as *multiplication of width by length*. This situation shows that the difficulties that prospective teachers face may stem from their learning experiences at school (Baturo & Nason, 1996).

Difficulties regarding area and perimeter are usually related to conservation of area and perimeter and using inappropriate units while calculating them (not using square units while reporting area measurements) (Baturo & Nason, 1996; Guner & Akyuz, 2017; Ma, 1999; Murphy, 2012; Yeo, 2008). It is also generally thought that two rectangles with the same area must have the same perimeter (Van de Walle, Karp, & Bay-Williams, 2014). However, this is not always valid. Similarly, two rectangles with the same perimeter measurements cannot be expected to have the same area, and this situation is not also limited to rectangles (Van de Walle, Karp, & Bay-Williams, 2014).

In fact, confusion of the topics of area and perimeter, how teachers teach these topics, and how they react to these errors are the basis of problems experienced by students related to area and perimeter. This is because teachers who have not exactly understood mathematical concepts cannot be expected to explain these concepts. What are more, teachers who have a good command of the subject matter but cannot present the topic in a way that students can understand also experience similar problems (Yeo, 2008). The main reason for this is that teachers use different types of knowledge for teaching mathematics (Rowland, Turner, Thwaites, & Huckstep, 2009; Shulman, 1987). They must not only be equipped with a good command of the subject matter but also have the knowledge of how to present it to enhance student comprehension in the most effective way and detect students' learning difficulties and mistakes (Gökkurt, 2014). These pieces of knowledge were first termed by Shulman (1986a) as *pedagogical content knowledge*. The concept of pedagogical content knowledge is used to express teachers' interpretations and transformations of subject-matter knowledge to support student learning. It especially involves understanding students' learning difficulties and prejudices (Van Driel, Verloop, & De Vos, 1998), and it represents certain strategies and approaches of the teacher while conveying mathematical knowledge to students (Van de Walle, Karp, & Bay-Williams, 2014).

Pedagogical content knowledge may be used effectively and flexibly during the interaction process between students and teachers. Teachers' actions while dealing with subjects largely depend on their pedagogical content knowledge (Van Driel, Verloop, & De Vos, 1998). Shulman (1986b) specified teachers' knowledge as subject matter knowledge, pedagogical content knowledge, and curriculum knowledge. Shulman (1987) also pointed out that pedagogical content knowledge has two components. These are the knowledge of *representations*, which involves instructional strategies that are used by teachers to make subject matters understandable to students, and the knowledge of students' learning difficulties, which is related to their misconceptions about subjects (Hume, 2011). Instructional strategies are the way subject matters are taught (Van Driel, Jong, & Verloop, 2002), as well as representations in the forms of pictures, analogies, and explanations to make a subject matter more comprehensible for students

(Shulman, 1987). Knowledge of students' understanding, on the other hand, comprises the knowledge related to student misconceptions, naive ideas obtained through the interpretation of prior learning experiences, and preconceived ideas about the subject matter (Shulman, 1987).

Looking at difficulties related to area and perimeter, it may be seen that these problems are not only experienced by students at schools (Livy, Muir, & Maher, 2012). In this case, inevitably, students often have misconceptions about area and perimeter. To cope with this situation, therefore, teachers should not only try to teach students in a way that avoids misunderstandings, but they should also have approaches to deal with the misconceptions that arise (Chick & Baker, 2005). On the other hand, prospective teachers with a limited understanding of area may fail to help children develop this notion because a prospective teacher's understanding of the nature of the area is seen as a key concept in their style of teaching (Murphy, 2012). To solve the difficulties experienced in the concepts of area and perimeter, it is important to raise prospective and in-service teachers' awareness of pedagogical content knowledge related to these topics. For this reason, this study aimed to investigate student errors in terms of the relationship between area and perimeter in rectangles, squares, and parallelograms regarding mathematics teachers' and prospective mathematic teachers' in accordance with their knowledge of students' understanding and knowledge of instructional strategies. This study will also reveal how the teaching experiences and mathematical backgrounds of in-service teachers and prospective teachers affect the knowledge of students' understanding and knowledge of instructional strategies.

Methodology

Research Model

One of the approaches of qualitative research, a case study, was employed in this study. Since the objective was to examine the participants' voice recordings on their explanations and answers to questions in a detailed way, a case study approach was used. Case studies serve to discover a phenomenon about which little is known or to examine it thoroughly. This approach owes its power to the researcher's ability to investigate the case in-depth and in detailed manner (Arthur, Waring, Coe, & Hedges, 2017). In this study, the aim is to investigate mathematics teachers' and prospective mathematics teachers' pedagogical content knowledge, their knowledge of students' understanding and knowledge of instructional strategies in relation to student errors in the relationship between perimeter and area in rectangles, squares and parallelograms in a detailed way.

Participants

This study was carried out with 10 pre-secondary school in-service mathematics teachers (years of service between 5 and 7 years) and 10 prospective mathematics teachers who were senior students. The reason why prospective teachers were chosen from among senior students was that they were assumed as knowledgeable and competent enough in the subject matter. The in-service and prospective teachers who participated in the study were chosen voluntarily. While choosing the participants, convenience sampling, which is a purposive sampling method, was utilized. The names of the in-service teachers and prospective teachers that took part in the study are kept confidential. While the in-service teachers are coded as T1, T2, T3, etc., the prospective teachers are coded as P1, P2, P3, etc.

Data Collection

As a means of data collection, an interview form involving four questions about student errors in perimeter and area was prepared. The interview schedule was in such a way that it could be possible to determine the participants' knowledge of students' understanding and knowledge of instructional strategies. If the interview questions are examined, it may be seen that the first question was prepared by inspiration from Ma's (1999) study, while the second and third ones were inspired by Tan Şişman and Aksu's study (2009). The fourth question was prepared by inspiration from Murphy's (2012) question, which had been adapted from the question in Tierney, Boyd, and Davis's study (1990). The first question was prepared to identify knowledge of students' understanding and knowledge of instructional strategies about changes in the perimeter and area of a rectangle. The second question aimed to test the notion of the variability of perimeter, while the third one sought to learn about knowledge of students' understanding and knowledge of instructional strategies related to area conservation. The last question was prepared to identify knowledge of students' understanding and knowledge of instructional strategies on the variability of the area and perimeter of parallelograms and rectangles. The reasons why the rectangle, the square, and the parallelogram were chosen for this study were that these are the topics students are taught on the pre-secondary school level. Moreover, they are interconnected, and there are very few studies in the literature dealing with the square, the rectangle and the parallelogram at the same time. The participants were given an interview form that includes a group of student errors. The participants' views were recorded by a voice recorder, and they were asked to write down their answers to the questions. Similar to the case in the study by Gökkurt, Şahin, Soylu, and Doğan (2015), by looking into whether the participants were able to detect student errors or not, the researchers tried to determine their *knowledge of students' understanding,* and by taking their suggestions on how to correct student errors into consideration, they tried to determine the participants' *knowledge of instructional strategies.*

Data Analysis

Descriptive analysis techniques were employed to analyze data. In the descriptive analysis, data are summarised and interpreted according to previously set themes (Yıldırım & Şimşek, 2013). For descriptive analysis, the framework prepared by Gökkurt, Şahin, Soylu, and Soylu (2013) was used after making some alterations as a result of the inconsistency of the present data with the codes they used. The reason why the researchers benefited from their framework was that, while forming this framework, Gökkurt, Sahin, Soylu, and Soylu (2013) formed certain draft themes and codes after collecting prospective teachers' written answers, and they reorganized them after reading these answers repeatedly. Later, they consulted an expert on whether they were comprehensible in terms of their validity, and consequently, after certain corrections, they made them clear and understandable enough for the reader. The framework created by Gökkurt, Sahin, Soylu, and Soylu (2013) was not used as the codes and themes in that study did not correspond to those in this one. Instead, in the light of the data obtained, new codes and themes were formed. An expert was consulted to check the validity of these codes and themes. In this study, the collected data were coded, and re-coded at different times by different researchers to increase reliability. These codes and categories are given in Table 1. The reliability percentage of the data coded according to the codes and categories in Table 1 was found to be 87%. For the uncommon codes, the researchers came together and negotiated. Uncompromised codes were removed, and some codes were changed. To ensure the validity of the study, the procedures in the study were described in a detailed way, and another researcher who is an expert in pedagogical content knowledge was consulted in the processes of preparation of the data collection tools and data analysis. Moreover, the researchers presented how they reached the results in a clear, understandable, and consistent way. In the section related to the codes obtained, direct quotes taken from the participants' answers are given. To ensure reliability, on the other hand, the researchers made sure that the results obtained were consistent with the data, and they explained the processes of data collection, forming categories and codes and analysis of these in a detailed way.

Table 1. Codes and Categories

Category	**Code**
Finding the error correctly	Finding the error correctly but no solution recommendations
	Finding the error correctly and recommending a partially correct solution
	Finding the error correctly and recommending a correct solution
Not finding the error correctly	No answer
	Finding the error incorrectly and no solution recommendations
	Finding the error incorrectly and recommending an incorrect solution
Finding the error partially correctly	Finding the error partially correctly and no solution recommendations
	Finding the error partially correctly and recommending an incorrect solution
	Finding the error partially correctly and recommending a partially correct solution

Results

In this section, the in-service and prospective mathematics teachers' ability to detect student errors, which is their knowledge of students' understanding, and their suggestions, methods, techniques, and strategies to correct these errors, which are called knowledge of instructional strategies, are investigated by looking into the explanations of the participants and their written solutions. The obtained data are presented with direct quotes, and they are also given in the tables featuring categories, codes, and frequencies. Table 2 shows the code, category and frequency information of the prospective teachers' answers to the first question.

Table 2. Prospective Teachers' Answers to the First Question

Category	**Code**	**Prospective Teachers**	**Frequency**
Finding the error correctly	Finding the error correctly but no solution recommendations	P7	1
	Finding the error correctly and recommending a correct solution	P5, P10	2
Not finding the error correctly	Finding the error incorrectly and no solution recommendations	P1, P2, P6	3
	Finding the error incorrectly and recommending an incorrect solution	P3, P4, P8, P9	4

When the prospective teachers' answers to the first question, asked in relation to the student error "If the perimeter of the rectangle increases, its area also increases" were investigated, the majority of the teachers agreed with the students. When the suggestions given by the prospective teachers who found the error correctly were examined, it was seen that only two of them offered a correct recommendation.

The following direct quote taken from the interview conducted with P10, who spotted the student error accurately and offered an accurate recommendation to correct the error, maybe given as an example to the answers given by the prospective teachers.

> "The student's idea that the perimeter of the rectangle increases when its area increases is definitely wrong. The student must have thought that the lengths of the sides also increased. We can show that this is not always the case. Let us consider a rectangle with a width of 4 cm and a length of 6 cm and compare it to a rectangle with a length of 13 cm and a width of 1 cm. The perimeter of the first rectangle is 20 cm, and the perimeter of the second one is 28, which means that the perimeter has increased. When we look at the areas of these two rectangles, we see that while the area of the first one is 24 cm^2, the area of the second one is 13 cm^2."

The ideas of P3, who could not detect the error or offer a correct solution to the students, were as follows.

"I think the way the student thought is right. Let's think of a rectangle with a width of 2 cm and a length of 4 cm, for example. If the sides of this rectangle increase by 2 cm, the width becomes 4 cm and the length becomes 6 cm. The perimeter of the first rectangle was 12 cm but now it is 20 cm, which means that it has increased. While the area of the first one is 8 cm^2, that of the second one is 24 cm^2. So, as the area increased, the perimeter also increased."

Table 3 shows the code, category, and frequency information of the teachers' answers to the first question.

Table 3. Teachers' Answers to the First Question

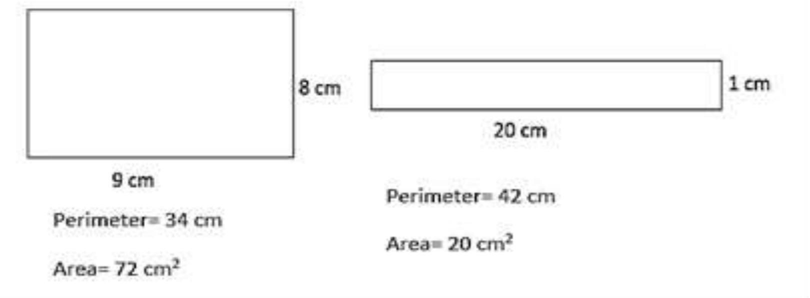
Category	Code	Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	T3, T7, T8, T9, T10	5
	Finding the error correctly and recommending a correct solution	T1, T5	2
Not finding the error correctly	Finding the error incorrectly and no solution recommendations	T4	1
	Finding the error incorrectly and recommending an incorrect solution	T2, T6	2

As seen in Table 3, seven of the mathematics teachers found the error correctly while answering the same question. In Table 3, one may see that two of the teachers recommended a correct solution to correct the student's error. These results show that in-service teachers' knowledge of students' understanding of the error was better than the prospective teachers. However, in-service teachers did not possess an adequate level of knowledge of the instructional strategies to offer the right solution.

To illustrate the answers given by the in-service teachers, the answer given by T1, who found the student error correctly and recommended an accurate solution to correct it, maybe quoted as follows.

"The student thought wrong. We can't generalize this situation. Let me show that it can be wrong with an example."

Figure 1. Answer of T1



The opinions of T6, who could not find the error correctly or come up with a recommendation for the students, were as follows.

> "I also think the student thought rightly, because the area increases when the perimeter increases. Take a rectangle with a width of 4 cm and a length of 7 cm, for example. Suppose that we have another rectangle with a width of 4 cm and a length of 10 cm. While the perimeter of the first rectangle is 22 cm, and its area is 28 cm^2, the perimeter of the second one is 28 cm, and its area is 40 cm^2."

Table 4 shows the code, category and frequency information of the prospective teachers' answers to the second question.

Table 4. Prospective Teachers' Answers to the Second Question

Category	**Code**	**Prospective Teachers**	**Frequency**
Finding the error correctly	Finding the error correctly and recommending a correct solution	P2, P3, P5, P8, P10	5
Not finding the error correctly	No answer	P4, P6, P9	3
	Finding the error incorrectly and recommending an incorrect solution	P1, P7	2

The prospective teachers had difficulty in understanding the third question and spent a lot of time in forming the parallelogram. Some of them even failed to form one. According to the results obtained in this question, half of the prospective teachers had the knowledge of students' understanding of the error and the knowledge of instructional strategies for making the right solution suggestion were an adequate level.

The response given by P5, who identified the student error correctly and offered an adequate solution, was as follows.

> "He thought that when the shapes changed, the sides would decrease, and thus, the perimeter would also decrease, or maybe, he was confused by $\sqrt{5}$ while making the calculations with square root expressions. The first shape is a square and its perimeter is 8a. The perimeter of the second shape is, on the other hand, $4a + 2a\sqrt{5}$. Let's have a look at the range of $\sqrt{5}$. It is closer to 2, and this makes it more than the perimeter of the square. Here, the child knows that this length is the hypotenuse length, and this is the longest edge."

The response of P1, who identified the student error incorrectly and could not offer a correct recommendation, was as follows.

"The student's answer is incorrect. We can't make a difference in the length of the shape by adding the cut-up part to other parts of the shape. We can explain it to the student by drawing it."

P4, who had no ideas as to why the student gave a wrong answer, provided the following response.

"…but how will it be possible? We can't place the triangles. The triangles have a right angle. How can I make a parallelogram with them?"

Figure 2. Answer of P5

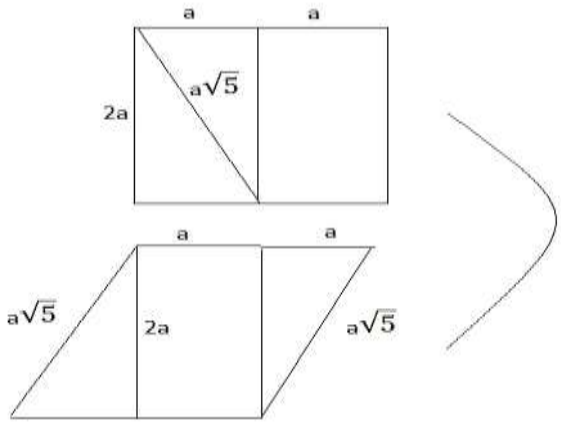


Table 5 shows the code, category, and frequency information of the teachers' answers to the second question.

Table 5. Teachers' Answers to the Second Question

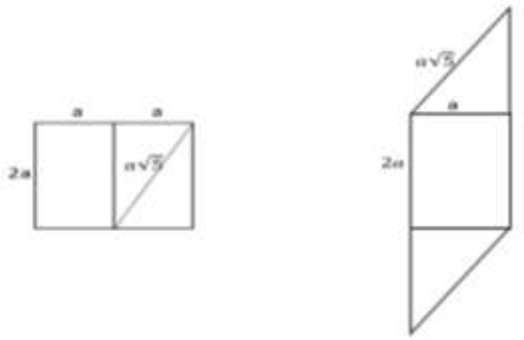
Category	Code	Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	T1, T8	2
	Finding the error correctly and recommending a correct solution	T2, T3, T5, T9, T10	5
Not finding the error correctly	No answer	T7	1
	Finding the error incorrectly and recommending an incorrect solution	T4, T6	2

When the in-service teachers' answers to the third question were examined, it was clear that seven of the teachers identified the student error correctly, two of them failed to do so, and one of them did not make any comments regarding it. These results showed that most of the in-service teachers had sufficient knowledge of students' understanding in determining the error, but they did not possess an adequate level of knowledge of the instructional strategies

The response of T5, who identified the student error correctly and gave an adequate recommendation, was as follows.

"Let me draw a square whose length is 2a cm. Its perimeter is 8a cm. Then let's form the expected shape. It's 8+2$\sqrt{5}$. That is to say, the perimeter has increased."

Figure 3. Answer of T5

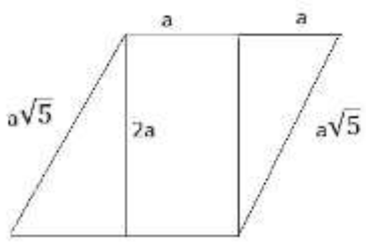


"Here, the student gave the wrong answer. He thought that the shape is narrow. We can show him how it is with the help of the Pythagorean theorem. …because if the sides are considered as the hypothenuse of the triangle, they have increased, and they became a$\sqrt{5}$."

T4, who identified the student error incorrectly and could not offer an accurate recommendation, gave the following answer.

"Let me draw it and see."

Figure 4. Answer of T4



"The perimeter of the parallelogram is less than that of the square."

Table 6 shows the code, category, and frequency information of the prospective teachers' answers to the third question.

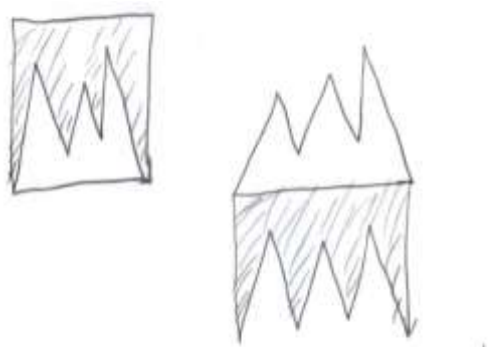
In the third question, the results indicate that most of the prospective teachers had sufficient knowledge of students' understanding to identify the error, however half of them had sufficient knowledge of instructional strategies to offer the right solution.

Table 6. Prospective Teachers' Answers to the Third Question

Category	Code	Prospective Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	P3, P5, P8	3
	Finding the error correctly and recommending a correct solution	P1, P2, P4, P9, P10	5
Not finding the error correctly	Finding the error incorrectly and recommending an incorrect solution	P6, P7	2

The opinions of P10, who spotted the error correctly and offered an accurate recommendation, were as follows.

Figure 5. Answer of P10



> "He made a mistake. He may have thought that there would be more sides. Confusing the area with the perimeter, he thought as if the number of the sides increased, and so did the perimeters. The area doesn't change if the shapes are shifted or placed somewhere else. …because the area is the surface covered by the shape. As the total area doesn't change, the areas are equal to each other. For instance, when you cover the floor of a rectangular room completely with a carpet, no matter how much you cut it into smaller pieces and place them side by side, the total [area] will definitely not change."

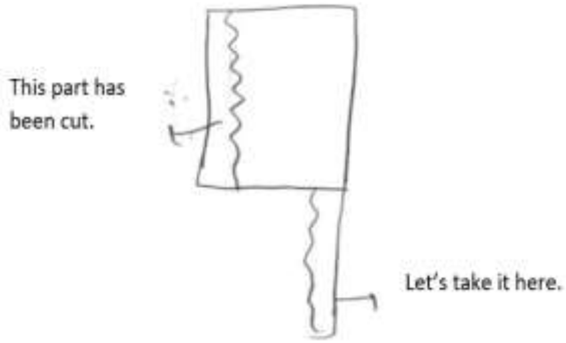
The ideas of P7, who thought the same way as the student and thus identified the student error incorrectly, were as follows.

> "I think his answer is right. …because the second shape covers a larger area. There are zigzags in the second one, so the area of the first shape is smaller."

Based on these explanations, it may be stated that P7 did not know that the area would not change when the shape is shifted or placed somewhere else.

Table 7 shows the code, category, and frequency information of the teachers' answers to the third question.

Figure 6. Answer of P7



It was observed that the majority of the in-service teachers identified the error correctly, and only one of them did not even do any reasoning related to it. As seen from Table 7, most of the in-service teachers had sufficient knowledge of students' understanding of the error and the knowledge of instructional strategies to suggest the right solution.

Table 7. Teachers' Answers to the Third Question

Category	Code	Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	T4, T8	2
	Finding the error correctly and recommending a correct solution	T2, T3, T5, T6, T7, T9, T10	7
Not finding the error correctly	No answer	T1	1

The following were the opinions of T10, who spotted the error correctly and recommended an appropriate solution.

> "The student thought wrong. The student might have confused the area with the perimeter here. It says zigzag here. Because it is indicated this way, most probably he thought that when the perimeter increases, the area will also increase. Additionally, he thought that the area was larger as there were two more parts. Based on the definition of the area, I try to eliminate the misconception between area and perimeter with concrete examples. For instance, I show the area covered by sugar cubes as a whole, and then, I leave the sugar cubes in a different place. I have the area of each sugar cube calculated. Therefore, the students will notice that the area has not changed."

Lastly, the fourth question will be examined at two stages. First of all, the findings related to how the teachers commented on the change in the perimeters of the two shapes with equal areas (Shapes 1 and 2), and secondly, how they viewed the change in the areas of the two shapes with equal perimeters (Shapes 1 and 3) will be given.

Table 8 shows the code, category, and frequency information of the prospective teachers' answers to the fourth question with respect to how the perimeters of two shapes with equal areas change.

Table 8. Prospective Teachers' Answers about how the Perimeters of Shapes Change

Category	Code	Prospective Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	P5	1
	Finding the error correctly and recommending a correct solution	P1, P3, P9, P10	4
Not finding the error correctly	Finding the error incorrectly and no solution recommendations	P8	1
	Finding the error incorrectly and recommending an incorrect solution	P2, P6, P7	3
Finding the error partially correctly	Finding the error partially correctly and recommending an incorrect solution	P4	1

When the explanations of the teachers on the student's answer with respect to how the perimeters of two shapes with equal areas (1 and 2) change were considered, half of the prospective teachers had sufficient knowledge of students' understanding in determining the error, and yet more than half had insufficient knowledge of instructional strategies to propose correct solutions.

The ideas of P10, who identified the student error correctly and offered a correct suggestion, were as below.

> "The areas of the first and second shapes are equal. The student couldn't figure out how the sides of the parallelogram would change while comparing their perimeters. Probably, he was inclined to find a relevant formula as the length of the vertical edge leg was given as 4 cm, but the lengths of the side legs were not given. The length of the base side was given as 9 cm, and because the lengths of the height and the side legs were not given, the student automatically thought there was missing information. To correct the error, I would emphasize that knowing the side lengths is not essential. I would remind the student that the side lengths of the second shape would be longer than 4 cm because of the Pythagorean theorem. That is, the perimeter would measure longer."

The following were the views of P6, who failed to identify the student error as she shared the same opinion as to the student and thus offered an incorrect suggestion.

> "In the second shape, the side lengths are not given. Oh no, I can't find the perimeter. I think something is missing. There is missing information in the base length as well... I can't say anything. I guess the student thought right."

The ideas of P4, who identified the error partially correctly but could not recommend a correct solution, were as follows.

> "I think the student thought wrong. A triangle is formed in the second shape, and the student could not find a connection between the side of the triangle and the parallelogram. No other information is needed."

Table 9 shows the code, category, and frequency information of the teachers' answers to the fourth question with respect to how the perimeters of two shapes with equal areas change.

Table 9. Teachers' Answers about how the Perimeters of Shapes Change

Category	Code	Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	T3	1
	Finding the error correctly and recommending a correct solution	T1, T5, T7, T10	4
Not finding the error correctly	No answer	T4, T6, T8	3
	Finding the error incorrectly and no solution recommendations	T2	1
Finding the error partially correctly	Finding the error partially correctly and no solution recommendations	T9	1

When the explanations of the in-service teachers on the student's answer with respect to how the perimeters of two shapes with equal areas (1 and 2) change were considered, three of the teachers could not make any comments and one of them had the same opinion as the student. These results show that half of the in-service teachers had sufficient knowledge of students' understanding to identify the error, but most of them had insufficient knowledge of instructional strategies to offer correct solutions.

T5, who spotted the student error accurately and offered a correct recommendation, expressed the following opinions.

> "He looked at the first and the second shapes. There is no missing information; the student is wrong. I would tell him that, when we compare the first and second shapes, the length of the side edge of the second shape is longer. While doing it, I would also remind him of the hypotenuse. In the second shape, if the vertical edge is 4 cm, because of the property of the hypotenuse, the side edges must be longer than 4."

T9, who partially identified the error but could not recommend a correct solution, expressed the following ideas.

> "The areas of the first and the second shapes are equal. When we compare their perimeters, we see that the student thought wrong. …because the lengths of the short sides of the parallelogram are different than those of a rectangle, their perimeters are also not equal to each other."

As it may be seen, even though T9 mentioned the existence of student error, she did not point out what this error stemmed from.

The answer given by T2, who identified the error in an incorrect way, was as follows.

> "The student gave the wrong answer…because the shapes with the same areas are 1 and 2. When their perimeters are taken into account, while the perimeter of the rectangle is 26, that of the parallelogram is smaller than 26."

In the fourth question, the teachers were also asked to comment on how the areas of two shapes with equal perimeters (Shapes 1 and 3) changed. The findings related to this were as follows.

Table 10 shows the code, category, and frequency information of the prospective teachers' answers to the fourth question with respect to how the areas of two shapes with equal perimeters change.

Table 10. Prospective Teachers' Answers about how the Areas of Shapes Change

Category	Code	Prospective Teachers	Frequency
Finding the error correctly	Finding the error correctly but no solution recommendations	P3	1
	Finding the error correctly and recommending a correct solution	P1, P5, P6, P8, P10	5
Not finding the error correctly	Finding the error incorrectly and recommending an incorrect solution	P2, P7, P9	3
Finding the error partially correctly	Finding the error partially correctly and recommending an incorrect solution	P4	1

When the prospective teachers' answers to the question related to the student error in finding the areas of shapes with the same perimeters were analyzed, it may be noted that most of the prospective teachers correctly identified the student error and half of them had sufficient knowledge of instructional strategies to suggest the correct solution.

The following was the answer given by P6, who spotted the error correctly and provided a correct recommendation.

> "Now, we see that the perimeters of the first and the third shapes are equal. Here, the student must have thought that he could not calculate the area as the height was not given. …but if he had drawn a vertical line starting from the corner, he could have found it, indeed. When we form a triangle by taking the vertical edge as the height and the short edge as the hypothenuse, the height is supposed to be smaller than the hypothenuse that is 4. …as the area of the third shape will be less than 36, that is, smaller than the area of the first shape."

The statements of P9, who could not identify the error as he thought the same way as the student, maybe quoted as follows.

> "The student will compare one to three. Now, the third shape is a parallelogram, and its base length is 9 cm, but its short edge is 4 cm, and these edges do not intersect vertically. I think the student said it right. …because the [height of the] first shape and the height of this shape will turn out to be different, and their areas will also be different. …but as we don't have the necessary information, we can't say anything about it now. Some more information should have been given."

The response given by P4, who disagreed with the student but still partially identified the error and provided incorrect recommendations, is quoted below.

> "The first and third shapes must be dealt with. I think what he said is wrong. First of all, when we look at them, the perimeters [of the first and third shapes] may be equal, but the area will change in parallel with the shape. The reason for this is that the area calculations for shapes such as the square, the rectangle, and the triangle are different."

Table 11 shows the code, category, and frequency information of the teachers' answers to the fourth question with respect to how the areas of two shapes with equal perimeters change.

The findings in Table 11 revealed that half of the in-service teachers had sufficient knowledge of students' understanding to determine the error, but only three of teachers had sufficient knowledge of instructional strategies to offer the right solution. When the findings were analyzed, it was found that the in-service teachers had the most difficulty with the fourth question as not all the side lengths of the parallelogram were given, and neither were the angles or height.

T1, who identified the student error correctly and offered correct recommendations, stated the following ideas.

> "The student wants to be given everything. When he compares the first and the third shapes, he thinks that since the height is not given in the third shape, he cannot calculate the area either. The student was not asked to calculate the area anyway. He was asked to make a comparison. To show it to the student, I would show him that the area of the rectangle is larger. If the short side of the parallelogram is 4 cm, the height of the long side will be less than 4 cm. ...because there is the hypotenuse."

Table 11. Teachers' Answers about how the Areas of Shapes Change

Category	**Code**	**Teachers**	**Frequency**
Finding the error correctly	Finding the error correctly but no solution recommendations	T3, T4	2
	Finding the error correctly and recommending a correct solution	T1, T5, T7	3
Not finding the error correctly	No answer	T8	1
	Finding the error incorrectly and no solution recommendations	T2, T10	2
	Finding the error incorrectly and recommending an incorrect solution	T6	1
Finding the error partially correctly	Finding the error partially correctly and recommending a partially correct solution	T9	1

T6, who wrongly identified the error of the student and offered incorrect recom-menddations, stated the following ideas.

> "The perimeters of one and three are equal. When we compare their areas, we see that the student gave a correct answer as the height is unknown in the third shape. Concerning this, even if the height is not given, if the angle had been given, we could have done something."

The ideas of T9, who partially accepted the wrong answer as correct and partially gave a correct recommendation, were as below.

> "There is no need for additional information. As the sides of the first and third shapes are equal to each other, their perimeters are equal, as well. …but as the width of the rectangle and the height which is used in the area calculations of the parallelogram in the third shape are different, their areas will also be different."

Discussion

This study aimed to investigate the pedagogical content knowledge of student errors in terms of the relationship between area and perimeter in rectangles, squares, and parallelograms by mathematics teachers and prospective mathematics teachers in accordance with the knowledge of students' understanding and instructional strategies. Data analysis revealed that in-service teachers' knowledge of students' understanding of errors and their knowledge of instructional strategies are better than prospective teachers. Findings of this study show that in-service teachers were more capable of detecting students' errors and suggesting the accurate solution than prospective teachers. When analyzed for each question, both in-service teachers and prospective teachers were able to correctly identify student errors in the third question about area conservation. Also, the knowledge of instructional strategies by both groups was not sufficient in the first question.

When the responses of the participants on the student misconception as the perimeter of the rectangle increases, its area also increases were analyzed, it was observed that most prospective teachers thought the same way as the student. Although the in-service teachers generally detected the error correctly, some of them were also found to detect the error incorrectly. In their study looking into the misconception that as the perimeter of the rectangle increases, its area also increases, Ma (1999) established that teachers had the same misconception as students. On the other hand, De Sousa, Gusmão, Font, and Lando (2020) found that teachers had some difficulties of understanding different ways to calculate an area. Livy, Muir, and Maher (2012) and Wanner (2019) also found that prospective teachers had the same mistakes as students. When the recommendations of the participants regarding the student error were examined, it was seen that the in-service and prospective teachers made recommendations by giving the rectangular examples where the areas decreased, despite the increase in the perimeters. Consequently, according to the answers given in response to the first question, the in-service mathematics teachers seemed to have more knowledge of students' understanding and instructional strategies than the prospective teachers. As regards this, Menon (1998) also pointed out that the pedagogical content knowledge of prospective teachers developed in time. As Menon put it, prospective teachers can teach certain concepts better, highlight the connections between subjects better and conceive better examples in time.

When a parallelogram was created from a square, it was seen that the in-service teachers perceived it more correctly compared to the prospective teachers in terms of the student error regarding how the perimeter lengths changed. This finding regarding the change in the perimeter lengths was in parallel with that in Tan Şişman and Aksu's study (2009), although their subjects were seventh graders. Their study revealed that seventh graders did not believe that the perimeter of the shape would change when a new shape was formed after it was cut into small pieces and reassembled using the same pieces. In another study parallel to these findings, Jirotková, Vighi, and Zemanová (2019) gave a particular trapezium to 10-11-year-old children and asked to compare areas and perimeters of three geometrical figures created by two trapezia congruent to the first, connected in three different ways. Students aged 10 to 11 thought that these shapes had the same perimeter since these shapes were made up of the same parts. According to Jirotková, Vighi, and Zemanová (2019), the reason of this confusion is related to students` visual perception of the area. Instead of deep reasoning, students predominantly visualize the area rather than perimeter and the visualization of perimeter might be eliminated (Jirotková, Vighi, & Zemanová 2019).

In the third question, the student was asked to form a new shape out of a rectangular sheet after it was cut downwards starting from its long side with zigzags by placing the cut-out part below the rectangle and then to compare the areas of the first and the second shapes. In this case, some prospective teachers stated that they increased the area because the second form took up more space. Similar to this result, Baturo and Nason (1996) found that prospective teachers had limited subject matter knowledge, and they were unaware of the fact that even if cut into separate parts, a two-dimensional area of a shape would stay the same when the same parts are reassembled forming a different shape. Here, the prospective teachers confuse the concepts of perimeter and area and the fact that they have misperceptions related to area conservation. In a similar vein, Lee (2009) also examined the views of elementary prospective teachers about the parallelogram area through shearing and squashing processes. In this study, it was also determined that the prospective teachers had the misconception that if the parallelogram perimeter increased, the area also decreased or if the perimeter increased, the area also increased. Although conducted with seventh graders, regarding this, according to Marshall (1997), 7th-grade students had a strong understanding of the concept of perimeter, but their understanding of the concept of area was not well-developed. In his study, he also found that the relationship between area and perimeter could not be understood. Tan Şişman and Aksu's study (2009) also supported these findings in that seventh graders did not have a conception of area conservation.

In the fourth one, the first subject in question was the misconception that the perimeters of the parallelogram and the rectangle cannot be compared when all sides are not known. Some of the prospective teachers thought that, as there were missing sides, nothing could be said about the length of one side of the parallelogram. Some of the in-service teachers, on the other hand, made a mistake in comparing the perimeters as one of the sides of the parallelogram was not given.

The second subject in question four was the misconception that the areas of parallelograms and rectangles whose perimeters are equal cannot be compared since all the side lengths or heights are not known. Some teachers and prospective teachers agreed with the student since the height was not given, a comparison could not be made, as there was missing information. When the obtained data were examined, the fourth question was found as the most difficult by the participant in-service and prospective teachers because of the fact that the angles, all side lengths and height were not given. In her study, to expose prospective teachers' subject matter knowledge, Murphy (2012) asked them to compare the perimeters of shapes with equal areas and the areas of shapes with equal perimeters. Although the two studies were similar in that, in both studies, prospective teachers were found to have difficulty finding the area of the parallelogram, this study was different from the other one as it was also found in this study that the prospective teachers compared the areas and perimeters by placing two shapes on each other. Regarding to this in her study, Herendiné-Kónya (2015) showed students at the ages of 7-11 two parallelograms with the same lengths but different areas and wanted the students to compare the areas of these parallelograms to the same edges, and it was found that, although it was easy for them to see that the areas were different, the students claimed that the area of these two parallelograms was the same.

Conclusion

Overall, when the answers given in response to the interview questions are taken into account, it may be stated that, even though they were on different levels of learning, neither seventh graders nor in-service teachers or prospective teachers had proper conceptual learning. The participants were observed to have the most difficulties in questions related to parallelograms. Marchis (2012) also established in her study that prospective teachers were less successful in tasks related to parallelograms. In this study, the prospective and in-service teachers tended to make a calculation on area and perimeter, but they had hardships explaining the concepts. This situation may have stemmed from the in-service teachers and prospective teachers focusing on computational knowledge rather than conceptual knowledge. Similar to these findings, Menon (1998) and Stemn (2020) conducted to investigate postgraduate and prospective elementary school teachers understanding of perimeter and area in rectangles and triangles and rectangles, also maintained that they had a computational understanding of them rather than a conceptual and relational one. In their studies on the subject matter knowledge of prospective teachers, Berenson et al. (1997), on the other hand, established that many of them had computational knowledge. According to Livy, Muir, and Maher (2012), this computational knowledge limits students' development of conceptual understanding and the potential of instructional strategies.

The results of this study indicate that prospective teachers had less pedagogical content knowledge than the in-service teachers related to area and perimeter. These differences were mostly related to knowledge of students' understanding of errors about area and perimeter. The reasons for these may be that lack of the prospective teachers' experiences or subject matter knowledge. To eliminate these problems, as Setyaningrum, Mahmudi, and Murdanu (2020) stated, students' misconceptions should be exposed more to the teacher training program. In this way, prospective teachers can learn how to spot mistakes of the students and how to identify students' mathematical thoughts during their teacher training programme at undergraduate level; and also have the opportunity to develop pedagogical content knowledge (Runnalls & Hong, 2019). In this study, in addition to pedagogical content knowledge, it was also observed that prospective teachers were incompetent in required mathematical subject matter knowledge compared to in-service mathematics teachers. In the existing literature, some studies highlight that prospective teachers and in-service teachers have superficial content and pedagogical knowledge about the relationship between area and perimeter. For instance, Livy, Muir, and Maher (2012) noticed that prospective teachers had similar strengths and weaknesses as regards to their subject matter knowledge and pedagogical content knowledge related to area and perimeter, and they stated that prospective teachers had deficiencies in subject matter knowledge, which is necessary for them to understand area and perimeter and make a connection between them. Kurt-Birel, Deniz, and Onel (2020) also found that primary school teachers had only superficial knowledge about area and perimeter of shapes. These researchers claimed that when teachers lacked mathematical subject matter knowledge, they also lacked pedagogical content knowledge and that a well-developed pedagogical content knowledge could be obtained with good mathematical subject matter knowledge. Therefore, in future studies, not only pedagogical content knowledge about perimeter and area, but also subject matter knowledge could be investigated.

Although prospective teachers take teaching classes during their undergraduate education, the instructional explanations they made in this study were not sufficient. In this respect, teaching classes may be revised both theoretically and practically to prepare prospective teachers as aimed by the mathematics curriculum. In addition to all this, when perimeter and area are first taught, teachers should teach students what these two concepts are through concrete materials or real-life examples instead of giving them formulae. It is possible to comprehend the perimeter-area relationship by presenting different geometric shapes consisting of the combination of the same number of unit cubes. Another activity recommended for a better understanding of the relationship between area and perimeter is puzzle game known as pentominoes (Wanner, 2019). By including these activities at all levels of education, the relationship between area and perimeter can be easily understood. Moreover, as pointed out by Tan Şişman and Aksu (2009), teachers could help reinforce the understanding of the changes in perimeter and area conservation through activities at school involving cutting, folding, and reassembling. Future studies should examine, in-service teachers' and prospective teachers' pedagogical content knowledge about the relation of area and perimeter of special quadrilaterals other than square, rectangle, and parallelogram can be examined more comprehensively.

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Appendix

Dear participants,

Please answer the following questions sincerely so that we can figure out your opinions. The length of our interview will be approximately 40 minutes. In this study, your identity will be kept confidential. Thank you for your participation.

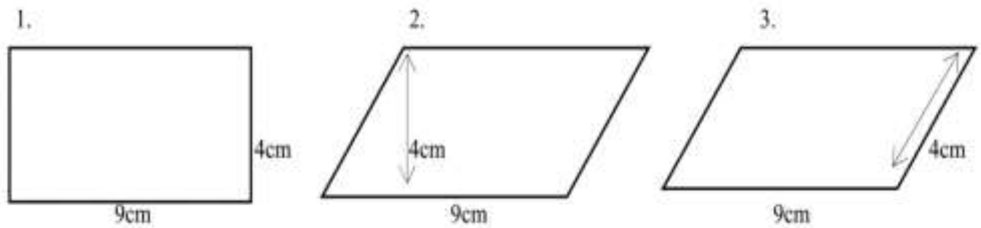
Interview Questions

1) One of the students said, "If the perimeter of the rectangle increases, its area also increases." Do you think this statement is correct? What would your answer be? Explain.
2) Form two rectangles by cutting a square sheet of paper into two equal parts. Cut one of these rectangles diagonally into two identical parts. Using all of the shapes that you have obtained (2 triangles and 1 rectangle), form a parallelogram. How did the area and the perimeter measurements of the first shape and the newly formed shape change? Explain."

   In response to this question, one student wrote "The perimeter of the parallelogram is less than that of the square."

   Do you think what the student says is right?
   Why might this student have thought in this way?
   Were you to encounter such a situation, how would you react in response to such an explanation?
3) After cutting a rectangular sheet of paper downwards starting from its long side with zig-zags, form a new shape by placing the cut-out part below the rectangle. How different are the areas of the first shape and the newly formed shape? Explain." In response to this question, one of the students said, "The area of the second shape is bigger than that of the first shape."

   Do you think what the student says is right?
   Why might this student have thought in this way?
   Were you to encounter such a situation, how would you react in response to such an explanation?
4) The students were asked to explain i) how the perimeters of the shapes with identical areas have changed and ii) how the areas of shapes with the same perimeters have changed by looking at these shapes. One of the students said that these shapes could not be compared, for not all of the sides or height measurements are known.



   Do you think the way this student thought is correct?
   Why might this student have thought in this way?
   Were you to encounter such a situation, how would you react in response to such an explanation?

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Effectiveness of the Big Math for Little Kids Program on the Early Mathematics Skills of Children with Risk Group

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The aim of this study is to determine the effect of Big Math for Little Kids (BMLK) Program on the early mathematics skills of children with lower socioeconomic level. The participants of the study consist of between 60-72 months of aged children with lower socioeconomic level recruited to kindergartens of Ministry of National Education from Turkey in Diyarbakır in the 2018-2019 academic year. The sample of the study consists of 40 children with above mentioned characteristics. Of these, 20 were assigned to the experimental group and 20 to the control group. Two schools were selected for experimental and control group, and the study was conducted after selecting ten children in a class of each school. In the study pretest-posttest with control group experimental model was used. The data of the study were collected through the Tests of Early Language Development Test (TELD-3) to determine children with adequate language skills of their own age group, and Test of Early Mathematical Ability (TEMA-3) to assess early mathematical development of children. As a result, BMLK program was determined to be effective in the mathematics development of children with lower socioeconomic level.

Keywords: early mathematics, early intervention, preschool period, big maths for little kids.

Introduction

Human development starts the moment when one is born and continues through lifetime. However, the early childhood period, which covers the ages of 0 to 8, is when the development is at its fastest and when humans are most open to the influence by their environment. The early childhood period has an important impact on a child's life in the future and children are ready to learn many skills during this period (Birkan, 2002). For this reason, early childhood is an important period in which foundations of many mathematical concepts are laid down, as in all areas of development, and in which children are open to develop mathematical skills (Clement & Sarama, 2007). Before school, concepts such as number, counting, measuring, shape, time and space come to mind when mathematics are considered. Such concepts contribute to the cognitive development of children and lay the foundation of early mathematical skills (Güven, 2005). From the moment he/she is born, the goal of a child is to explore the world.

During the early childhood period, children learn and start to use countless math-related concepts. This acquisition happens naturally as the child interacts with his/her surroundings and people around. During the pre-school period, children gain a lot of experience on numbers and quantity. They compare

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quantities, perceive the concepts of few and many and come up with various solutions to balance a tall building made of blocks. A few examples would be counting how many olives he/she ate, matching the number of plates to the number of napkins, sharing cookies equally among friends and counting fingers. Mathematical skills acquired during the early childhood period serve as the foundation of more complex mathematical information to be learned throughout school by supporting abilities to solve problems, making analyses and hypotheses, which form the basis of children's scientific thinking skills. In addition, children's early math skills are directly related to their future academic success, possibility to graduate high school with higher grades, higher employment rates, and professional achievement (Kroesbergen et al., 2009). In addition, a child's experience with pre-school math has a direct influence on that child to not develop a fear of math throughout school life, feeling excited for learning math and to develop a positive attitude towards math (Starkey & Klein, 2008; Williams & Coles, 2007). For this reason, early mathematical skills are a part of pre-school education programs in many developed countries.

Early mathematics skills need to be developed in pre-school period and children may exhibit different developmental characteristics in these skills. Some children lag, unable to develop their skills as good as their peers. The most important group among such are children of low socioeconomic status (Brooks-Gunn & Duncan, 1997; Ginsburg, Lee & Boyd, 2008; Schreier & Chen, 2013). Research shows that children of low SES families lag one development year behind children of high SES families when pre-school math performance is considered (Hughes, 1986; Klein, Shim, Scales & DeFlorio, 2002). This SES gap presents itself as early as age 3 and continues to widen during the school year if not intervened (Klein et al., 2008). Starkey et al. (2004) identified a significant gap against low SES between math performances of low SES and mid SES pre-school students despite similar ethnic backgrounds. In their study, Griffin et al. (1994) observed that 5 to 6-year-old children of low SES families exhibit similar level of math skills compared to 4 to 5 year-old children of mid SES families. In their study in the US, Denton and West (2002) indicated that children with low SES scored half a standard deviation less than the average compared to other SES groups in a nation-wide standard math exam. In another study, pre-school students of low SES scored less in math than their peers of other SES groups and this score gap kept widening up until the 8th grade (Schoenfeld & Stipek, 2011). In summary, it appears that SES is an important factor in children's mathematical success.

Another factor affecting children's early math skills is the type of school the children attend. The effect of the school factor is that teachers in public and private schools have different expectations about the child (Sparkes, 1999). Teachers and families in disadvantaged schools often have low expectations about children (Sparkes, 1999). Research shows that when teachers' expectations are low, students are less motivated and, as a result, perform less in math skills (Eamon 2005). Another factor in children's math skills is the low participation of families in the school. Regardless of income groups, it is known that the participation of families in children's school activities increases the academic performance of their children (Galindo & Sheldon, 2012). Another reason for having difficulties in math skills is cultural differences and bilingualism (Keels & Raver, 2009). Studies show that the difference between children's mother tongue and school language negatively affects the academic achievement of children, and the level of school readiness of bilingual and immigrant children is lower than other children (Fuligni & Yoshikawa, 2004; Magnuson, Lahaie & Waldfoger, 2006).

Even though the family's income is the most important factor influencing the socioeconomic status, it also includes variables such as the educational level of the family, resources at hand (a studying room, a computer), and the professional status of the family (Bradley & Corwyn, 2002). The fact that children with low SES are less exposed to or deprived of care, rich learning environments and stimulating environments such as friends and school causes them to display a poorer performance than children of other income groups (Case, Lubotsky & Paxson, 2002; Ginsburg et al., 2008; Schreier & Chen, 2013).When socioeconomic status is considered in Turkey, it is safe to say that there is a significant population with low income status in the Southeastern Anatolia Region. In this region, development levels of all provinces are lower than the country average, except for Gaziantep (TUİK, 2015). There are no intervention programs or additional support in Turkey's educational system targeting low SES children. This results in an opportunity gap in education and the gap between academic performances of children at different socioeconomic levels widens (Myers, 1992). Children of low SES scored less than their peers in all academic fields in mathematics in particular at the Trends in International Mathematics and Science Study carried out by the International Association for the Evaluation of Educational Achievement on students in fourth and eighth grades and at the Program for International Student Assessment carried out on 15-year-old students. According to the PISA 2015 report, children of low SES lag one school year behind children of high SES (OECD, 2017). The average score of students of low SES was 376 while this figure was 480 among students of high SES (OECD, 2017). According to quantitative research on primary school attendance, primary school attendance of children of low SES is lower than those of other SES groups (ACEV, 2006).

In Turkey, it is highly important to support children of low SES on all fields of development on mathematics. Children living in low SES regions are unable reach their cognitive potential because they are not stimulated enough by their environment. Early and long-lasting interventions can prevent the adverse impacts of a negative environment on a child's potential (Thompson & Nelson, 2001). Studies also suggest that early interventions improve children's early mathematical success and impacts of such interventions are felt for years (Watts et al., 2016).

There are many studies suggesting that early intervention programs ensure school readiness and eliminate socio-economic based gaps (DeLoach 2012; Opel, Zaman, Khanom & Aboud 2012; Presser, Clements, Ginsburg & Ertle 2012; Starkey, Klein & Wakaley, 2004). High quality early childhood programs that consider the individual characteristics of a child, his/her needs and social and cultural environment are of importance in the education of children of the low SES (Griffin et al., 1994). Children of the low SES, who attend pre-school institutions that do not offer such an education, often experience persistent failure since they are unable to benefit from an intervention on pre-school math skills (National Council of Teachers of Mathematics [NCTM], 2006; National Research Council, 2009). In line with this need, many countries developed and included pre-school intervention programs targeting disadvantaged children into their mathematical curricula.

It is noted that children, who complete this education, are academically more successful, work in higher quality jobs and benefit more efficiently from healthcare services. For the long term, a higher quality labor force also contributes to the economic development of the society (Coble & Allen, 2005). Mathematical skills are vital for training employable citizens and approximately 90% of all professions require a certain level of math skills (House 2006; Mikulski, 2001; Steen, 2001). For this reason, it is highly important to support children in the early period with necessary mathematical skills and design programs accordingly.

The BMLK Educational Program to be covered in this study is used in many countries especially with children of the low-income group and has been proven to be effective. Therefore, implementing the BMLK program in Turkey would be instrumental in developing the mathematical skills of children of the low SES, helping them to start school prepared and preventing academic failures in the future. This program is also thought to be important in terms of addressing the lack of intervention programs on pre-school mathematical skills.

As a result of the study, it is believed that information on the BMLK's effectiveness will not only serve to improve the future academic success of children of the low SES but also of children with developmental disabilities. The study is also significant in terms of setting out the mathematical training needs of pre-school children of the low SES. Finally, the study is thought to shed light on other studies on children's mathematical development and contribute to the introduction of early intervention programs.

Method

Research Design

This study adopted the experimental model with pretest/posttest/control test design with a control group. In experimental designs with pretest and posttest control group, experimental and control groups are formed and these groups undergo pre-experiment and post-experiment measurements (Büyüköztürk, 2012).

In the study, children in the experimental group were administered the BMLK program, while children in the control group continued receiving their regular education curriculum. For this study, the dependent variable is the "mathematical development" of children in the study, while the independent variable is the "Big Math for Little Kids Educational Program", the impact of which on children's mathematical development is being analyzed.

Study Group

The study group comprised children who attend kindergartens that are affiliated to the Ministry of National Education in Diyarbakır city center in the educational year of 2018-2019. A list of central district kindergartens was received from Diyarbakır Provincial Directorate of National Education to create the sample group. The lists were reviewed, another list of schools of the low socio-economic status at the same district was drawn up and four schools were randomly selected. Then, information on the schools' SESs was verified through interviews held with administrators and teachers of the listed schools and the researcher identified 4 classes in 4 schools, in which the study will be carried out, after providing information on the educational program to be administered. This study included children of the low socioeconomic status with average recipient language skills according to the TELD scores, without any medical diagnosis and are reported to have an average success level by teachers. There are 11 girls and 9 boys in both test and control groups. The mean age of children in test and control groups is 64 months.

Data Collection Tools

In the study, the Test of Early Mathematics Ability (TEMA-3) and the Test of Early Turkish Language Development (TELD) were performed to evaluate children's early mathematical development and level of Turkish recipient language skills, respectively. After the application, the researcher asked questions about the educational program to hear the views of children, families, and teachers by making use of the social validity questionnaire developed by the researcher.

Test of Early Mathematics Ability (TEMA-3; Ginsburg & Broody, 2003)

TEMA-3 was developed to evaluate the mathematical abilities of children between the ages of 3-0 through 8-11. TEMA-3 consists of 72 questions measuring early mathematical skills such as numbering, numeral literacy, and number comparison.

Each question is marked as correct or incorrect and the number of correct answers gives raw scores. The raw score obtained from the test can be converted into age equivalents, percentile ranks and a standard score (Math Ability Score). The increase in math ability score points to an improvement in a child's math skills (Ginsburg & Baroody 2003). An increase of at least 4 points suggests that the increase in Math Ability Score is statistically significant. In the original form of the test, a standard score of 69 and below is considered very poor, 70 to 84 below average, 85 to 92 average, 93 to 107 above average, 108 to 115 superior and 161 and above very superior mathematical skills. A normed study for this test has yet to be conducted in Turkey.

Test of Early Turkish Language Development (TELD; Topbaş & Güven, 2013)

TELD is the Turkish adaptation of the language development test titled the Test of Early Language Development (TELD-3). The test was developed to assess the language development of children between the ages of 2-0 through 7-11. TELD is used to identify children with language development problems, to come up with an intervention program for such a problem and to ensure early identification of children at risk of academic failure (Topbaş & Güven, 2013). The test contains a total of 76 questions assessing semantics and grammar.

The internal consistency coefficient for the recipient language test used in this study was found to be .93. The test is administered individually to each child. For each question, the score is 1 for correct or 0 for incorrect. Raw scores are converted into standard scores based on age. For the TELD, a standard score of 131 to 165 is considered very good, 121 to 130 good, 111 to 120 above average, 90 to 110 average, 80 to 89 below average, 70 to 70 poor and 35 to 69 very poor language skills.

The Social Validity Questionnaire

Social validity data that is required to demonstrate the effectiveness and functionality of the BMLK program were acquired from answers of Social Validity Data Collection Questionnaires developed for children, families, and teachers. For this purpose, three questionnaires have been developed and presented below.

Social Validity Questionnaire for Families has 5 questions. In this questionnaire, families were asked if they were content with the fact that early mathematical skills were being taught, if there was anything they would like to change in the program, if the program had a contribution in developing children's early mathematical skills, if home activities had a contribution in developing children's mathematical skills and if they see any change in their children in terms of their interest in and motivation for mathematics.

Social Validity Questionnaire for Teachers has 5 questions. In this questionnaire, teachers were asked if they were content with the fact that early mathematical skills were being taught, if there was anything they would like to change in the program, if the program had a contribution in developing children's early mathematical skills, if they would like to implement this program in their classrooms and if they see any change in children in terms of their interest in and motivation for mathematics.

Social Validity Questionnaire for Children has 3 questions. In this questionnaire, children were asked if the program was fun, if they would like these activities to be implemented in their own classrooms and what they liked and disliked in the program.

Families and teachers were asked to answer "yes, no, or partially" to the questions and then to elaborate on their answers. Face-to-face interviews were held with all participants to collect social validity data. In addition to questions in the questionnaire, all participants were asked whether they would like to add anything else.

Data Collection

Implementation Process

Big Math for Little Kids Educational Program to be Implemented. The BMLK Educational Program is based on supporting the mathematical development of pre-school children aged 48 to 72 months. It is a research-based developmental program funded by the National Science Foundation and developed by Ginsburg et al. in 2003 as a kindergarten and pre-school program to prepare children to primary school. The activities in the program were developed in six fields namely, number, shape, pattern, logical reasoning, measurement, operations on numbers, and space.

The materials/activities in the program were designed specifically for the needs of children at a specific level of development. The program consists of a teacher resource binder, a general program overview and teacher guide booklets for each 6 unit. Units included in the program are: "What Are Numbers?", "The Shape of Things", "Patterns Plus", "Measure Up!", "Working With Numbers" and "Getting Around". Program that was designed for each development level also includes 6 colored storybooks for each unit. Homework storybooks, which are black and white versions of classroom storybooks, were prepared for children to fill out with their families at home (Ginsburg, Greenes & Balfanz, 2003). Materials in the program were designed to be low-cost, to meet the needs of all children and to be accessible for children of all economic levels. The program also includes teaching activities, evaluation materials and reproducible activity pages. The program that was designed for each development level also includes 6 colored storybooks for each unit. Homework story books are for children to use at home with their parents. The BMLK Educational Program has been used in Turkey since its adaptation.

Before implementing each activity in the BMLK, the educational environment was arranged. Based on the type of activity, either a roundtable setting was established or desks and chairs were moved aside to facilitate mobility. Before each activity, the researcher brings to class pre-copied activity sheets, assessment forms, and necessary materials. Before the implementation, the researcher has a little chat with students and informs them about the activities. At the end of every activity, what was learned that day was summarized and children were told what to do for the next activity. Four units in the Big Math for Little Kids Educational Program were administered over the course of 11 weeks.

After the implementation of the BMLK, TEMA-3 was administered to experimental and control groups as a post test. The test was read ministered to the experimental group to see if the effectiveness of the BMLK was permanent. After the implementation of the follow-up test, children, families, and teachers were asked their opinions about the program.

Data Analysis

Data collected within the framework of the study was analyzed using the IBM SPPS 22 package program. To serve the purposes of the study, first it was checked if data showed normal distribution. The coefficient of skewness was found to be between -3.4 and -4.5 and the coefficient of kurtosis to be between -8.0 and -9.3. According to the Kolmogorov-Smirnov test, not all measurements showed normal distribution (K-S(z)=0.00; p<.00) (Pallant, 2015).

Since TEMA-3 pretest/post test scores of experimental and control group children and TEMA-3 posttest and follow-up test score averages of experimental group children did not show a normal distribution, the *Wilcoxon Signed Rank Test (matched pair)* was use. The *Mann Whitney U Test* was used to compare TEMA-3 pretest and post test score averages of experimental and control group children.

The effect sizes were also measured during group comparisons. For the Mann Whitley U and Wilcoxon Signed Rank Tests, the effect size was calculated by dividing the *z* value by the square root of the sample size (Pallant, 2015). According to Cohen's criteria, .1 indicates small, .3 medium and .7 large effect size (Cohen, 1992).

Findings

The aim of this study was to examine the effectiveness of the BMLK Educational Program in developing the mathematical skills of kindergarten children of low socio-economic status. In line with this aim, findings came out from the study are presented below in relation to the research questions.

Any possible significant difference between the TEMA-3 pretest and posttest score averages of experiment and control groups was analyzed by making use of the *Mann Whitney U-Test*. The average scores, standard deviations, rank averages, rank sums, *U* and *p* values and effect sizes of groups are listed in Table 1 and 2.

Table 1. Pretest Results of TEMA-3 Scores of Children Participated in the Study

Group	N.	$\bar{X}$	*SD*	Rank Average	Rank Sum	*U*	*p*	Effect
Experimental	20	80.95	7.67	19.30	386	176	.51	.01
Control	20	81.25	8.12	21.70	434			

Table 1 shows that TEMA-3 pretest scores of experimental and control groups are close. *Mann Whitney U-Test* did not find a significant difference between average TEMA-3 pretest scores of control and experimental groups (*U*=176, *p*=.51). These results show that children are at a similar level in terms of their early mathematical skills.

Table 2. Posttest Results of TEMA-3 Scores of Children Participated in the Study

Group	N.	$\bar{X}$	*SD*	Rank Average	Rank Sum	*U*	*p*	*Effect*
Experimental	20	93.85	2.36	29.13	582.50	27.50	.000	.68
Control	20	81.35	6.72	11.88	237.50			

The analysis results presented in Table 2 showed significant differences between TEMA-3 scores (U=27.50, p<.001). TEMA-3 scores and equivalent ranks of the experimental group were higher than of the control group. It is striking that the effect size between groups was high (.68).

Any possible significant difference between the TEMA-3 pretest and posttest score averages of the control group was analyzed by making use of the Wilcoxon Signed Rank Test. Mean ranks, mean sums, effect sizes and z and p values of experimental and control groups for TEMA-3 are listed in Table 3 and Table 4.

Table 3. Comparison of TEMA-3 Pretest and Posttest Score Averages of the Control Group

Posttest - Pretest	N.	Rank Average	Rank Sum	*z*	*p*	*Effect*
Negative Rank	4	3.75	15			
Positive Rank	3	4.33	13	-.17	.86	.03
Equal	13					

According to the results obtained from the Wilcoxon signed rank test in Table 3, there was not a significant difference between TEMA-3 pretest and posttest scores of students of the control group (z= -.17, p> .86).

Table 4. Comparison of TEMA-3 Posttest and Control Test Score Averages of the Experimental Group

Posttest - Follow-Up Test	N.	Rank Average	Rank Sum	*z*	*p*	*Effect*
Negative Rank	3	3.5-1	14-1			
Positive Rank	2			-1.83	.08	.01
Equal	15					

According to Table 4, there is no significant difference between the posttest and control test scores of the experimental group children (z=-1.83, p>.05). However, there was a slight decrease in the mean ranks of children in the follow-up test. Fifteen students received the same scores from the post and follow-up tests while two students scored higher in the follow-up test and three scored less.

Social Validity Findings

After the completion of the study, the Social Validity Data Collection Questionnaire was used to evaluate the effectiveness and functionality of the program through the eyes of children, teachers and families.

Families were positive towards the program answering "yes" to all questions but "is there anything you would like to change with the program?". Families pointed out that the program developed their children's mathematical skills. They also mentioned that games played, and storybooks read at home also attracted the attention of other children at home and improved their motivation towards school.

Teachers were positive towards the program answering "yes" to all questions but "is there anything you would like to change with the program?". After the application, teachers expressed that children wanted more math activities, were looking forward to activity time, were more interested in doing homework, worked more in harmony in groups, the activities improved the math skills of children and that they were surprised to see children liked these activities because they were bored with previous activities. Both teachers noted that they would like to use this program in their own classes if they were to receive a training on the program.

All the children in the experimental group stated that the activity was fun and that they would like to do a similar thing in their classrooms. All the children stated that there was not nothing they did not like with the administration.

Discussion

In this study, it was investigated whether the BMLK training program was effective in the development of math skills of children from lower socioeconomic level preschool child. In the findings obtained, the experimental group children participating in the training program were more successful in the tests evaluating their early mathematics development than the control group children who did not participate in the training program. As a result, the applied training program was found to be effective in the mathematics development of children from the lower SES. Findings for the research questions are discussed below,

Pre-intervention early mathematical skills of children were analyzed using the TEMA-3 and the pretest scores of children were found to be low and like one another. The average TEMA-3 score of the experimental group children and control group children were 80.95 and 81.25, respectively. Çakır's study (2019) found the average TEMA-3 score of 100 children, who attend kindergarten in Ankara and exhibit normal development, to be 98.5 while Çavdarcı (2016) found the average TEMA-3 score of 32 children between the ages of 48 and 72 months who attend kindergarten in Ankara to be 97.2. Even though there is no norm research on the test in Turkey, it can be concluded that children scored below average taken into consideration the scores in the original form (a score between 70 and 84 corresponds to below average) and recent studies. Bearing in mind the importance of early mathematical skills on children's future academic performance, the poor early math skills performance of children in this study of the low SES offers a justification for why this intervention program should be implemented.

As a result of the administration of the BMLK program, this poor performance of experimental group children considerably got better after the administration and when compared with the control group in the posttest. This result is consistent with the relevant literature (Aber, Jones, & Raver, 2007; Jordan, Huttenlocher, & Levine, 1992; Starkey & Klein, 2008; Wakeley, 2002; Young & Loveridge, 2004). These studies also found quality mathematical intervention programs to be effective in the early mathematical skills of children of low SES.

It is thought that the effectiveness of the BMLK program on children's early mathematical skills is due to its content, methodology and administration. It is noted that early childhood programs aimed at improving the interest of children in mathematics and developing their experience and knowledge and that contains sequential activities and integrates math with other activities can teach children many mathematical skills (NCTM, 2006). According to Skwarchuk (2009), child-centered, interesting, systematic, sequential and play-based math activities are effective in improving the mathematical skills of economically disadvantaged children. Research suggests that play-based learning improves children's academic and social skills (Katz, 2019). Through play, children can explore, experiment and solve problems through creative and fun ways (Skwarchuk, 2009). Since the BMLK program is a child-centered and play-based program encompassing all these features, it was effective in the development of children. Another proof that the program was effective is the fact that the experimental group children found the BMLK program to be fun and that they would like to do the same activities in the BMLK in their classrooms.

The social validity questionnaire and notes taken throughout the study show that teachers opt for teacher-centered teaching methods rather than play-based ones for teaching early math and literacy skills. In many countries, there has been a shift from a play-based experiential approach to more academic ones and from hands-on activities to worksheets and teacher leadership in kindergarten education. TEDMEM (2016) report found in Turkey that "pre-school education concentrates on teacher-centered practices and teaching methods thus leading to kindergartens that look like primary schools." In contrast, it is noted that pre-school children gain more out of play-based programs than academically oriented ones (Carlsson, McLaughlin & Almon, 2015). Children learn best when they are engaged in play-based activities targeting their development levels, previous experiences and current needs (Carlsson et al., 2015). Play-based education also enables children to engage in flexible and high-level thinking processes that are considered necessary for the 21st century student (Greenes, 2003). These processes are problem-solving, analyzing, assessment, social skills-gaining and knowledge and creativity application. For this reason, the early mathematical development of children, who took part in the child-centered BMLK educational program that involves play-based mathematical activities, was significantly greater than the ones.

One possible reason why BMLK has an impact on children's early mathematical skills is the involvement of families in the education process. It is inevitable for cognitive gains to remain short-term if families, who are capable of supporting children constantly, are not included in intervention efforts (Kağıtçıbaşı, 1998). It is thought that interventions that include families would not only benefit the child but also his/her immediate circle, especially in Turkey, where family ties are quite tight (Kağıtçıbaşı, 2010). In this study, parents stated that the administered program improved their children's mathematical skills. They also mentioned that games played and storybooks read at home also attracted the attention of other children at home and improved their motivation towards school. For the reasons explained above, it will be particularly important for teachers, working in low SES regions, to get informed on how families can contribute to their children's mathematical development at home through in-service training.

There are some limitations to take into consideration when evaluating the outcomes of this study. First, this study was conducted with 40 children. This limits the generalizability of the findings. Therefore, similar studies should be conducted with larger groups. Secondly, study findings are limited to data collected from children studying in kindergartens affiliated to Diyarbakır Provincial Directorate of National Education. Therefore, it is important to support the results by conducting studies in different regions.

Even though it seems unnatural to re-apply scientific-based intervention programs developed in developed countries in developing countries, there are no alternatives to such programs (Woodhead, 1985). Proving the effectiveness of early intervention programs is costly and requires at least 20 years of effort (Kağıtçıbaşı, 2010). Therefore, it is of importance to take a closer look at the results of intervention programs, which are adapted to Turkey from other countries, and try to evaluate their wider applicability. There seems to be no comprehensive study aiming to develop early mathematical skills of pre-school children with low income level. This study is significant since it is the first comprehensive intervention effort in Turkey to develop early mathematical skills of children of the low-income level. Administering the BMLK program to a larger sample group and conducting new studies that show the long-term effects of the achieved results can offer policymakers a more attractive set of findings to introduce the necessary changes and can lead the way for amendments to the pre-school curriculum.

Conclusion

In the study pretest-posttest with control group experimental model was used. The data of the study were collected through the Tests of Early Language Development Test (TELD-3) to determine children with adequate language skills of their own age group, Continuing Assessment and Checking Up Forms to evaluate the implementation process of the implemented program and Test of Early Mathematical Ability (TEMA-3) to assess early mathematical development of children. After applying the TEMA-3 test and the Continuing Assessment Form as a pre-test, four units of BMLK program were applied to the experimental group for 11 weeks by the researcher. The main themes of the four units are numbers, shapes, patterns and measurement skills. After the experiment, TEMA-3 vii test and Continuing Assessment Form were applied to the experimental and control groups as post-test. The obtained data were analyzed by using Mann-Whitney U and Wilcoxon tests. The results of analysis showed that the children in the experimental group scored better in the tests evaluating the early math skills when compared the control group children. As for the assessment of the implementation process of BMLK, the children in the experimental group were determined to have gained most of the program's goals. As a result, BMLK program was determined to be effective in the mathematics development of children with lower socioeconomic level.

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The Opinions of Mathematics Teacher Candidates Who Have Received a STEM Training on STEM and the Activities they Designed in the Class

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This study has been conducted to investigate the opinions of mathematics teacher candidates on science, technology, engineering and mathematics (STEM) training and the activities designed by them. It has been carried out using the case study method, which is one of the qualitative research patterns. In this study, semi-structured interview forms and activity cards have been used to gather data. While semi-structured interview forms served to elicit teacher candidates' opinions on STEM, activity cards were employed to ask them to design an activity in which they could put their training on STEM into practice. The study has been conducted with 34 senior mathematics education students and the data obtained have been analyzed using the content analysis method. It has been concluded that mathematics teacher candidates have positive views on STEM training. The activities designed by the candidate teachers can be categorized under four categories, which are interdisciplinary, the engineering field it is related to, the preferred method and the activities that are not suitable for STEM. Teacher candidates should be trained on how to integrate STEM education into their lessons. Stem activity examples should be presented to teacher candidates for applicability.

Keywords: Mathematics teacher candidate, stem, activity, training, opinion

Introduction

It is a widely accepted fact that knowledge is a necessity in an age of rapid advancement for everyone wanting to keep up with these advances because we are living in an age when knowledge is considered as power. In today's economy, which is relatively more global, more technological and more competitive, it is critical to progress in the disciplines of science, technology, engineering and mathematics (Raines, 2012). The necessity of combined efforts in such fields as science, technology, engineering and mathematics to find solutions to the problems in this ever-globalizing world is a natural result of this situation (Moore et al., 2014). In order to compete in the 21^{st} century, countries need an innovative STEM workforce (Çorlu, Capraro & Capraro, 2014).

STEM is a term which first started to be used in the 1990s by the National Science Foundation (NSF) as an acronym that stands for science, technology, engineering and mathematics (Bybee, 2013). STEM education, whose importance has been increasing in recent years, involves the integration of science, technology, engineering and mathematics with one another. As STEM blurs the line between disciplines by nature, integration is thought to be more in harmony with the nature of STEM (Wang, 2012). STEM is a whole new discipline which is connected to other disciplines (Morrison, 2006). Based on the integration of disciplines, STEM education aims to make a connection between disciplines so that learning can be connected, focused, meaningful and

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relevant to learners (Smith & Karr-Kidwell). It is an integrative approach which is translated into Turkish as FeTEMM (standing for Fen, Teknoloji, Matematik, Mühendislik Eğitimi), and helps students to adopt creative problem solving techniques (Akgündüz et al., 2015; Gülhan & Şahin, 2016). STEM education is an integrative approach aiming to make students, who are going to be the future innovators, embrace creative problem solving techniques (Roberts, 2012). STEM education covers the knowledge, skills and beliefs which are collaboratively formed at the intersection of more than one STEM area (Çorlu, Capraro & Capraro, 2014). While it mainly focuses on the disciplines of science and mathematics, STEM education also includes the areas of technology and engineering (Bybee, 2010b). Being able to align with contemporary educational standards and direct major components of educational reform, STEM has a significant potential for innovation in education (Bybee, 2013). STEM education aims for students to gain a multidisciplinary perspective when faced with problems and to acquire the necessary knowledge and skills (Şahin, Ayar & Adıgüzel, 2014). Instructional programs which are formed by joining more than one discipline in an interconnected way enable students to be informed about various areas and develop their problem solving and cooperative learning skills as well as increase their interest and motivation (Niess, 2005). As stated by Çorlu et al. (2012), STEM education is at the core of the reforms intending to raise a generation with the ability to innovate and thus, the scope and the theories of this approach should be looked into at school and university levels.

The Ministry of National Education (MoNE) in Turkey, which took action in 2016 to design the curricula in accordance with STEM education goals, announced that the learning outcomes in STEM-based education could be selected from among those specified by the Head Council of Education and Morality (Talim ve Terbiye Kurulu Başkanlığı-TTKB) and belonging to elective courses such as Environmental Education, Media Literacy, Creative Thinking, Scientific Applications, Mathematical Applications, Graphic Design (Integrated Teaching Project, 2016).

Overall, in STEM education, the connection between real life and the course content is established and the disciplines of science, mathematics, technology and engineering are aimed to be integrated (Yamak, Bulut & Dündar, 2014). According to Çorlu (2014), it is essential to develop research-based STEM strategies in our schools to foster cooperation among mathematics, sciences and technology and design teachers and to promote students' critical and creative thinking skills. STEM activities provides students with an opportunity to learn actively (Bransford, Brown & Cocking, 2000). A significant aspect of STEM education is that it has been integrated in such a way that the focus is on the real life applications of the fields of science, technology, engineering and mathematics and their approach in dealing with the complicated problems emerging in daily lives rather than teaching each discipline separately (Johnson, 2013; Roehrig, Moore, Wang, & Park, 2012; Bybee, 2010).

Another advantage of a STEM-based education is stated as students' seeking ways of solving problems using their existing knowledge when they encounter an unfamiliar situation (Wang, 2012). Previous research claimed that teachers' beliefs and attitudes about mathematics and sciences have had an impact on their classroom applications (Handal & Herrington, 2003; Levitt, 2002; Roehrig & Luft, 2004; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wilkins & Ma, 2003). Morrison (2006) stated that individuals who have received STEM training are problem solvers, innovative, creative, self-confident, able to think logically, technology literate, and can connect their own culture and history with education. Similarly, Bybee (2010) defined these individuals as adaptable, self-controlled individuals with high decision making, communication, problem solving and social skills.

The number of studies on STEM in Turkey has been increasing notably in the past few years (Baran, Canbazoğlu-Bilici, & Mesutoğlu, 2015; Karahan, Canbazoglu Bilici & Ünal, 2014; Şahin, Ayar, & Adıgüzel, 2014; Yamak, Bulut, & Dündar, 2014).

The number of studies investigating teachers' views on STEM-based activities is quite few (Siew, Amir, & Chong, 2015; Wang, Moore, Roehrig, & Park, 2011). Sümen and Çalışıcı (2016) taught the environmental literacy course with STEM activities and took teachers' opinions. According to the results of the research, it was determined that teachers think these activities are effective, easy and fun. Çınar, Pırasa, and Sadoğlu (2016) investigated the pre-service science and mathematics teachers' views on STEM education, and according to the findings, prospective teachers stated that STEM applications are fun, improve psycho-motor and spatial skills, support cooperative learning, provide effective and cooperative learning. In another study Özbilen (2018) conducted with 5 science and a mathematics teacher defined the STEM education model as one of the indispensable building blocks of science and mathematics, but stated that they were reluctant to apply it due to reasons such as teacher competencies, materials and lack of cooperation.

In order to determine the deficienencies of teacher candidates towards STEM and STEM education, first of all, their perceptions should be revealed. Determining teachers' attitudes and perceptions about STEM is important for them to make up for their deficiencies on these issues (Morrison, 2006; Harris, Lowery-Moore ve Farrow, 2008).Teachers who have received the essential education about STEM and are proficient at it play a crucial role in making STEM education extensive throughout the world (Wang, 2012).

This study has been conducted to investigate mathematics teacher candidates' views on STEM education and the activities that they design. In this study, answers to the following questions were sought:

1. What are the opinions of mathematics teacher candidates on STEM education?
2. How are the activities to be used in STEM education, which are designed by mathematics teacher candidates ?

Method

Research Model

The study is handled with the "comparative case study" design, one of the qualitative research approaches. By dealing with more than one limited case, this study aims to analyse candidate mathematics teachers' views on STEM and the activities that they designed in comparison to each other as well as on their own. To this end, comparative case study, which encompasses the analysis of more than one case, has been employed. A case study analysing multi-cases (at least two) comparatively is called as a comparative case study (Stake, 2006).

Participants

Table 1. The Procedures of the Education

Week	Procedures	Content
1	The introduction of STEM as a notion	Students were presented some introductory information about the interdisciplinary nature of STEM (science, technology, engineering and mathematics)
2	The science aspect of STEM	The science aspect of STEM was introduced. The participants were given explanatory information on the science aspect of STEM.
3	The technological aspect of STEM	The technological aspect of STEM was introduced. The participants were given explanatory information on the technology aspect of STEM. Also, programs in which technology is used were introduced.
4	The engineering aspect of STEM	The engineering aspect of STEM was introduced. The participants were given explanatory information on the engineering aspect of STEM.
5	The mathematical aspect of STEM	The mathematical aspect of STEM was introduced. The participants were given explanatory information on the mathematical aspect of STEM.
6	An analysis of STEM-based activities	Sample STEM-based activities were analysed.. These activities were evaluated.
7	The design of STEM-based activities	Students were asked to design STEM-based activities. The missing and/or faulty parts in these activities were spotted and analysed by the researcher and the participants were given feedback on them.
8	An evaluation of the STEM-based activities	In that week, the researcher conducted interviews.

This study has been conducted with 34 senior mathematics education students in total. When determining the participants, appropriate sampling, which is one of the purposeful sampling methods, was used. Purposeful sampling method is determining a small group as a sample group in a way that will make gathering data easier (McMillan & Schumacher, 2014). The interviews were conducted with six volunteers.

The main objective of this study is to analyse the STEM activities devised by candidate teachers who have received STEM education. To this end, STEM education was given to candidate teachers in two classroom hours a week in a period of eight weeks. The procedures of the education given to candidate teachers is given in Table 1.

Data Collection Tools

The data of this study were collected in two stages.. In the first stage, activity cards were used. The activities designed by candidate teachers were analysed by using activity cards. In the second stage, interviews were conducted with volunteer candidate teachers by using semi-structured interview forms. The semi-structured interview form was prepared by benefiting from the body of literature. The prepared form was given to two experts in the field, which had their PhDs in the field of mathematics education and the experts were requested to evaluate the appropriateness of the questions in the form. According to the feedback received from them, some minor corrections were made and the form was given to three candidate mathematics teachers to check its language validity. The participants were asked the questions *"What do you think of the STEM education that you have received? Can you evaluate it?", "What do you think about the applicability of STEM education in secondary schools?", "Can you design activities suitable for STEM? What are the principles that you will take into account when designing these activities?", "What are the student skills that are fostered in STEM education?" and "Do you have some additional remarks on STEM education?"*. According to the feedback obtained from teacher candidates, some additional questions were added to the form and it was revised to its final form.

To evaluate the candidate teachers' activities, an activity card was used. An activity card is a tool employed when evaluating the performance of participants. On the activity card, the statement "Design a STEM activity to enable fractions outcome which is in the curriculum of the fifth graders." was written.

Data Analysis

The data obtained in the study was analysed using the content analysis method. First, the obtained data was transcribed. Next, the transcripts were coded. Then, categories were formed by classifying codes according to their common features. The same process was followed both in the analysis of the interview form and the activity card. Coding was done by the first researcher and the codes were marked as "appropriate" or "inappropriate" by the field expert.

Validity and Reliability

The validity of the study has been dealt with as external validity and internal validity. To provide internal validity, the codes and categories are defined in detail. The most frequently used method to reflect the accuracy of the results obtained in content analysis is to include "direct quotes". To reach external validity, the research process has been explained and presented in a detailed way. Moreover, the theoretical framework is discussed in a detailed way in the introduction, and in the discussion section, the findings of this study is compared to those of the previously mentioned studies.

Reliability in content analysis especially depends on the coding process. If the category determination process is carried out meticulously, it is highly likely to perform a highly reliable study (Tavsancıl and Aslan, 2001). The interviews, after being transcribed, are based on the responses given to the interview questions by the researcher and an expert evaluation has been made. Evaluation was determined as "common opinion" and "difference of opinion". Reliability = common opinion/ (common opinion + difference of opinion) x 100 ”calculated using(Miles & Huberman, 1994). Using this formula, the percentage of agreement was calculated as 0.82. Since this value is above 70%, it indicates that the coding made is reliable (Yıldırım & Şimşek, 2003).

Findings

Findings on Students’ Views on STEM Approach

In this section, the findings obtained from the analysis of the students’ views on STEM are presented in the form of tables by using content analysis. In Table 1, the students’ views on STEM approach are given. The findings in Table 2 are grouped under the category of “advantages.”

Table 2. Students’ Views on the STEM Approach, which they had a Training on

Category	Code	Frequency
	Beneficial	6
	Leading students to do research	3
	Encouraging group work	3
	Enabling to make interdisciplinary connections	2
	Teaching robotic coding	2
	Student-centered	2
	Enabling permanent learning	2
Advantage	Fostering creative thinking	2
	Integrative	1
	Enabling a good understanding of the subject	1
	Arousing interest	1
	Addressing kinaesthetic intelligence	1

	Increasing work force	1
	Necessary	1
	Saving time	1
	Fostering inventive skills	1

As seen in the advantages category in Table 2, students mostly amphasized the idea that STEM approach is beneficial.

A student who thinks that STEM approach is beneficial expresses his views as follows:

> *"In terms of education, I think it is really beneficial because we are trying to teach multidisciplines all together. This enables us to give students more information in shorter period of time. Also, going to the Center of Science and Art and meeting robots there, and realizing that we can mobilize robots with just simple software affected me in a positive way."*

A student who is of the opinion that the STEM approach will lead students to do research expresses his thoughts as in the following quotation:

> *"I mean, as we are not inclined to research, we don't do much research. That's why we can lead students to do more research with this method."*

A student who holds the view that the STEM approach leads people to creative thinking expresses himself as follows:

> *"Because we are trying to synthesize mathematics, engineering and sciences in this approach, I think it will lead students to think differently and creatively. In that way, different points of view may emerge."*

The views of a student thinking that the STEM approach will contribute to the development of kinaesthetic intelligence in students are as follows:

> *"This will improve students' kinaesthetic intelligence. If kinaesthetics is followed by verbal expressions and visual application, the student will be a gainer in every sense. In this way, multiple intelligence will be triggered and as it addresses various types of intelligence, everybody can benefit from it."*

As can be seen in Table 3, the student views on the applicability of the STEM approach in secondary schools can be categorized under two headings: applicable and non-applicable. Students thinking that it is applicable hold the view that implementing the STEM approach in secondary schools is beneficial as it increases permanent learning.

Table 3. Student Views on the Applicability of the STEM Approach in Secondary Schools

Category	Code	Frequency
Applicable	Increasing permanent learning	4
	Leading students to do research	3
	Enabling students to think differently (creatively)	3
	Enabling students to make connections between disciplines	2
	Arousing curiosity	1
Non-applicable	Difficult to apply in crowded classes	1

Below are the views of a student thinking that the implementation of the STEM approach in secondary schools will enhance permanent learning:

> *"I think it's applicable. If it is integrated into subjects within a process, it can be applied quite well. Because it increases students' permanent learning. As it addresses more than one sense of the student, it is more permanent."*

A student who expressed a negative view on the applicability of the STEM approach in schools, however, explains his opinion as follows:

> *"Overall, I don't think it is applicable to the curriculum in secondary schools. At high schools, skills are more differentiated but in secondary schools, there is no such differentiation yet, so I don't think it's applicable. Not all students can learn advanced mathematics. This is more like leading everyone forcibly to a high level of the numeric fields. Every student can learn information that is enough for himself but they shouldn't be pushed to advanced levels. Apart from this, it is difficult to apply it in crowded classrooms. Since STEM relies on application, I don't know how efficient it can be to apply it in crowded classes."*

Table 4. Student's Views on Whether They Can Design Activities Suitable for the STEM Approach

Category	Code	Frequency
I can design it.	Suitable learning outcome	6
	Immediate circle	1
	Observation	1
	Daily life problems	1
	Interesting	1

As can be seen in Table 4, candidate teachers' views on whether they can design an activity suitable for the STEM approach are categorized under the heading "I can design it." Most of the candidate teachers state that they can design an activity to achieve a suitable learning outcome.

Below is a quotation clarifying the views of a student who thinks he can design an activity to reach an appropriate learning outcome.

> *"I think I can design one. I would be careful to use it for suitable learning outcomes. I would pay attention to not forcing it on students. I mean, I wouldn't*

say I will adapt every subject to technology or to engineering. If it would be a forced attempt, I wouldn't try to cover that objective. I would prepare an activity to concretize [the subject]. As one of the aims of STEM is to prepare the student for the daily life, I would concretize."

Table 5. Student Views on the Skills Considered to Be Improved by the STEM Approach

Category	**Code**	**Frequency**
Skill	Motivation	5
	Knowledge	4
	Cognitive ability	2

As can be seen in Table 5, teacher candidates hold the opinion that the STEM approach improves motivation (46%), knowledge (36%) and cognitive ability (18%).

The opinions of a student who thinks that the STEM approach increases both knowledge and the motivation of the students expresses his thoughts as follows:

"It would increase their knowledge because they do research. It would help them explore their areas of interest. Because they browse their motivation would increase. There wouldn't be such thing as boredom. Because there is a lot to do research on. It would enable them to think and to improve themselves. As we will be the ones to design the STEM curriculum they may invent something without being aware of it."

Below are the statements made by a student who is of the opinion that the STEM approach will increase students' motivation as well as improve their cognitive skills:

"It will increase motivation. Because the student will participate in the lesson more actively... Whether it is right or wrong, he participates in some way and as the teacher supports him, his motivation increases. As the student forms his own knowledge system about concepts, we can say that there is an improvement in both language [skills] and cognitive development. Because the student forms his own dictionary of concepts in his mind. Forming his own knowledge system about concepts... I mean the student reaches the concept by himself and that's why he systematizes the information that is required to be conveyed to him.

Table 6. Student Suggestions to the STEM Approach

Category	**Code**	**Frequency**
	Informing teachers (Seminars)	2
	Courses to be offered at faculties	2
	A platform of sharing information	1
	Technological facilities	1
	Visual application	1

As can be seen in Table 6, 2 students (%29) suggested seminars about the STEM approach and 2 students (%29) suggested teaching it as a course at faculties.. One student said that a platform to share information is essential, one student stated that to maket he STEM approach a more effective one, technological facilities need to be increased and one other student suggested placing emphasis on visual applications.

The views of the student who helds the opinion that it is necessary to organize seminars about the STEM approach and to found a platform to share information on are as follows:

> *"If teachers are made more knowledgeable about STEM, if more seminars are given, if more emphasis is given on this subject at faculties of education, better outcomes can be achieved. A platform like EBA can be prepared on the internet, too. I mean, in terms of sharing of the samples with everyone, a teacher can design an activity about STEM and other teachers may contribute to it and enrich it. That is, a sharing platform can be established."*

Findings Related to Student Activities that are Prepared based on the STEM Approach

When the STEM-based activities of the mathematics teacher candidates are analysed, four categories emerge: the interdisciplinary link, the engineering field it is linked to, the preferred method and the activity not suitable for STEM. The codes reached under the theme interdisciplinary link and the activity visions of some of the teachers are presented in Table 7.

Table 7. The Codes Reached under the Theme of Interdisciplinary Link

Category	**Code**	**f**
The type of the interdisciplinу link	Science-Mathematics	5
	Engineering-Mathematics	1
	Mathematics-Technology	4
	Mathematics-Engineering	1

Table 7 indicates that the activities prepared by 5 of the candidate teachers (46%) cover the link between science and mathematics. Activities prepared by 4 candidate teachers (36%) deal with the relationship between engineering and technology. While the activity of 1 candidate teacher (9%) involve the relationship among engineering, mathematics and technology, the one prepared by the other candidate (9%) is about the connection between mathematics and engineering.

Table 8 below shows the codes reached in the category of the linked engineering field and the activity visions of some of the candidate teachers.

Table 8. The Codes Reached in the Category of the Linked Engineering Field

Category	**Code**	**f**
The linked engineering field	Geological Engineering	1
	Electrical Engineering	1
	Civil Engineering	2

As can be seen in Table 8, while two candidate teachers linked the activities they prepared to civil engineering, one of them linked it to geological engineering and another one linked it to electrical engineering.

The codes reached under the category of the preferred method and the activity visions of some of the teachers are given below in Table 9.

Table 9. The Codes Reached under the Category of the Preferred Method

Category	**Code**	**f**
The preferred method	Modelling	5
	Inquiry	1
	Game	2
	Presentation	1
	Question& Answer	1
	Invention	3
	Constructive approach	1
	Creative drama	2
	Material use	4

As can be seen in Table 9, 5 of the the participant candidate teachers (25%) used modelling in the STEM-based activities they designed. 4 of them (20%) designed activities based on material use 3 of the candidates (15%) designed activities using the method of invention. 2 of the candidate teachers (10%) designed an activity using creative drama. 2 of the candidate teachers designed the activity (10%) with a game. One of them (5%) designed it with the inquiry technique.. One of the candidates (5%) designed the activity with the presentation technique. One of the candidate teachers (5%) designed it using the question and answer technique. 1 of the candidate teachers (5%) designed it adopting the constructive approach.

The codes reached under the category of activities not suitable for STEM and the activity visions of some of the teacher candidates are given in Table 10.

Table 10. The Codes Reached under the Category of Activities Unsuitable for STEM

Category	**Code**	**f**
Activity unsuitable for STEM	Interdisciplinary link	2
	Low feasibility	6
	Knowledge-based Real-Life Problem	2
	Product	2
	Evaluation	1

Table 10 shows that 13 of the candidate mathematics teachers who participated in the study prepared an activity that is not suitable for the STEM approach. Six of the teacher candidates (46%) designed activities with low feasibility. The activities designed by two teachers did not cover a knowledge-based real life problem. Two of the candidates designed their activities without sticking to the principle of making a connection between disciplines. In two of the activities designed by candidate teachers, there was no production.. One other activity prepared by a candidate lacked an evaluation.

Conclusion and Discussion

The majority of the candidate mathematics teachers who participated in this study, which aims to investigate their views on STEM education and the STEM activities that they designed, expressed positive views on STEM education. The body of literature also supports the findings of this study (Eroğlu & Bektaş, 2016; Siew, Amir & Chong, 2015; Wang, 2012; Wang, Moore, Roehrig & Park, 2011).

Except for one of the teacher candidates, all of them expressed positive views on the applicability of STEM education in secondary schools. In Wayne Long's study (2012), it is argued that it is essential to give STEM education in primary and secondary schools. Similarly, Huneycutt (2013) and Yıldırım and Selvi (2013) state that it will be beneficial to give STEM education at an early age. Weyrick (2010) contends that giving STEM education in every phase of the educational process will provide the students with the opportunities to acquire such skills as cooperative learning, critical thinking, interpersonal communication skills, which are regarded as 21st century skills and engineering skills. It is also argued that STEM education facilitates students' acquisition of 21st century skills such as the ability to solve complex problems, communication and cooperation, the importance of which are increasing day by day (Bybee, 2010).

Most of the candidate teachers stated that they could design activities suitable for STEM to reach the desired learning outcomes. A good STEM lesson not only covers the subject previously taught by the teacher but also must be connected to other subjects. This enables the teacher to teach the STEM integration more effectively and makes them more enthusiastic about the application of STEM integration in their classes.

The skill that the candidate teachers think the STEM approach improves the most is motivation. Most of them also hold the opinion that the STEM approach increases knowledge and improves cognitive skills. The findings of this study on this subjecet are supported by the body of literature. In their study aiming to investigate teachers' and candidate teachers' views on the use of STEM approach in science classes, Siew, Amir and Chong (2015) state that the participants thought a project-based STEM approach contributed to an increase in students' interest and motivation in science classes after a 8-week workshop held for them. Furthermore, teachers and candidate teachers expressed their view that due to the applications done, STEM education improves scientific process skills. Likewise, Eroğlu and Bektaş (2016) investigated the opinions of the science teachers, who received a STEM education, about STEM-based classroom activities and this study found that science teachers thought STEM and STEM-based activities contributed to an increase in student motivation and interest in addition to improving their scientific process skills. Another study investigating the effect of science-technology-engineering and mathematics activities on the fifth graders' scientific process skills and their attitudes to science, which was carried out by Yamak, Bulut and Dündar (2014), concluded that students' mental and cognitive skills improve as they employ the skills of observation, experiment design and determination of the variables in mini-design applications.

When the findings about candidate teachers' suggestions on the STEM approach are analyzed, it is seen that some teacher candidates suggested seminars about the STEM approach, while some others underlined the necessity of introducing the STEM approach in courses at faculties. In their study, Aslan-Tutak, Akaygün and Tezsezen (2017) introduced a Cooperative STEM Education Module, which was prepared according to the STEM Education Approach and they did research into the effects of the module on the candidate teachers' perceptions of STEM education. When asked what kind of a support as regards STEM they would like to be given, the participants responded by saying that they needed to observe sample projects and attend seminars. On the other hand, for the integration of the holistic and interdisciplinary outlook brought about by the STEM education into Turkey's educational system, it is necessary to raise the awareness of teachers, who are one of the main pillars of the educational system, on STEM when they are studying at the faculty of education (Buyruk and Korkmaz, 2016)

The first category obtained through the analysis of the activities designed by teacher candidates is the type of the interdisciplinary link. When the activities designed by the candidate teachers are analyzed, it can be seen that they mostly prepared activities based on the relationship between science and mathematics. In the model presented by Çorlu et al. (2014) on STEM education, mathematics and science have central roles, whereas the fields of engineering and technology are restricted to supportive roles. The same number of teacher candidates, on the other hand, prepared activities focusing on the relationship between mathematics and technology. Only one of the students prepared an activity using the link among mathematics, engineering and technology.

Another code reached through the activities designed by teacher candidates is the linked engineering field. Under the category of the linked engineering field are the codes of civil engineering, electrical engineering and geological engineering.

Still another category obtained via the activities designed by candidate teachers is the preferred method. The teacher candidates used various menthods in the activities that they designed. The methods that were used most were modeling and material use.

The last category obtained through the activities designed by teacher candidates is activities unsuitable for the STEM approach. Most of the activities designed by candidate teachers have low feasibility. Some candidate teachers did not fulfill the requirement of covering a knowledge-based real-life problem, while some others did not make the interdisciplinary connection. In some other activities, on the other hand, there was not a product.

Considering the opinions of the teacher candidates participating in the study teacher candidates should be trained on how to integrate STEM education into their lessons and stem activity examples should be presented to teacher candidates for applicability.In future research, after analyzing the other studies in the body of literature, a survey can be prepared and a wider sample can be used. In this study, candidate teachers were asked to design an activity to teach mathematics. However, these activities can be designed for subjects other than mathematics.

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The Effectiveness of Inquiry-based Learning on Middle School Students' Mathematics Reasoning Skill

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This study investigated the effectiveness of inquiry-based learning (IBL) approach in ratio and proportion on the mathematics reasoning skill of seventh-grade students. The study was carried out in a seventh-grade mathematics course in a middle school located in the Central Anatolia region of Turkey during the 2016-2017 academic year. The IBL content was prepared and implemented about the ratio and proportion topics on which the reasoning skill is effective in the 7th grade curriculum. The IBL teaching implementations were conducted with 30 seventh grade students, but nine students, who represented different math achievement levels, were selected for the study's analysis. Course video recordings, worksheets, student interviews, and diaries were used as data collection tools. The results showed that the students' predictive, explanation, generalization and justification skills emerged as indicators of reasoning skill. Students made different predictions and generalizations based on their existing knowledge and they developed solutions to problems using different strategies in IBL process. According to these findings, it was concluded that students' reasoning skill were effective during IBL.

Keywords: Inquiry-Based Learning in Mathematics (IBL-M), Reasoning skill, Ratio and proportion, Middle school students

Introduction

IBL is a training strategy in which students develop their own methods and practices in structuring scientific knowledge. IBL is a problem-solving approach used by learners from early childhood which aims to enhance their inquiry and reasoning skills within cause-and-effect relationships to learn concepts and develop in-depth understanding (Keselman, 2003). IBL in mathematics education (IBL-M) involves an instructional process in which semi structured questions involving real-life problems are asked to encourage learning and stimulate curiosity in students (Blair, 2008). IBL-M emphasizes the importance of feedback while constructing new mathematical ideas.

The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the importance of mathematical problem-solving in the IBL-M process in which students are engaged in mathematical argument-making, assumptions, question-formulating, and solution development for realistic problems. Therefore, IBL-M is student-focused, inquiry-based, and problem/ question-based, and the use of communication, cooperation, and reflection skills are essential in the instructional process. IBL-M offers the opportunity to generate and create new perspectives on mathematical problems and content by

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giving students questions in the context of the curriculum, starting from their previous experience and experimenting with multiple solutions (Chapman, 2011). The IBL mathematics classroom is based on the construction of mathematical rules, concepts, and principles by students beyond the direct presentation of mathematical concepts and processes by the teacher. IBL-M can help students to develop alternative mathematical explanations through interaction with their teacher and peers, the creation of socio-mathematical norms, and the free circulation of ideas in the classroom. In IBL-M, students are asked questions such as "If happens, what should we do?" to encourage them to question and explore (Slavit & Lesseig, 2016). Utilizing such questions in the teaching process may improve students' ability to recognize and adopt mathematical concepts and rules. Many studies (e.g. Chapman, 2011; NCTM, 2000; Stein, Engle, Smith, & Hughes, 2008) have indicated that IBL-M is effective in solving non-routine problems, as well as in researching and exploring mathematical rules and concepts. Kwon, Park, & Park (2006) emphasized that IBL-M can develop students' in-depth mathematical understanding and mathematical thinking and positively contribute to their problem-solving and creativity skills. However, despite having been shown to have positive effect on mathematics instruction, the IBL-M approach is still under-utilized by mathematics teachers (Jacobs et al., 2006). Among the common reasons why teachers do not apply the IBL approach in their mathematics lessons include the lack of time for students to explore and research due to exam preparation demands, a lack of independent working skills among students, and the inadequacy of students (Engeln, Euler, & Maass, 2013). Furthermore, Handal (2003) found that many teachers chose not to use IBLM because of their belief that mathematics is made up of rules and principles, and thus mathematical knowledge is based on memorization rather than exploration.

Reasoning Skill in IBL-M

The effectiveness of students' reasoning skill can emerge during in IBL-M process. Schoenfeld (1992) definers reasoning skill "*the ability to use mathematical tools in mathematicalisation and abstraction and understanding mathematical structure using these tools*" (p. 1). In addition, NCTM (2009) defined reasoning as the "*process of conclusion based on evidence or assumptions*" (p. 56). According to many researchers (Kasmer & Kim, 2011; Martin & Kasmer, 2010), reasoning skill involve analysing a problem, choosing a strategy for its solution, applying the strategy to find a solution to the problem, and supporting the process through their thinking in this respect, providing support for the development of reasoning skill is an important element of meaningful mathematics instruction.

Reasoning skill is a discussion tool which is carried out with the aim of making inferences based on logic (Sperber, 2000). Based on this definition, justification through discussion, evidence-based explanation, argument evaluation and persuasion are indicators of reasoning skill (Mercier & Sperber, 2011). In a discussion environment, explanations based on justification are offered for the acceptance of ideas and thoughts by the community and there is an effort to persuade the community. A common generalization or group thoughts with different opinions are revealed by critical evaluation of the shared views and thoughts. IBL-M takes the form of classroom discussions and educational strategies in which students' thought processes are supported in seeking solutions to problems to discover mathematical concepts and, finally, make generalizations (Hähkiöniemi, 2013).

For the present study, ratio and proportion were chosen as the topic for the IBL-M instruction. The importance of ratio and proportion were emphasized by NCTM (2000), which stated that "*students throughout the fifth to eighth grades are given reasoning skill developments and that a great effort should be made to develop this skill*" (p. 144). Many students have difficulty internalizing the concepts of ratio and proportion (Baron, 2010). Lanius and Williams (2003) emphasized the importance of understanding ratio and proportion, as they form the basis of understanding many concepts in mathematics and other disciplines. Students who encounter problems with ratio and proportion have the opportunity to develop different problem-solving strategies, as well as create common solutions (Shield & Dole, 2008). It follows that the teaching of ratio and proportion presents the students with the opportunities to be engaged in a discussion environment and to express their thoughts considerably. Furthermore, with this topic, it is necessary for students to shift from addictive comparison to multiplicative comparison.

There are many real-life problem situations related to ratio, proportion, and proportioning. Ratio and proportion are related to many areas of everyday life such as drawing, trading, converting currencies, adjusting and following recipes, among others (Tourniaire & Pulos, 1985). It has been suggested that real-life problems be used to teach ratio and proportion, and that students should be encouraged to use different strategies and develop written and oral explanations (Ilany, Keret, & BenChaim, 2004). In this way, solving with different strategies of real-life problems can contribute to the development of students' reasoning skills.

Ministry of National Education (MoNE, 2009) adopts the constructivist approach in mathematics education. The present study thus aims to demonstrate the effectiveness of the IBL-M approach, on which the constructivism emphasis, in improving students' reasoning skill in mathematics. For this purpose, the present study focused on the teaching of ratio and proportion to seventh-grade mathematics students to examine the effectiveness of IBL-M on their reasoning skill. The study thus posed the following research question:

1. How effective is the IBL-M approach in the teaching of ratio and proportion on students' reasoning skill?
2. What are the views of students on the effectiveness of the IBL approach in mathematics instruction?

Methodology

Study Design

This research model is based on qualitative research methods that provide in-depth analysis of the study (Bogdan & Biklen, 1998). The present study used a case study design. Case studies are appropriate in the examination of special cases in a certain context and include in-depth research on specific phenomena such as programs, people, processes, or social groups in education (Creswell, 2008).

Participants

The study participants consisted of nine students who had different levels of mathematics abilities among 30 seventh-grade students studying at a middle school in the Central Anatolia region of Turkey during the 2016-2017 academic year spring semester. According to the classroom teacher's opinion, the nine participants were selected based on their varying mathematical achievement levels to provide diversity for the study. With maximum variation sampling, it can be show up whether there are common phenomena between diversified situations and different dimensions of problem (Yıldırım & Şimşek, 2013).

After informing the students about the contents of the study, volunteer students were selected as participants. To assess changes in the students' reasoning skill the participants' work was observed and interviews were conducted. Each participant was assigned a pseudonym by the researchers to ensure confidentiality.

Study Instruments

The present study used qualitative data collection methods such as interviews, observations, and documents. The study instruments included interview forms, observation forms, and worksheets. In this way, it was possible to prevent, compare, and confirm the loss of data as many different data sources were included. Triangulation of the study data was achieved with the use of different qualitative data collecting tools (Guba & Lincoln, 1982).

Interviews. In the study, semi-structured interviews with students were conducted to determine the extent of changes in their reasoning skill during the IBL-M. The interview form was created by the researchers based on the related literature (Slavit & Lesseig, 2016; Brown & Walter, 2014). The prepared draft form content was sent to three mathematics education experts to determine the scope, content, and suitability of the form. The draft form was prepared for pilot implementation to obtain the advice of experts. Interview content included questions regarding IBL-M implementation, activities used, and the evaluation of individual structures in the study's implementation processes.

Following the completion of the implementations, pilot interviews were conducted to determine the views of students. After the pilot implementation, some questions were changed and the final version of the interview form, in which eight semi-structured questions were included, was provided (Appendix-1). The actual interviews were also carried out following the same process used in the pilot implementation.

Interviews were completed two days after the end of the study implementation. The interviews were conducted between the researcher and the participants individually, so that the students would not be affected by each other and would share their views sincerely. The interviews were held in a quiet and disturbance-free environment in vice principal's office with an "interviewing" sign hung on the door to prevent interruptions. Each interview lasted about 15-20 minutes and was audio recorded. The participants were informed prior to the interviews that they would be audio recorded and that their identity would be kept confidential. Each participant consented to participate in the interview. After, the interviews recordings were transcribed, the participants were asked to confirm whether the recording understood correctly.

Observations. The semi-structured observation form was filled in by a non-participant observer throughout the study implementation process, because the researcher was busy teaching and observing the participants. The observation form was developed by the researchers and was first applied in a pilot practice session to obtain the opinions of experts; changes were made after. The observation form was prepared based on the development of reasoning skill described in the MoNE (2013) Middle School Mathematics Curriculum. Using the observation form, it was possible to record the data in an objective manner. Thus, it was provided with the support of the data through observations and opinions as well as the help of the assistant teacher's observations.

Documents. Students' worksheets were used in the present study as data collection tools. The worksheets consisted of sections that guided students in each activity in the IBL-M process and included questions for them to answer. The student worksheets also included homework questions.

Data Analysis

Data were obtained in two ways; data from video recordings during IBL-M and interviews with participants at the end of the IBL-M process. Below is a detailed description of the analysis of the data obtained in two ways.

Video Recordings Analysis. During IBL-M, problem solving activities were carried out with the students. Problem solutions and explanations of students were evaluated within the scope of reasoning skill in this process. This implementation process was recorded with video recording. The solutions, explanations and discussion process of the students to the problems were analysed within the scope of reasoning skill.

The data were then classified under the headings of reasoning skill, as per the aim of the present study. Literature is examined in the formation of these skills and sub-skills and skill list has been revealed. Reasoning skill include predicting and inferring; defending the accuracy and validity of the inference; presenting data in logical generalizations; and informal proofing such as recognizing mathematical relationships and establishing cause-effect relationships, assumptions, and generalizations (Stylianides & Stylianides, 2009). In this study, student reasoning skill were analysed and evaluated according to this framework.

The analysis of the data thus obtained in accordance with the predetermined themes is defined as descriptive analysis (Yıldırım & Şimşek, 2013). For this purpose, a four-step path was followed. Firstly, literature on reasoning skill has been searched and analysis framework related to skills and sub-skills has been formed. Then, the data analysed from the data set were processed into the identified skills and sub-skills. At this stage, the obtained data for reasoning skill was coded according to their common characteristics, and the codes are listed. The codes were categorized into the categories that emerged. Coding was completed by two different researchers and provided a comparative end-of-view consensus. To provide descriptive support for the submitted codes and categories, direct citations from participant opinions, and worksheets were included. The analysis of the content was directly supported by descriptive explanations.

Interview Analysis. At the end of the IBL-M, individual interviews were conducted with the students to evaluate the IBL-M process. The data obtained from the individual interviews were analysed through content analysis and the findings were supported descriptively with direct quotations. Content analysis is a technique that enables indirect analysis of human behaviours that cannot be directly observed and cannot be measured (Fraenkel & Wallen, 2009). The audio and video recordings obtained from the individual interviews were converted to written computer documents. In the individual interviews conducted after IBL-M, a student was coded together with a researcher specialized in mathematics education. Coding for other students was analysed separately by expert educator and researcher; codes and categories were compared and mutually negotiated.

The IBL-M Process in Ratio and Proportion Instruction

The course content was prepared by the researchers based on the curriculum content for the topic of ratio and proportion. In addition, the creation of content was modelled after the sample content obtained from Van de Walle, Karp and Bay-Williams (2013) and related literature. To determine the appropriateness of the course content in terms of the curriculum, IBL-M, and the students' learning needs, an evaluation form was prepared based on the related literature and then sent to three researchers who are experts in mathematics education to obtain their opinions. The lesson contents were arranged according to the experts' opinions, and 10 one-hour sessions were planned. The IBL-M ratio and proportion instruction was implemented during the spring semester of the 2016-2017 academic year following the completion of the pilot study.

The researcher assumed an observational role and taught the lessons. Furthermore, one assistant teacher was involved in the study as an assistant to assist with video recording and taking photos during the classes. At the end of the IBL-M implementation, interviews were held to determine the students' views regarding IBL-M. During IB-M, video recording was used to record all the data and student behaviours. The camera was placed at the rear of the classroom to prevent the students from changing their natural classroom behaviours. The IBL-M process should include the following elements:

- The identification of students' prior knowledge
- The creation of a learning environment in which students can share their ideas with one another
- The fostering of students' "why, what, how, etc." questions
- The evaluation of student explanations using alternative explanations from the teacher
- Efforts to develop alternative solutions to a problem

During the study implementation, each student worked independently and created their own' solutions for each problem. The students were asked to share their solutions with their peers and provide justifications for their answers. Throughout this process, the researcher asked the students to elaborate on why, what, and how they chose to solve the problems. An outline of the IBL-M implementation process from the first lesson is provided below.

The IBL-M Implementation Process during the First Session. In the first session, the students were given the following instructions:

- Instead of receiving direct descriptions and explanations, the students should focus on their relationships among concepts.
- Individual and group work will be done
- The activity, worksheets, and reflective logs will be included in the lesson
- Everyone's ideas are important, and everyone should engage in discussion rather than be concerned about right and wrong answers

After the above instructions were given, a discussion environment was created by asking the class questions to assess their existing knowledge and establish a comfortable environment in which they could express their views freely. Student answers to the question "What is ratio?" were written on the board. The views of the students were revealed additive or multiplicative comparison with this question. Then, the students were asked to give a general explanation of proportion. Then, meaningful problems that related to the students' daily lives were presented.

Example 1. Affixing images of apples, oranges, and watermelons to the blackboard, the teacher stated, “A farmer gathered ripe apples, oranges, and watermelons from the field and put them in the basket.” The teacher then asked about the meaning of the expressions “apples to oranges ratio” or “apples to watermelon,” and what their mathematical representations would be. Then, “why” questions related to the representations were posed. The students were asked to make explanations and justifications.

How the mathematical representation of the rate is expressed, how many different representations can be made $\frac{a}{b}$, and the a/b and a:b representations were mentioned. For example, the apple to orange ratio is said to be the simplest notation, as 8/6. It is emphasized that the ratio is constant and different multiples express the same ratio. For proportion, the simplest notation simplification is used. Through this process, the students learned about ratio and proportion.

Example 2. By affixing a picture of two persons to the board, in which one person is tall (185 cm) and the other is short (110 cm), the students are asked to describe the proportions.

The teacher attempted to associate the simplification operations used in Example 1 with the problem described in Example 2. For this purpose, the students were asked questions such as “How can the proportions be written in their simplest form?”

Findings

The Effectiveness of IBL-M on Students’ Reasoning Skill

Ratio and proportion problems necessitate the development of reasoning skill and involve the transition from additive comparison to multiplicative comparison. Table 1 displays the analysis of the students’ reasoning skill observed at the beginning and end of the IBL-M implementation process, taken from all classroom observations.

Table 1. Process Analysis for the Development of Reasoning Skill

Beginning of IBL-M	**Process IBL-M**
Additive comparison	Multiplicative comparison
Multiplicative comparison	Convincing
	Exploring
	Making connections
	Justification
	Generalizing
	Systematizing
	Symbolic representation

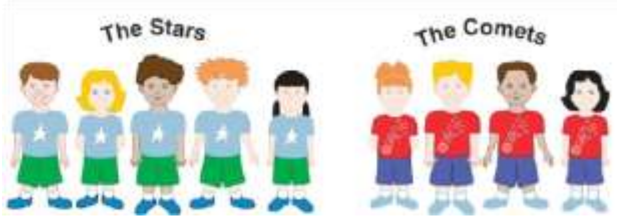
Analysing the class discussions during different sessions of the IBL-M training, it was found that the students’ reasoning skill improved during the process. It was observed that the students used justifications for their solutions, which contributed positively to their reasoning skill development. This result was supported by the information provided in the assistant teacher observation forms. The observations showed that the students improved in predicting, establishing cause and effect relationships, explaining and justifying, generalizing, and abstracting. The following is an example from the class discussion of the first session:

Researcher (R): *A farmer gathered ripe apples, oranges, and watermelons from the field and put them in the basket. What is the ratio of apples to oranges and watermelons to apples?*
Samet: *There are mostly apples, then oranges, and last watermelons.*
Berk: *Teacher, when we say proportion, do we look at the differences between the items?*
Ali: *When we say "ratio," we are dividing, so we use a slash (/).*
Fatih: *Yes, when we are asked for the ratio, we write the first one on the bottom, and the second one on the top.*
Mehmet: *No, I think it is the opposite: the first is on the top, and the second one is on the bottom.*
R: *Yes, when we are asked to find the ratio, we write the first one on the bottom, and the second one on the top.*
Mehmet: *In this way our teacher taught us for not to mix them up.*

Most students thought that finding the ratio required addition or subtraction. It can be said that the students in the direction of the answers are inadequate about the ratio, proportion and the reasoning skill. At the same time, it was seemed that students' answers were informed by their teacher's prior instruction. Thus, it was observed that students were lack of conceptualization of ratio as well as their multiplicative comparison. For this reason, multiplicative comparison was first emphasized, and then the transition from ratio to proportion was achieved. Next, to assess the additive and multiplicative comparison of the students, open-ended, real-life problems were created, and a class discussion ensued. Below, an excerpt from the discussion transcript of the second session is provided.

Question: *What can you say about the numbers of girls in the two groups? (see Figure 1)*

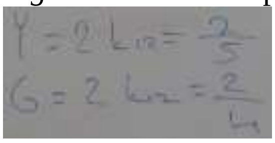
Figure 1. Question's Visual



Ali: *Teacher, do we say which group has the most girls, or do we compare the groups?*
Burcu: *If we are asked in which group there are more/fewer girls, we would only have to look at the numbers, then we would have said 2 since the number is equal in both groups.*
İdil: *If the number of girls is more than the number of the boys, we could say 3-2 = 1 for the stars group and 4-2 = 2 for the coments group.*

It can be seen from the transcript that when the students tried to provide explanations, they were not sure of whether to use addition or multiplication in their solutions. Below, the transcript from this class discussion continues:

R: *In both cases, the number of girls is equal, so why did you solve it like this?(see Fig. 2)*
Suna: *Yes, the numbers of girls in the two groups are equal, but we are supposed to make comparisons. So, if the ratio between the first and second group is 2/5, there are 2 girls in the second group but 2 girls out of 4 people. So, the girls' ratio in the second group is 2/4.*

Figure 2. Suna's Response


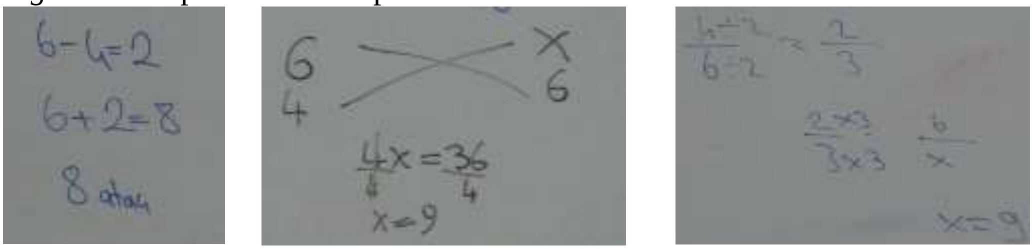
Researcher: *But I did not say the ratio of the number of girls to the total number of individuals in the group.*
Aslı: *If you asked for the ratio of girls to boys, we would write 2/3 for the first group and 2/2 for the second group.*
Fatih: *Teacher, whether we rate the number of girls to the number of boys in the group or to the total number, it is higher in the second group. Nothing changes.*

The students' responses to the example problem in the second session show that they had already developed some multiplicative reasoning skills, and they realized that proportion involves comparison and not addition. At the same time, it seems that they had discovered the concept of ratio constant in an informal way.

Inadequate and appropriate strategies for the use of multiplicative comparison to elicit students' ideas in the IBL-M process are presented in an excerpt from the transcript of the fourth session below.

Question**:** *Mr. Short has a friend named Mr. Tall. When the height of the Mr. Short was measured using paperclips, it was found that he was 6 paperclips tall. When Mr. Tall and Mr. Short are measured using buttons, it was found that Mr. Tall was 6 buttons tall and Mr. Short was 4 buttons tall. According to this, how tall is Mr. Tall in paperclips?*

Figure 3. Sample Student Responses



Before strategies were presented directly by the teacher, it was observed that the students were thinking critically, criticized different solution methods for ratio and proportion, and justified their chosen solutions in Figure 3. Some student solutions given during the class discussion are displayed below.

> ***Suna:*** *When we look at the height of Mr. Short, it increased by 6-4 = 2. So, Mr. Tall's height will also increase by 2 and will be 6 + 2 = 8.*
> ***Mehmet:*** *The first one is measured with paper clips and the second one is with buttons. So, it is not the same. We have to do it this way, because the two situations are different.*
> ***Samet:*** *There is a comparison of Mr. Short and Mr. Long here, so it is a ratio.*
> ***Ali:*** *There is also a comparison between the first case (paper clip measurement) and the second case (button measurement). So, proportions should be used.*
> ***Aslı:*** *It is measured with the paperclips at first, then with buttons. Mr. Short was 6 paperclips and 4 buttons in height, so the value is reduced. Then, Mr. Tall, who measured 6 buttons, should be a larger number when measured with paperclips.*

From the transcript excerpt above, it can be noted that the students were thinking critically and using reasoning skill. Student explanations and statements using the cross-multiplication strategy are presented below.

> ***R:*** *What does the cross you drew here mean?*
> ***Burcu:*** *It means that we cross-multiply.*
> ***R:*** *Why do we cross-multiply? What is the logic behind it?*
> ***Burcu:*** *If the question is about proportion, we write 6/4 for Mr. Short as the length in paperclips and buttons, and the ratio for Mr. Tall as x/6. That is 6/4=x/6. We then cross-multiply to get the solution.*

The student's explanation shows appropriate strategy-use and the justification of his thinking in the context of cause-effect relationships; furthermore, the student displayed the ability to make generalizations. A student's statement related the use of the unit rate strategy is presented below.

> ***Mehmet:*** *When comparing two things, we were writing the ratio. In this question, there is a measurement with paperclips and buttons. So, I wrote the lengths of the buttons/paperclips, 4/6. This reduces to 2/3 in its simplest*

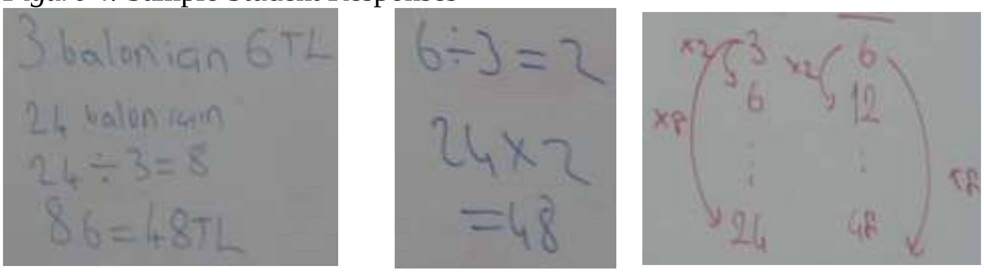
> *form. Since the ratio does not change, I wrote 2/3=6/x, then cross-multiplied to get x = 9.*

The above statement shows that the student solved the problem by going beyond simple proportion and calculating the product of the proportion. Thus, many different solution strategies can be explored by the students by trying different ways of solving a problem, even if the definition or usage of the equivalent fraction strategy is not known by the student.

Through reasoning skill development, it was aimed to establish proportionality without proportioning in the fifth and sixth sessions, which were intended to foster students' reasoning skill development. For this purpose, problems with direct proportions were given first. The transcript excerpt below shows student reasoning using direct proportions.

Question: *Arda, who went to an amusement park, paid 6 ₺ for 3 balloons. So, how much does it cost to buy 24 balloons?*

Figure 4. Sample Student Responses



As shown in Figure 4, the students used different solutions to arrive at the correct answer. Afterwards, different solutions were demonstrated by the students on the board and they were explained by the teacher.

Even though the definition of direct proportion was not given, the students demonstrated an awareness of this in the solutions they gave for the problem and showed an understanding of the logic of direct proportion.

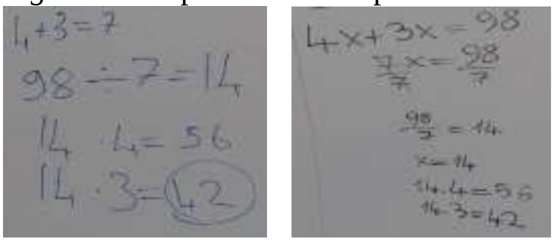
> ***Mehmet:*** *I divided the price by the number of balloons (6/3 = 2). So, I found that the price of one balloon was 2 ₺. Then, I calculated that 24×2=48 ₺. So, 48 ₺ is needed to buy 24 balloons.*

Therefore, although no problem-solving strategy was given to the students, they discovered many ways to solve the problem on their own. In this way, the students discovered the existence of the proportionality constant. At the same time, it seems that the students realized the solution from the unit rate strategy.

In the sixth session, the students were first introduced to the expression "x," which had not yet been taught in the class. Problems with an unknown were then presented to the students and their solutions were examined.

Question: *The ratio of two numbers is ¾. Since the sum of these numbers is 98, what is the smallest number?*

Figure 5. Sample Student Responses



> ***Ali:*** *I thought that, if these numbers were not a ratio that is 3 or 4, then the sum would be 7.*
> ***Samet:*** *The numbers are not mentioned here, but the ratio is. So, I think addition is not suitable here.*
> ***Ali:*** *Here, the total is not 7, it is 98, and 14 times the sum of these numbers is (98/7 = 14). That is, if 14 times the sum is calculated, and 14 times those numbers are also calculated, then the answer can be found by taking 14 times 3 as 14 x 3 = 42, and 14 times 4 as 14 x 4 = 56.*
> ***Berk:*** *I do not understand this way.*
> ***R:*** *So, what are we supposed to do, as your friends are not convinced about the solution?*
> ***Ali:*** *Then I'll show you that it's true by reversing. Look, (turning to his friend), now we've got the products and we found the numbers 42 and 56. Rate them, 42/56 = ¾, did you see? I am correct.*

In the Figure 5, it seems that the student did the numeration without using the unknown and solved the problem. At the same time, the student justified his answer by explaining the reasons to his friends who did not understand the solution. The student used the word "proof" to convince his friend that his solution was correct. Although the process that the student used is not considered to be formal proof, it involved the backward strategy.

> ***Burcu:*** *When the ratio of the two numbers is given, it means that the simplest form is ¾. That is, one is a product of 3 and the other is a product of 4. But the products of 3 and 4 are the same, so it is simplified. Then, we call one 3x and the other 4x. Their total is 98. I wrote 4x + 3x = 98. I multiplied by 7x = 98, unknown 7. Then, to find the unknown, we have to divide both sides of the equation by 7. I found that x = 14; that is, 14 times 3 and 4.*

In his solution using x, the student showed the ability to use abstract thinking. In this sense, the result of the instructional process addressed the students' higher-order thinking skills. It was thus observed that even though the students were not provided with rules and methods directly, they were able to develop their own strategies to solve problems related to ratio and proportion.

In the seventh session, inverse proportions were explored. The researcher aimed for the students to solve the problems using the ratio information without teaching them the definition of direct proportions. In order to establish the relationship between direct and inverse proportions without giving the information to the students directly, a problem about inverse proportions was given to the students and classroom discussion took place. During the discussion process, the researcher aimed to assess the students' reasoning processes rather than getting the correct answers to the problems. The following transcript is an excerpt from the class discussion of the seventh session:

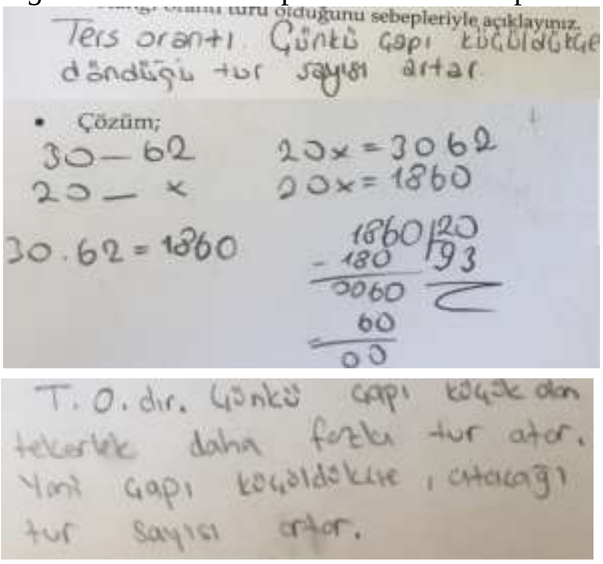
> ***R:*** *In a garden there are apple trees, and the apples are gathered in 6 hours by 4 workers. Do you think the apples would be gathered by 2 workers in more time or less time?*
> ***Samet:*** *Teacher, it will take longer because the number of people decreases.*
> ***Berk:*** *The time will increase, because 4 people can do it faster. So, with only 2 people it will take longer.*
> ***R:*** *So, what kind of a relationship do you think there is between the number of workers and the time in which the work is done?*
> ***Berk:*** *One rises while the other decreases.*
> ***Mehmet:*** *Yes, it is the opposite. Like a seesaw, someone goes up and someone goes down.*
> ***Fatih:*** *Teacher, this is inverse proportion.*
> ***R:*** *What sort of mathematical description can we make if we compare the problems that we have solved up to now (the inverse proportion example) and the present example, if we get it together?*
> ***Aslı:*** *If one of the given items increases while the other increases, it is direct proportion. If one of the given items decreases while the other increases, it is inverse proportion.*

Looking at the sample discussion process that took place during the seventh session, it was observed that the students tried to understand inverse proportion by starting with their understanding of direct proportion. When looking at the process of establishing students' understanding of direct and inverse proportionality relationships, it was observed that they were able to justify the similarities and differences between the problem situations and the reasoning that arose from the problems in the previous sessions. The process of associating the concepts of direct and inverse proportion was followed by the inverse proportion problem given in the eighth session.

For the following problem, the students were asked to find out what kind of proportion it was, find a solution, and justify their answer.

Question: *The diameter of a bicycle front tire is 30 cm and the diameter of the rear tire is 20 cm. How many times does the front wheel rotate when the rear wheel rotates 62 times along a road?*

Figure 6. Student Responses to the Sample Problem from the Eighth Session



Upon examining the students' explanations in Fig. 6, it is clear that they determined the relationship between the variables using inverse proportions. When we look at the example mathematical solution, it seems that the solution is made by going beyond the cross-multiplication strategy used with direct proportions. The students who came up with the solution were asked to explain how they solved the problem and justify their ideas.

> ***Burcu:*** *In our previous examples, there was direct proportion, and some of our friends solved the problem of direct proportionality by doing cross-multiplication. For this question I thought like that, when we rate two things in the direct proportion, for example, we were writing 3 apples/5 oranges as the ratio of apples to oranges.*
> ***R:*** *How would you like to express this in general terms, not just for the apple-orange example?*
> ***Burcu:*** *By using a, b, or something? If we want to rate two things, then we write a/b so we can multiply and simplify the numerator and denominator with the same number. I thought it could be written as a product in the inverse proportion.*
> ***R:*** *Why do you think the inverse can be expressed using multiplication?*
> ***Burcu:*** *For example, we say in the direct proportion if 4 apples cost 5 liras, how much do 8 apples cost, we can write it like this 4/8=5/x. So, what is actually the same direct proportion between apple and price is written as 4 and 8 parts, so if there was an inverse proportion, the multiplication would be 4x8.*
> ***R:*** *It is not only for this example but also in general let us say, how you express there is an inverse proportion between two variables can be like the example that you gave.*
> ***Burcu:*** *So, we can say a x b if it is for any situation.*
> ***R:*** *Yes, it means that when expressing the direct proportion between the two expressions we can say a/b and we can say a x b when expressing the*

inverse proportion. Well, if you were going to describe the solution to the problem, why did you do side-by-side multiplication?
Burcu: *Now if we write 4/8=5/x we cross-multiply, if I generalize it as you wanted to be and if I write a/b=c/d, the cross product would be a × d = b × d. For the inverse ratio, I said a x b for the ratio display of two things. Then, I can write a × b = c × d. That's what I did for the wheel; I wrote the wheel diameter times the number of turns. That is like 30×62, I can explain if it is not understood.*

Looking at the solution of the student for the problem in the eighth session and his explanations, it seems that he has conceptually understood the relationship between direct proportion and inverse proportion. The student generalized and abstracted from the sample for the direct proportion, and also expressed the inverse proportion from the relation between the variables in the direct proportion at the same time. In the same way, for the inverse proportion, the student expressed himself using mathematical expressions correctly and making generalizations and abstractions independently of the sample case. It seems that the student has justified the explanations they have made and supported them with examples. The student stated that he/she could repeat his/her explanation, which shows that the student has the skill of persuasion.

The findings obtained from the interviews with the participants are consistent with those of the observations presented in Table 2. Reasoning, cause-effect relationship establishment, and reasoning and verification codes, which were obtained from the interview analysis, are presented alongside student statements.

Table 2. Reasoning Skill Scope of Student's Views

Code	Participants	Sample Student' Answers
Reasoning	Aslı, Berk, İdil, Mehmet, Samet	***Samet:*** *Before, I did not know what ratio was, or what I needed to do. But you showed different examples, such as lemonade and trucks, and I learned about proportion from these questions.*
		Berk: *In sixth grade, we were taught that "the comparison of two things means that it is a division problem," but the first time this concept was clear to me was when we were learning about proportion.*
		İdil: *They make us memorize that "Cross-multiplying is for the direct proportion, side-by-side multiplying is for the inverse proportion." But for the first time in our class we saw why this happened, and I understood why.*
		Aslı: *...you did not give the definition of direct and inverse proportions but when I saw the problem, I understood the relationship between inverse and direct proportions.*
Cause Effect Association	Ali, Burcu, Fatih, Mehmet, Suna	***Mehmet:*** *You asked for a question and all of us to solve it. You have looked at our solutions from our individual notebooks. So, I solved the questions forcibly. Because I do not know anyway, I thought "how can I do this with what I know."*
		Fatih: *We found that our friends were solving problems in different ways and then they explained their solutions. When I see a question, I used my mind, thinking "how shall I solve this, and which method is easiest?"*

Justification	Ali, Aslı, Burcu, Samet, Suna	***Suna:*** *...For example, if I solved the question on the board, I have to explain my solution to my friends to convince them that I solved it correctly.*
		Ali: *Our solutions should have a logical explanation that we have solved it correctly...*
		Samet: *You say "why," when solving at the board, and sometimes I realized that my solution did not make sense to me either and that it was wrong. How do I know it's wrong? It is wrong if I cannot explain it or if the solution path is not correct in mathematics. I cannot persuade my friends if I cannot give them a good explanation.*
		Burcu: *It's important to know the reason for something instead of just memorizing. We forget when memorizing, and if we cannot explain why, friends will not accept that method. I also do not believe something when it is not explained to me.*
Verification	Ali	***Ali:*** *I proved the correctness of the solution I made.*

Therefore, it is clear that the IBL-M process had a positive effect on the students' reasoning skill. Upon examining the solutions given by students of different mathematical achievement levels, it seems that improvement in the reasoning skill of students at all levels occurred. In particular, students at lower mathematics achievement levels (i.e., Berk, Mehmet, and Samet) who struggled with multiplicative comparison in ratio and proportionality problems, were able to solve ratio problems using the unit rate strategy were able to solve ratio problems using the unit rate strategy. Students with moderate and high math achievement levels were able to make inferences, provide justification, and explain their reasoning. Furthermore, they were able to understand the cause-and-effect relationship between variables. These students also discovered effective fractional and cross-multiplication strategies and provided effective and persuasive explanations for their solutions. These findings were supported by student opinions and can be said to be positively influential in the development of reasoning skill, which are regarded as a basic skill required for ratio and proportion.

Student Opinions of IBL in Mathematics Education

The data obtained from interviews conducted to determine students' thoughts concerning IBL in mathematics education are presented in Table 3. Through the student interviews, the following codes were identified: positive attitude toward mathematics, discovery of different methods without giving rules, lack of memorization as a result of reasoning, and concretization. Table 3 presents these codes with direct student quotations.

Table 3. Student Opinions of IBL in Mathematics Education

Code	Participants	Sample Responses
Attitude	Mehmet, Samet	***Mehmet:*** *I was afraid of math before, but this was more fun.*
		Samet: *When I was able to develop my own method, I saw that I could solve mathematics problems in my own way.*
Discovering Different Methods	Ali, Aslı, Burcu, Berk, Fatih, İdil, Mehmet, Suna, Samet	***İdil:*** *Since you did not give us direct rules, we all tried to find solutions by thinking and experimenting.*
		Suna: *That we solved the problems as we wanted allowed us not to feel restricted. When I encountered a problem, I saw that I could produce a solution even though I did not know the rule.*
		Burcu: *Normally in our lessons our teacher gives the rule and then we solve the questions applying that rule. We learned afterwards what the solution was when you were solving the questions in different ways.*
		Berk: *After I solved each problem with you in different ways, when I encountered a question, solutions that you made came to my mind. If it makes it easier to solve the problem, I apply it.*
		Mehmet: *Since in the lesson we did not learn the rule, we were forced to solve the problems on our own. At first, I did not know what to do and it was so difficult, but you wanted to bring all of us up to the board. I saw where I did the right thing or wrong thing when I did my solutions. I do not even try to solve any questions in our other lessons.*
Not Memorizing	Berk, Burcu, Fatih, Mehmet, Suna	***Fatih:*** *In the first lesson, you asked us what a ratio was, and you waited for us to answer. Then we wrote all our answers and created the definition of ratio ourselves.* *We did not take the definition from a textbook.*
		Suna: *Why we did something was very important. If I did not tell you why I was doing something, my friends would not accept my method.*
		Burcu: *You asked me why after you brought me up to the board to solve a problem, which I thought was better. I can solve the problem by understanding it as direct/inverse proportion, but I also learned the reason for the numerations. I have not memorized the solution.*
		Berk: *When my teacher told us the method, I kept it in my mind step by step. I can solve most of the questions as well. But you gave me different questions and you asked me and other friends why it is done like that. I learned after hearing the explanations.*
Concretization	İdil, Samet	***Samet:*** *You brought a lot of different materials to the course; they were permanent in my mind. For example, you brought fruit to teach us ratio, we talked about tall and short people, and we learned the unit /unitless rates.*
		İdil: *The use of pictures and the smart board was more colourful. It was both memorable and visually pleasing. If we use the smart board, we can see different kinds of problems, not just written questions.*

The IBL-M implementation showed that the enriching teaching content appears to be effective in improving the learning outcomes of students. The results of the study showed that the students were challenged in the process that they were actively involved in, which fostered their positive attitudes toward mathematics and improved their confidence by making their own solutions. In light of these findings, it can be said that IBL-M has a positive effect on student reasoning skill, a finding that was reinforced by the students' opinions of the teaching method.

Conclusions

Reasoning skill is expressed by the NCTM (2000) as mathematical skills that must be developed at the beginning of primary education. It is stated that with the provision of reasoning skill development, it can also be effective in making proofs that are accepted as the basis for developing mathematical understanding in the upper grades (Stylianides, 2007). It has been stated that students often meet with difficulties in reasoning, ratiocinating, and generalization (Healy & Hoyles, 2000; Healy & Hoyles, 2007). For this purpose, it is possible to develop the elements of assumption, reasoning, and generalization as informal proof by the form of teaching that will be effective in the development of reasoning skill.

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The findings of this study showed that by using the IBL-M, the students showed improvement in their reasoning skill, the inferences they made based on reasoning, and in developing and generalizing operational strategies using mathematical expressions. The students were able to produce solutions for real-life problems by solving problems through deductive and inductive reasoning based on their existing knowledge and experiences. It can be said that the IBL-M approach provides the opportunity for the identification, discovery, and development of students' reasoning skill. Similarly, Leatham, Peterson, Stockero, & Zoest (2015) stated that IBL-M provides students with opportunities for rich forms of reasoning as part of classroom discourse in individual or small group work. The IBL-M approach requires students to make logical explanations for their solutions or provide justification for their answers by answering "why." It can be said that additive and multiplicative situations are learned by the students in this way. Proof can be given in informal ways, such as persuading their friends that their solution is correct. According to Wilhelm and Wilhelm (2010), IBL-M emphasizes that middle school students become effective at constructing and explaining their arguments. Likewise, Epstein (2007) found that adopting the IBL-M approach in mathematics education can positively affect students' mathematical concept development, data collection techniques, data representation skills, and critical thinking skills. Rasmussen and Kwon (2007) found that IBL-M supports students' learning through mathematical discussions, explaining their thoughts, providing evidence, and using mathematical knowledge in different problems. Kuster, Johnson, Keene, & Andrews-Larson (2017) reached the conclusion that IBL-M provides students with opportunities to generalize their existing knowledge through structuring and reasoning.

Schoenfeld (1996) stated that students should be able to answer questions through questioning. In the period of reasoning skill development, it is seen that students use different problem-solving strategies for realistic problems and explain their solutions using logical reasoning. Furthermore, it can be said that IBL-M is effective in developing students' critical and reflective thinking. Similarly, Pratt and Woods (2007), in the framework of the Postgraduate Certificate of Education (PGCE), stated that IBL-M is effective in developing problem-solving skills and mathematical reasoning, and fosters the creation of supportive learning environments. Studies have shown that questioning students allows them to better understand their solutions and contributes positively to the development of problem-solving skills (Chouinard, 2007; Mills et al., 2011).

The results of the present study were found to be based on the basic idea of mathematics by the students, followed by inquiry and reasoning, and mathematical language and demonstrations, and a formal development. In this context, as described by Rasmussen, Zandieh, and Wawro (2009), the mathematical terminology developed by IBL-M suggests that students' mathematical thinking and solutions contribute to the development of mathematical language, representation, and ideas that depend on certain mathematical standards. Jackson et al. (2013) emphasized the development of mathematical expressions and representations through the IBL-M approach, in which the expressions used by students in their solutions, mathematical ideas, and justifications are within the framework of certain mathematical norms.

In the present study, the students seemed to be more comfortable expressing themselves using mathematical language in IBL. They demonstrated confidence and were actively involved in the lessons. Previous research has demonstrated the positive effect on learning outcomes when students take an active role in the classroom (e.g., Polya, 2014; Schoenfeld, 1985; Schoenfeld, 1987). In addition, adopting the IBL approach in mathematics education was shown in this study to help develop students' positive attitudes toward mathematics and meaningful learning. This finding is consistent with the results of Wilhelm and Wilhelm (2010), who found that middle school students who were reluctant to participate in classroom activities, became more willing to do so in an IBL teaching environment.

Stein et al. (2008) stated that getting students to take active roles in the classroom is among the most challenging aspects of teaching. Stein et al. (2008) found that IBL in mathematics education is effective in ensuring that students participate in class and that students are actively involved in learning activities. Stephan and Rasmussen (2002) emphasized that IBL provides an environment in which students can share important mathematical ideas and ways of reasoning through individual or group work. Similarly, Kuster et al. (2017) suggested that students are encouraged to think about alternative approaches and ways of thinking in the IBL process, and that student skills and achievement development are supported, where opportunities for mathematical expression of students' views are provided.

Among the goals of the MoNE (2013) Middle School Mathematics Curriculum, it is stated that opportunities for students to be active in the teaching environment, conduct research and inquiry, think critically and justify, develop different solutions for problems, and express their opinions and thoughts should be encouraged. Based on the findings of the present study, it can be said that these goals can be achieved though the IBL-M approach. Thus, it can be said that the use of the IBL-M approach in mathematics education will contribute positively to the reasoning skill development of the students, among many other benefits. Based on findings of the present study, IBL-M should be investigated in different subjects and contexts to further investigate its effects. By examining the effect of IBL-M on different skills, the general applicability and effectiveness of the method can be demonstrated.

Recommendations

As a result of the study, recommendations for researchers and educators are presented below.

- IBL can be included in secondary school mathematics education. In this way, students are made more active and a positive effect can be achieved on both their math achievement and motivation.
- IBL-M was applied to ratio and proportion in this study. Studies on different mathematical subjects can also be carried out. Likewise, this study focused on students' reasoning skills in the IBL-M process. Comparisons can be made for mathematics skills thanks to the work to be done on different skills.
- Since this study is a qualitative research, the IBL-M process on ratio and proportion has been described in detail. In this way, it is thought that the content is useful and applicable for students with similar features. In addition, it is thought that contribution can be made to practitioners with the contents of IBL-M to be realized by different researchers.

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Appendix-1: Individual Interview Form

This interview aims to acquire information about your opinions on the practices and the content of IBL-M in which you participated. For this purpose, I will pose some specific questions. During the interview, your identity and your statements will be kept completely confidential. I guess it will take approximately 15 minutes to complete our interview. I would like to start the interview with your permission.

Questions

1. Can you evaluate the IBL-M process?
2. What are the positive/negative aspects?
3. What opportunities did it provide?
4. Do you want your other lessons/subjects to be conducted with this method?