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Learners' Experiences in Solving Probability Problems in Grade 11 in selected Schools in Mthatha Area

By Nkwezana Esona[○]*, Tirivanhu Muchuweni*[*] *& Benjamin Tatira*[±]

This study investigated the experiences of Grade 11 learners in solving probability problems in secondary schools in Mthatha, Eastern Cape, South Africa. Probability is a key strand in South Africa's Curriculum and Assessment Policy Statement (CAPS), yet performance remains consistently low. Guided by constructivist learning theory, the study examined how learners' intuitive reasoning, use of representations, and classroom environments shaped their engagement with probability. A mixed-methods design was used with 490 learners and five teachers across three schools. Data included a diagnostic test adapted from past examination papers, a structured survey on perceptions and strategies, and qualitative coding of learners' written responses. Findings showed persistent misconceptions about sample space, independence, conditionality, and the use of probability rules. Learners struggled to move between symbolic, diagrammatic, and verbal forms and often misread probability language. These difficulties were compounded by limited scaffolding, compressed curriculum time, and a procedural focus. The study concludes that targeted instructional scaffolds, teacher professional development, and deliberate integration of multiple representations are needed to strengthen conceptual understanding and improve learner outcomes in probability.

Keywords: Probability, Grade 11 learners, learning challenges, misconceptions, mathematics education

Introduction

Probability supports logical reasoning, problem solving, and decision making across school and everyday contexts (Blitzstein & Hwang, 2019; Ghahramani, 2024). In South Africa's Curriculum and Assessment Policy Statement (CAPS), key probability ideas are introduced in Grade 10 and extended in Grade 11. Despite this emphasis, many learners still find probability abstract and achieve below expected levels (Batanero & Álvarez-Arroyo, 2024; Kaplar, Lužanin, & Verbić, 2021). Understanding why these difficulties persist requires looking closely at learners' reasoning patterns and classroom experiences.

[○]Graduate Student, Bachelor of Education in Mathematics, Walter Sisulu University, Mthatha, South Africa.
[*]PhD Candidate, Rhodes University, South Africa (Corresponding Author).
[±]Senior Lecturer, Department of Mathematics and Science Education, Walter Sisulu University, Mthatha, South Africa.

This study explores how Grade 11 learners in selected Mthatha schools experience and make sense of probability tasks. By analysing diagnostic tests, surveys, and written responses, it identifies common challenges and strategies within real classroom contexts. The paper contributes a South African perspective to global research on probability learning, highlighting how language, pacing, and instructional support influence learner understanding. Unlike many international studies conducted in resource-rich or monolingual settings, this study provides empirical evidence from multilingual, under-resourced classrooms, showing how contextual factors such as language of instruction, limited time, and teacher scaffolding shape learners' experiences with probability. This local focus adds new insight into how global patterns of misconception appear in South African classrooms.

Problem Statement

Despite its central role in the South African mathematics curriculum, probability continues to be one of the least understood and most poorly performed topics in secondary schools. Reports from the Department of Basic Education (DBE, 2011) and recent research (Batanero & Álvarez-Arroyo, 2024; Kaplar, Lužanin, & Verbić, 2021) confirm that many learners view probability as abstract and disconnected from real-world reasoning. Local studies further highlight that both teachers and learners regard probability as one of the most difficult strands in mathematics (Brijlall, 2020). If these challenges are not addressed, learners' mathematical literacy and overall performance in Grades 10–12 will continue to be compromised, restricting their access to higher education and career pathways in science, technology, and related fields (Nguyen & Tran, 2023; Grigaliūnienė, Lehtinen, Verschaffel, & Depaepe, 2025).

Prior research emphasizes that targeted scaffolding, explicit work with multiple representations, and carefully sequenced tasks can improve learner understanding. At the same time, teacher knowledge of common misconceptions and task design plays a central role in shaping outcomes (He & Xin, 2025; Post & Prediger, 2024; Batanero, 2022). This study, therefore, positions probability not only as a curriculum requirement but also as a gateway to broader mathematical reasoning. By examining learner challenges, strategies, and instructional influences, the paper contributes a South African perspective to the international conversation on how probability can be taught more effectively.

Research Questions

This study sought to address the following questions:

a) What challenges do Grade 11 learners face in solving probability problems?
b) What strategies do learners employ to overcome these challenges?
c) How do teaching methods and classroom environments influence their experiences?

Literature Review

This section reviews prior research related to the research questions of the study. Firstly, we consider the challenges learners typically face when engaging with probability concepts, representations, and language. Secondly, we examine how learners' strategies are shaped by instructional approaches and classroom environments, highlighting the roles of task design, scaffolding, technology, and teacher knowledge.

Challenges Learners Face When Solving Probability Problems

Research such as Batanero (2022) and Batanero and Álvarez-Arroyo (2024) consistently shows that learners experience major difficulties with probability, and these challenges have been documented across different contexts. One of the most persistent issues is the conflict between everyday intuition and formal probability concepts (Ghahramani, 2024; Blitzstein & Hwang, 2019). Learners often confuse independence with mutual exclusivity, struggle to coordinate conditional information, or assume outcomes are equally likely even when they are not (Kaplar, Lužanin, & Verbić, 2021; Tillé, 2023). This "equiprobability bias" leads to systematic errors, particularly in tasks involving sampling without replacement (Batanero & Álvarez-Arroyo, 2024; Kaplar et al., 2021; Tillé, 2023). Studies such as Papaieronymou (2017) confirm that targeted instructional approaches in probability can significantly improve learner outcomes. Such findings show that learners bring strong intuitive ideas to class, but these intuitions often clash with formal probability rules.

Another well-documented challenge is representational fluency (Post & Prediger, 2024; Kazak & Pratt, 2021). Probability requires learners to shift between verbal statements, symbolic notation, tree diagrams, Venn diagrams, and tables (He & Xin, 2025; Brnic, Greefrath, & Reinhold, 2024). At these transition points, information is frequently lost or misapplied. For example, learners may draw a Venn diagram but fail to link it to symbolic expressions or construct a tree diagram but apply rules incorrectly. This reflects the tendency to treat representations as disconnected procedures rather than as tools for reasoning (Post & Prediger, 2024; Kazak & Pratt, 2021). Studies further show that foundational skills such as estimation, which support probabilistic reasoning, are not always consistently emphasized in curricula (Xenofontos, Hizli Alkan, & Andrews, 2023).

Language is also a barrier (Can, 2020; Verschaffel, Schukajlow, Star, & Van Dooren, 2020). Technical terms such as given, at least, independent, or have precise mathematical meanings, but learners frequently interpret them in their everyday sense (Xu & Ball, 2024; Staffel, 2023). This mismatch raises cognitive load and contributes to systematic errors even in otherwise straightforward tasks (Verschaffel et al., 2020; Can, 2020). For instance, when "or" is treated as exclusive rather than inclusive, valid outcomes are incorrectly excluded from the sample space.

Instructional and systemic constraints add to these challenges (Brijlall, 2020; Batanero, 2022). In many schools, probability is taught late in the year, leaving little time for exploration (Nguyen & Tran, 2023; Meng & Wu, 2025). Under pressure to finish the syllabus, teachers may rely on procedural demonstrations instead of fostering conceptual understanding (Amedume, Bukari, & Mifetu, 2022). Limited pedagogical content knowledge (PCK) makes it harder for teachers to anticipate and address common misconceptions effectively (Batanero, 2022; Brijlall, 2020). As Staffel (2023) observes, when probability is presented as a rushed and abstract topic, learners are left without meaningful connections to real-world contexts.

Taken together, the literature identifies four recurring areas of difficulty: (1) conceptual misunderstandings of independence, conditionality, and sample space; (2) weak representational fluency; (3) language-related misinterpretations; and (4) instructional and systemic constraints. These categories explain why many learners approach probability with low confidence, provide incomplete responses, or avoid probability tasks altogether.

While these challenges are well known in international studies, very few have explored how they appear in South African classrooms. This study adds to the literature by showing how these problems look in Mthatha schools, where learners study in multilingual settings and teachers work with limited time and resources. The mix of English and isiXhosa, along with fast curriculum pacing and minimal scaffolding, creates unique learning conditions that shape how students understand and experience probability.

Learner Strategies for Solving Probability Problems

Although many learners struggle with probability, research such as Cai and Gu (2019) and He and Xin (2025) shows that they attempt a range of strategies to make sense of problems. Some learners draw diagrams, such as Venn diagrams or tree diagrams, or use tables to organize information (Post & Prediger, 2024; Kazak & Pratt, 2021). Others break complex tasks into smaller steps or apply general rules such as the complement rule. However, these strategies are often partial or incomplete, and learners frequently abandon them before arriving at correct solutions (Kaplar, Lužanin, & Verbić, 2021; Staffel, 2023). This suggests that while learners are aware of possible tools, they may lack the knowledge or confidence to use them effectively (Cai & Gu, 2019; He & Xin, 2025). Collaboration with peers is another strategy reported in the literature. Learners may share partial solutions or compare their reasoning with classmates, which can sometimes help them monitor and refine their thinking. However, such exchanges are often mathematically imprecise, and without teacher facilitation, they do not always lead to more accurate solutions (Nguyen & Tran, 2023).

Metacognitive strategies also play a role in supporting problem-solving. Prompts that encourage learners to plan steps, keep track of given information, and explain their reasoning aloud can help them regulate their thinking (Fan, Song, & Guan, 2021; Bloomfield & Fisher, 2019). These strategies reduce unproductive "trial and error" and guide learners toward more structured approaches (He & Xin, 2025). Yet, studies show that learners rarely adopt such strategies independently and usually do so only when they are explicitly scaffolded by teachers.

The use of technology provides another promising avenue for strategy development (Weigand, Trgalova, & Tabach, 2024; Sánchez, García-Ríos, & Sepúlveda, 2024). Digital tools such as simulations and visualizations allow learners to experiment with probability scenarios, observe long-run frequencies, and compare their intuitive judgments with formal models (Weigand et al., 2024; Sánchez et al., 2024; Brnic, Greefrath, & Reinhold, 2024). Although access remains a challenge in many schools, learners often express curiosity and motivation when exposed to these resources. Curriculum design also plays a role, since limited early emphasis on estimation, a competence closely tied to probabilistic reasoning, can reduce the strategies available to learners later on (Xenofontos, Hizli Alkan, & Andrews, 2023). While learners do attempt strategies such as diagramming, breaking problems down, collaborating with peers, and experimenting with digital tools, these strategies remain fragile and are rarely sustained without structured teacher support. The literature indicates that the most effective strategy used occurs when teachers provide scaffolds that prompt learners to externalize, monitor, and refine their reasoning processes.

Most studies on learner strategies have been done in schools with good technology and where all learners speak the same language. This study adds new insight from Mthatha, where learners study in multilingual classrooms and have limited access to digital tools. Looking at how these local conditions affect problem-solving strategies helps to explain how teaching support and language use shape learners' engagement with probability in South African schools.

Instructional and Classroom Influences on Learner Experiences

Research such as Batanero (2022) and Brijlall (2020) consistently shows that learners' experiences with probability are deeply shaped by instructional approaches and classroom environments. Task design is critical (Meng & Wu, 2025; Vásquez & Alsina, 2021). When probability tasks are sequenced around central ideas and highlight structural connections, learners are more likely to see probability as a coherent topic rather than as isolated rules. In contrast, procedural task design often leads students to focus on memorization without understanding underlying concepts.

Instructional scaffolds further influence learners' approaches to problem solving (He & Xin, 2025; Cai & Gu, 2019). Prompts that encourage them to plan solution steps, record given information, and justify their reasoning help them organize their thinking and reduce reliance on guesswork (Fan, Song, & Guan, 2021). Scaffolds are especially important in probability, where intuitive ideas often conflict with formal concepts. Teachers' PCK is essential in this process. Teachers who can anticipate common misconceptions, such as confusing independence with mutual exclusivity, are better able to design interventions that directly address them (Batanero, 2022; Grigaliūnienė, Lehtinen, Verschaffel, & Depaepe, 2025). Professional learning opportunities, including lesson study and collaborative planning, have been shown to strengthen teachers' PCK and improve classroom practice (Nguyen & Tran, 2023).

Classroom climate also shapes learners' willingness to persist with challenging tasks (Xu & Ball, 2024; Amedume, Bukari, & Mifetu, 2022). Environments that value multiple solution paths, normalize mistakes, and connect probability tasks to meaningful contexts promote confidence and resilience. In contrast, classrooms where teachers rush to final answers or discourage exploration increase the likelihood that learners will abandon tasks.

Technology integration provides further instructional opportunities (Weigand, Trgalova, & Tabach, 2024; Sánchez, García-Ríos, & Sepúlveda, 2024). Simulations, interactive diagrams, and digital tools make abstract structures such as conditionality and long-run frequencies more visible and tangible. They also offer immediate feedback, helping learners reconcile intuitive reasoning with formal probability models. However, limited access in many schools reduces the impact of such tools. In essence, research highlights that task design, scaffolding, teacher PCK, supportive classroom climates, and purposeful technology all play key roles in shaping learners' experiences with probability.

While these ideas are well supported in international studies, there is little research showing how they appear in South African classrooms. In Mthatha, teachers face challenges such as limited teaching time, large classes, and few digital resources. Lessons are often fast-paced, and most learners study in English even though their first language is isiXhosa. These conditions affect how teachers plan, explain, and support probability lessons. By focusing on these local realities, this study adds new insight into how classroom practices and language contexts influence how learners experience probability.

Theoretical Framework

This study is grounded in Constructivist Learning Theory, originally advanced by Vygotsky (1978). The theory posits that learners actively construct knowledge by engaging with tasks and connecting new ideas to their prior experiences. In mathematics learning, and particularly in probability, this often means that intuitive reasoning drawn from everyday contexts conflicts with formal concepts. For example, learners may assume equal likelihood, confuse independence with mutual exclusivity, or misinterpret terms such as given. Such misconceptions frequently persist unless they are deliberately challenged through structured opportunities for reconstruction (Batanero & Álvarez-Arroyo, 2024; Kaplar, Lužanin, & Verbić, 2021).

Constructivism emphasizes the role of instructional practices that confront misconceptions, encourage reflection, and foster conceptual restructuring (Vásquez & Alsina, 2021; He & Xin, 2025). In probability learning, representations such as Venn diagrams, tree diagrams, and tables become especially valuable when used as sense-making tools rather than procedural shortcuts. However, many learners in this study engaged with these representations only superficially, highlighting the need for teaching that integrates representational use with explicit opportunities for dialogue, reflection, and guided problem solving (Post & Prediger, 2024; Nguyen & Tran, 2023; Xu & Ball, 2024). This constructivist lens, therefore, shaped both the analysis and interpretation of learner struggles, underscoring how probability misconceptions persist unless instruction is deliberately designed to help learners rebuild their conceptual frameworks.

Methodology

This study employed a mixed-methods design, combining quantitative and qualitative approaches to provide a fuller picture of learners' experiences with probability. Quantitative elements aligned with a positivist paradigm and included the use of diagnostic tests and surveys to generate numerical data that could be analyzed statistically. At the same time, qualitative elements, grounded in interpretive traditions, were used to analyze learners' written responses for patterns of errors and reasoning strategies. This combination allowed the study not only to measure performance outcomes but also to interpret the meanings behind learners' errors and solution attempts.

The study involved 490 Grade 11 learners and five mathematics teachers from three public secondary schools in the Mthatha district of the Eastern Cape. These schools were purposively selected to reflect the typical diversity of learner backgrounds and instructional conditions in the region, including both township and peri-urban contexts. The choice of Mthatha was informed by its documented performance challenges in mathematics and its representativeness of many South African districts facing similar resource and pacing constraints. The intention was therefore not to generalize nationally but to generate context-specific insights that could inform localized interventions and contribute to comparative discussions in mathematics education. This focused sampling approach aligns with the study's constructivist orientation, emphasizing depth of understanding over statistical generalization. Figure 1 summarizes the participant selection and sampling process.

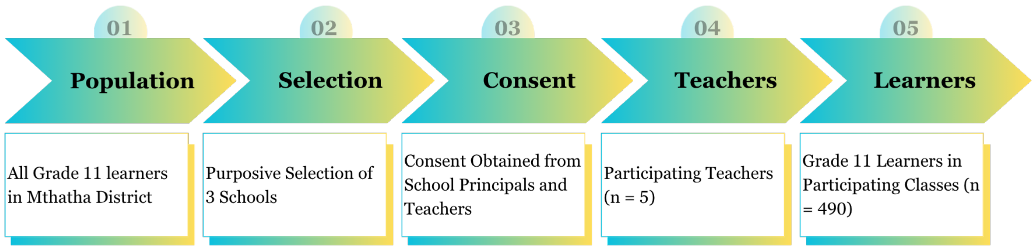
In this study, quantitative analysis emphasized measurable outcomes such as error frequencies and success rates. Qualitative analysis, through content analysis of written responses, focused on recurring errors and misconceptions that could not be reduced to numbers alone. Together, these methods offered both breadth and depth in understanding the problem.

Data Analysis. Data were analyzed using both quantitative and qualitative techniques. Quantitative analysis included descriptive statistics, which summarized accuracy rates and error frequencies, as well as inferential tests such as t-tests and chi-square to examine differences and relationships between demographic, instructional, and performance variables (Croucher & Cronn-Mills, 2021). Qualitative analysis involved content analysis of learners' written responses. Errors were identified, coded, and classified using an a priori scheme synthesized from probability-education research, with categories for (i) conceptual misunderstandings (independence, conditionality, sample space), (ii) representational disconnects (verbal–symbolic–diagrammatic), (iii) language-related misinterpretations, and (iv) procedural violations (e.g., without-replacement denominators). The scheme and exemplars drew on

Batanero and Álvarez-Arroyo (2024), Post & Prediger (2024), Kazak & Pratt (2021), Kaplar, Lužanin, & Verbić (2021), and Tillé (2023). In keeping with the study's constructivist framework, qualitative interpretation focused on how learners constructed, modified, or relied on prior conceptions while solving tasks. Patterns were examined not only as errors but as evidence of learners' meaning-making processes, consistent with constructivist views of knowledge reconstruction. While frequencies of error categories were later reported, the process of identifying and interpreting these errors was qualitative in nature. Validity and reliability were addressed using standardized and moderated test items, expert review, and pilot testing. Reliability was supported by consistency checks in both the pilot and main studies, while qualitative trustworthiness was enhanced through systematic coding and triangulation of test and survey data.

Ethical Considerations. Ethical approval for the study was obtained from the relevant university research ethics committee and the district education office. Permission to conduct research in the selected schools was granted by school principals. Participation was voluntary, and learners and teachers were informed about the study's purpose and procedures. No identifying information was collected, and all responses were anonymized. Parental consent and learner assent were obtained in line with ethical research standards for school-based studies.

Figure 1. Participant Selection and Sampling Process (*developed by the authors*)



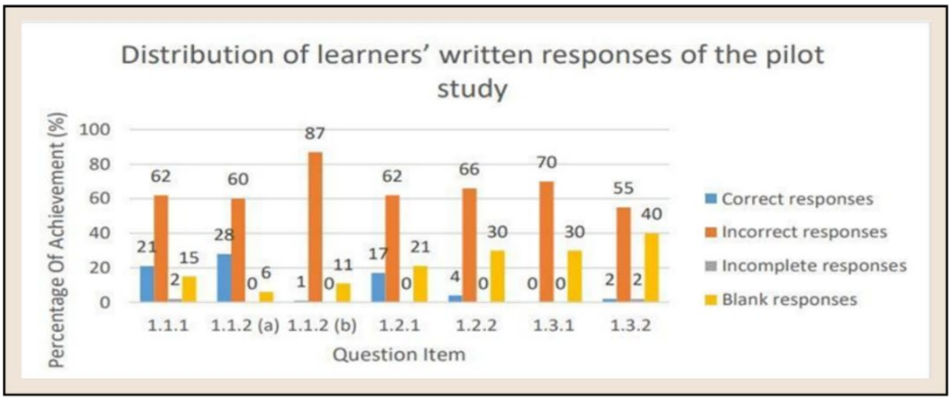
Results

This section presents the findings of the diagnostic assessment in relation to the three research questions. First, we report the challenges learners faced when solving probability problems, focusing on performance patterns, error types, and misconceptions. Second, we describe the strategies learners employed when attempting to solve these problems. Finally, we consider how teaching methods and classroom environments influenced learners' experiences with probability.

Challenges Learners Face in Solving Probability Problems

The diagnostic test confirmed that Grade 11 learners in this study experienced significant difficulty with probability. The overall mean score was 11% (SD = 7), reflecting consistently low performance across items. Correct responses were especially scarce on questions requiring interpretation of Venn diagrams (21%), calculation of compound events (1–2%), and application of probability rules. Many learners left items blank or provided incomplete responses, and fully correct solutions were rare. These performance patterns are summarized in Figure 2, which shows the distribution of learner-written answers and follow-up questions.

Figure 2. Distribution of Learner-written Answers and Follow-up Questions (developed by the authors).



To summarize these results more clearly across related task categories, Table 1 presents overall success, error, and no-response rates. While Figure 2 highlights individual item-level differences, Table 1 groups these into conceptual areas such as compound events, conditional probability, and representational fluency.

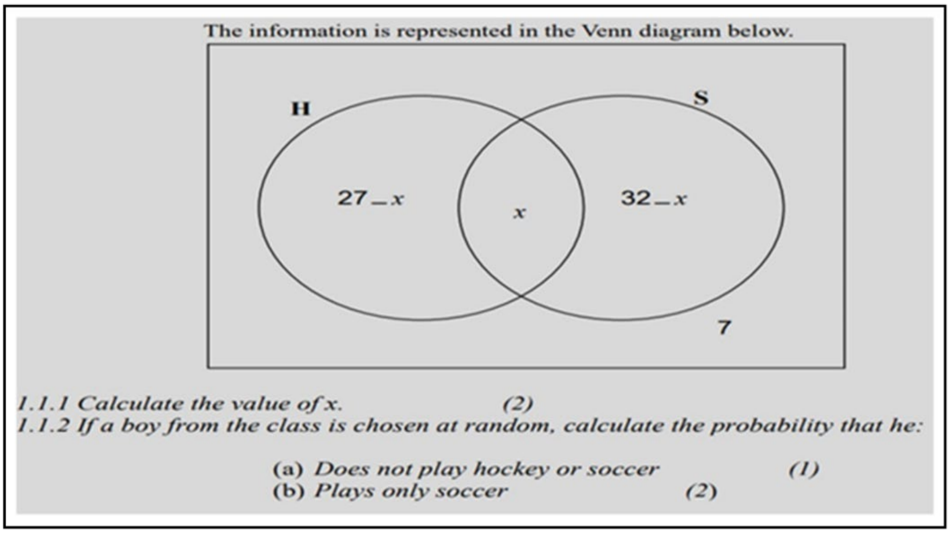
Table 1. Summary of Learners' Performance on Diagnostic Probability Items (N = 490)

Diagnostic Focus	Concept Tested	% Correct	% Incorrect	% No Response
1. Sample space identification	Understanding possible outcomes	16	68	16
2. Independence vs. mutual exclusivity	Distinguishing conceptual relationships	2	70	28
3. Compound events	Multi-step reasoning (AND/OR)	1	69	30
4. Venn-diagram interpretation	Representational fluency	21	60	19
5. Conditional probability	Interpreting P(A | B) and dependency	7	66	27
6. Sampling without replacement	Adjusting denominators across draws	8	55	37
7. Verbal–symbolic translation	Linking words to notation	10	52	38

As shown in Table 1, correct responses were lowest for compound and conditional probability items, consistent with the very low overall mean score of 11 percent (SD = 7). Items requiring representational fluency, such as interpreting Venn diagrams, showed slightly higher performance but remained well below mastery. Percentages were rounded to the nearest whole number, and the data were summarized from learners' diagnostic responses (M = 11%, SD = 7). These quantitative findings mirror the patterns discussed later in Section 6.

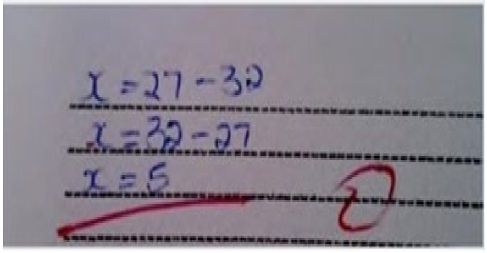
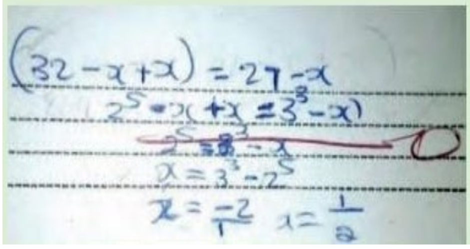
At the item level, performance remained weak. For Question 1, only 16% of learners answered item 1.1.1 correctly, and just 1% answered items 1.1.2 and 1.1.3 correctly. Item 1.2.1 recorded the highest success rate at 24%. Incorrect responses averaged 60.4%, and incomplete responses were frequent. For Question 2, correct answers were similarly rare: only 7% of learners answered item 2.1.1 correctly. Nonresponses were particularly high, with 67% for items 2.1.1 and 2.1.2, 75% for item 2.1.3, 50% for item 2.2, and 40% for item 2.3. An example of one of the diagnostic test items is shown in Figure 3.

Figure 3. Example of Diagnostic Test item used in the Study (developed by the authors)



Learners' written responses revealed recurring errors. Many misinterpreted statements involving and/or given, failed to account for the total sample space, or produced probabilities outside the valid range of 0 to 1. Others neglected to update denominators in without-replacement contexts, which distorted probability calculations. Illustrative examples of such errors are presented in Figure 4, which shows responses to Question 1.1.1.

Figure 4. Learner Responses to Question 1.1.1 Illustrate a Misinterpretation of the Sample Space (developed by the authors)

<table>
<tr><th>Learner 1 (L1)</th><th>Learner 2 (L2)</th></tr>
<tr><td>
</td><td>
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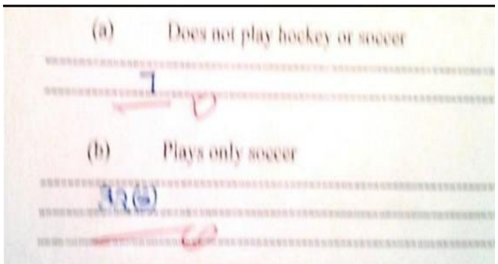
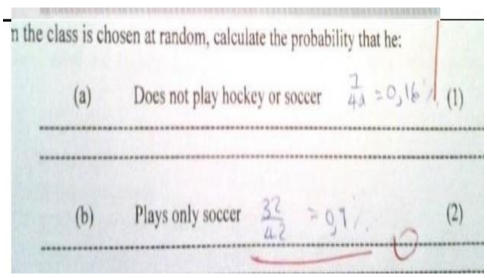
</table>

Additional misconceptions appeared in responses to Question 1.1.2, where some confused conditional probability with mutual exclusivity, applying rules inappropriately or abandoning solutions midway. Errors were also common when learners attempted to translate verbal problems into diagrams or symbolic notation. In many cases, diagrams or equations were disconnected from the problem statement, indicating weak representational fluency. Examples of these misconceptions are shown in Figure 5.

Figure 5. Learner Responses to Question 1.1.2 showing Confusion between Conditionality and Mutual Exclusivity (developed by the authors)

<table>
<tr><th>Learner 3 (L3)</th><th>Learner 4 (L4)</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Finally, technical terms such as independent and at least were often interpreted in their everyday sense rather than their mathematical meaning, which compounded errors even in relatively simple tasks. While numerical scores revealed low performance, qualitative coding of written responses provided deeper insight into the types of misconceptions that shaped learner reasoning.

Quantitative Summary and Relationships

To complement the descriptive findings, basic inferential analyses were conducted to explore relationships between learner characteristics, instructional contexts, and probability performance. Independent-samples *t*-tests indicated no significant difference between male and female learners ($t(488) = 1.07$, $p = .28$), confirming that performance challenges were broadly consistent across gender.

However, a one-way ANOVA showed significant between-school variation ($F(2, 487) = 4.21$, $p < .05$), suggesting that contextual factors such as pacing, teacher support, or resource access may have influenced outcomes.

Pearson's correlation analysis further revealed a small but statistically significant relationship between learners' self-reported confidence in mathematics and their probability scores ($r = .27$, $p < .01$). No meaningful association was found between prior mathematics achievement (based on school records) and diagnostic scores ($r = .09$, $p = .11$). These patterns align with qualitative evidence indicating that differences in classroom practices and learner attitudes, rather than demographic factors, were key contributors to the observed performance disparities. A summary of the main quantitative indicators, including descriptive and inferential results, is presented in Table 2.

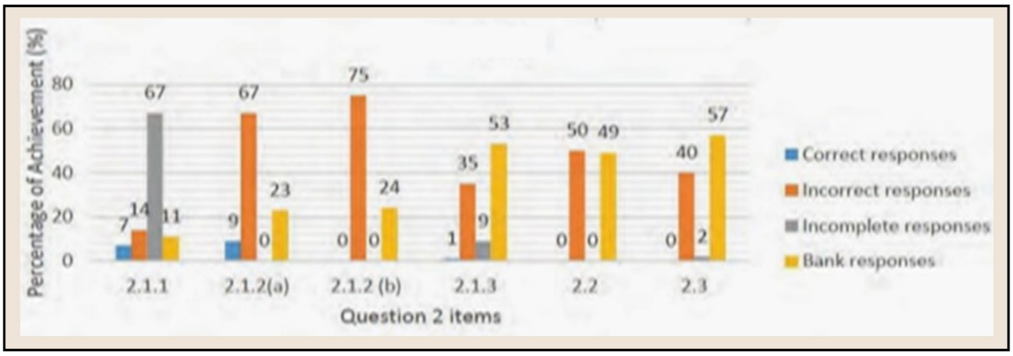
Table 2. Summary of Quantitative Performance Indicators

Measure	Mean / % (SD)	Range	Test / Association	p-value	Interpretation
Overall diagnostic test score	11 (7)	0–42	—	—	Consistently low performance
Venn-diagram interpretation items	21% (9)	5–45	—	—	Weak representational fluency
Compound-event calculation items	2% (2.3)	0–10	—	—	Minimal conceptual understanding
Between-school differences	—	—	ANOVA F(2,487)=4.21	< .05	Contextual variation significant
Gender comparison	—	—	t(488)=1.07	0.28	No gender difference
Confidence–score correlation	—	—	r=.27	< .01	Confidence linked to achievement
Prior achievement–score correlation	—	—	r=.09	0.11	No significant link

Learner Strategies for Solving Probability Problems

The diagnostic test revealed that learners demonstrated limited and fragile use of problem-solving strategies. Many items were left blank or contained only partial steps that did not progress to valid calculations. A small number of learners attempted strategies such as drawing Venn diagrams or applying the complement rule, but these efforts were rare and often unsuccessful. More commonly, learners substituted numbers without defining events, applied probability rules inconsistently, or abandoned solutions midway. Examples of such representational difficulties and incomplete strategies can be seen in Figure 6.

Figure 6. Learner Responses to Question 2 Illustrating Representational Difficulties and Incomplete Strategies (developed by the authors)



Written responses to Question 2 provided further evidence of these weaknesses. Blank spaces, incomplete setups, and inconsistent representations were frequent, while clear and logically sequenced solutions were scarce. Some learners attempted to represent events using diagrams, but these were not connected to symbolic notation or calculations. Others applied probability rules without clarifying the conditions under which the rules were valid, which led to errors or incomplete work. Even when diagrams, tables, or symbolic notation were used, they were often misapplied. For example, some learners drew tree diagrams but left branches unlabeled, while others began Venn diagrams but failed to complete them in ways that matched the task requirements. These patterns suggest that representational tools were not being used as reasoning aids but rather as procedures to be attempted. Instances of fully correct or well-developed strategies were rare. Most responses reflected procedural beginnings without conceptual grounding, resulting in incomplete or incorrect solutions. This indicates that while learners were aware of potential strategies, they lacked either the knowledge, confidence, or instructional support to apply them effectively.

Instructional and Classroom Influences on Learner Experiences

Learners' responses reflected the influence of classroom practices and teaching approaches. Many reported, through survey items and informal comments, that probability lessons were delivered at a fast pace, often toward the end of the school year, which left little time for exploration or revision. This compressed pacing contributed to the high number of nonresponses in the diagnostic test, as learners tended to abandon problems when they were unable to follow procedural explanations.

The findings further showed that probability was often taught in a procedural manner rather than with a conceptual focus. Teachers typically demonstrated rules for compound events and Venn diagram tasks but provided few opportunities for learners to engage with the underlying structures. As a result, learners rarely attempted strategies such as planning solution steps, annotating givens, or validating answers. The absence of such scaffolding contributed to incomplete and fragmented responses in the diagnostic tasks.

Teachers' survey and interview responses supported these findings. Several teachers reported that they had limited time to cover probability and often taught it close to examination periods, leaving little room for practice or exploration. Others noted that language barriers made it difficult to explain key probability terms to learners whose home language was not English. Large classes and a lack of visual or digital resources also limited opportunities for interactive or exploratory teaching. These insights help explain the learning patterns observed in learners' diagnostic responses and highlight the classroom realities shaping instruction in Mthatha. Figure 7 summarizes the main challenges teachers described when teaching probability, showing how time pressure, language barriers, large classes, and limited resources interact to shape learners' experiences in Mthatha classrooms.

Figure 7. Concept Map of Teacher-reported Challenges in Teaching Probability

Survey responses also highlighted the importance of classroom climate. Learners noted that mistakes were frequently treated as final outcomes rather than as opportunities for discussion. This discouraged persistence when tasks became difficult. In classrooms where teachers moved quickly to final answers, learners were less likely to attempt alternative approaches. Conversely, some learners expressed a preference for tasks that allowed exploration and reflection, which they felt supported their understanding.

Technology did not appear in learners' diagnostic responses, reflecting the limited access to digital tools in the participating schools. However, learners indicated curiosity about the use of simulations and interactive resources, suggesting that these could help make abstract probability concepts more concrete. The results show that classroom practices and instructional environments strongly shaped learners' experiences. Fast pacing, limited scaffolding, and a procedural emphasis reinforced misconceptions and incomplete strategies, while supportive climates and the potential use of technology offered pathways for improvement.

Discussion

Challenges Learners Face

The results confirm that probability remains a highly challenging strand for Grade 11 learners. Quantitatively, the diagnostic test produced a mean score of 11 percent (SD = 7), with significant between-school variation ($F(2, 487) = 4.21$, $p < .05$). These low averages mirror the qualitative evidence of deep conceptual fragility observed in learners' written work. The very weak performance on compound and multi-step items indicates that many learners relied on everyday reasoning patterns that were not adequate for coordinating events across multiple steps. From a constructivist perspective, this reflects assimilation of new tasks into pre-existing intuitive frameworks rather than accommodation and restructuring toward formal probability concepts. Similar patterns have been reported in prior research, where learners default to additive reasoning instead of developing multiplicative reasoning for compound events (De Keersmaeker et al., 2024).

Recurring misconceptions observed in this study, including confusion between independence and mutual exclusivity, misinterpretation of sample spaces, and production of invalid probabilities, suggest that entrenched prior conceptions were not effectively confronted during instruction. These qualitative patterns correspond closely with the quantitative finding that only 1–2 percent of learners solved compound-event items correctly, reinforcing how specific misconceptions translated into measurable performance deficits. This aligns with earlier findings that such misconceptions persist when learners are not provided with structured opportunities for conceptual reconstruction (Batanero & Álvarez-Arroyo, 2024; Kaplar et al., 2021; Tillé, 2023).

Learners' difficulties in moving between verbal, diagrammatic, and symbolic forms also point to representational fluency as a key weakness. The 21 percent success rate on Venn-diagram items quantifies this limitation, confirming that representational errors identified qualitatively were not isolated incidents but systematic obstacles. These difficulties echo evidence that students often treat diagrams and notations as disconnected procedures rather than as tools for reorganizing understanding (Post & Prediger, 2024; Kazak & Pratt, 2021). Conditional probability proved especially fragile, showing that without explicit teaching that links representations, learners struggle to sustain reasoning across steps.

Language demands further compounded these challenges. Everyday interpretations of terms such as and, or, and independent often blocked progress before calculations could begin. Constructivist learning theory highlights the role of language in shaping thought. Without explicit reconstruction of such terms, learners' intuitive meanings continue to dominate problem solving (Verschaffel et al., 2020; Can, 2020). Together, the numerical data and qualitative coding reveal three interrelated clusters of difficulty: conceptual misunderstanding, weak representational fluency, and linguistic confusion, each supported by both statistical and interpretive evidence. Constructivist theory explains why these challenges persisted. Learners' intuitive frameworks were not confronted and rebuilt during instruction. Anticipating these misconceptions and deliberately creating opportunities for representational and linguistic reconstruction should therefore be central to teachers' pedagogical content knowledge in probability (Batanero, 2022; Grigaliūnienė et al., 2025).

In the Mthatha classrooms studied, these challenges were made worse by the language of instruction. Most learners spoke isiXhosa at home but learned mathematics in English, which increased confusion around probability terms and symbols. The fast pace of lessons and limited class time also gave few chances for teachers to address these misunderstandings. This shows how multilingual and time-pressured learning environments can deepen the same conceptual and language barriers seen internationally. Viewed through a constructivist lens, these patterns show that learners were building new knowledge on top of incomplete or intuitive ideas. Without enough time or scaffolding to reorganize their thinking, misconceptions were carried forward instead of reconstructed, which helps explain the persistence of these difficulties.

These findings confirm international patterns of misconception and language difficulty but also extend them by showing how they interact with multilingual and time-pressured classroom contexts. While studies in higher-income settings often attribute errors mainly to conceptual misunderstanding, this study reveals how limited instructional time, dual-language learning, and scarce visual supports can magnify those same misconceptions. In this way, the Mthatha results enrich global discussions by highlighting how resource and language factors shape the persistence of known probability misconceptions.

Learner Strategies

The diagnostic results show that learners' strategies for solving probability problems were limited, fragile, and often abandoned. From a constructivist perspective, this reflects learners' assimilation of new tasks into familiar but inadequate intuitive habits, rather than a reorganization of their thinking into conceptually grounded strategies. Attempts at Venn diagrams or the complement rule frequently broke down when learners were required to align these with symbolic notation, confirming that representations were treated as disconnected procedures rather than as reasoning tools (Post & Prediger, 2024; Kazak & Pratt, 2021).

The prevalence of incomplete setups, number substitution, and abandoned solutions points to weak metacognitive regulation. Constructivist theory emphasizes the importance of scaffolds that help learners externalize their reasoning and monitor their progress. Prompts that require students to plan solution steps, record givens, and justify their operations can support the development of monitoring routines and reduce reliance on trial and error (He & Xin, 2025; Cai & Gu, 2019). The lack of such scaffolding in the participating schools may explain why learner strategies often remained incoherent and unstable.

Language difficulties further constrained strategy development. Many learners interpreted terms such as independent and or in everyday rather than technical ways, which disrupted attempts to apply strategies consistently. This finding supports earlier research showing that misinterpretation of probability language undermines the connection between diagrams, symbolic notation, and problem contexts (Verschaffel, Schukajlow, Star, & Van Dooren, 2020).

The potential role of technology also emerges from the literature. Interactive simulations and visualizations can make hidden structures such as conditionality and long-run frequencies visible, while providing immediate feedback to reconcile intuitive reasoning with formal models (Weigand, Trgalova, & Tabach, 2024; Sánchez, García-Ríos, & Sepúlveda, 2024; Brnic, Greefrath, & Reinhold, 2024). Although technology was not available in the studied schools, learners' curiosity suggests that its integration could help scaffold more effective strategies.

In summary, the strategies observed in this study reflect learners' reliance on intuitive heuristics rather than coherent, conceptually grounded methods. Constructivist learning theory suggests that progress requires instruction that deliberately surfaces learners' current strategies, scaffolds reflection and monitoring, clarifies technical probability language, and integrates representational and technological supports to foster the gradual reconstruction of more effective problem-solving approaches.

In the Mthatha context, limited access to computers or calculators meant learners could not use visual or simulation tools to test their ideas. Lessons were also teacher-led, leaving little time for exploration or group discussion. These conditions help explain why most learners depended on trial-and-error strategies and rarely checked or refined their reasoning.

International research often reports that learners use incomplete or inconsistent strategies even when resources are abundant (Cai & Gu, 2019; He & Xin, 2025). The Mthatha data extend this evidence by showing how fragile strategies become when access to technological and representational tools is limited. Rather than contradicting global trends, these results reveal a contextual amplification of known problems, where resource scarcity and teacher-led instruction reduce learners' opportunities to test and refine their reasoning.

Instructional and Classroom Influences

The findings show that learners' difficulties with probability were strongly shaped by the instructional and classroom contexts in which they learned. Probability lessons were often taught quickly and near the end of the school year, limiting opportunities for exploration and review. From a constructivist perspective, this pacing reduced the chances for learners to reorganize their intuitive frameworks and reinforced the procedural and fragmented approaches observed in their responses. Prior studies confirm that rushed instruction and compressed coverage of probability undermine conceptual reconstruction and leave misconceptions unaddressed (Batanero, 2022; Brijlall, 2020).

The lack of scaffolding further constrained learners' engagement. Few learners planned solution steps, annotated givens, or validated answers, suggesting that these practices were not consistently modeled in classrooms. Constructivist theory highlights that scaffolds are essential for externalizing learners' ideas, testing them, and guiding conceptual reorganization (He & Xin, 2025). The present findings align with research showing that without structured scaffolding, learners often abandon tasks prematurely or rely on superficial procedures (Post & Prediger, 2024).

Classroom climate also played a central role. Learners reported that mistakes were often treated as final outcomes, which discouraged persistence. Constructivist learning emphasizes that errors should be used as resources for discussion and reconstruction. Supportive environments that normalize mistakes and encourage exploration have been shown to increase resilience and promote deeper engagement with probability (Xu & Ball, 2024; Amedume, Bukari, & Mifetu, 2022). The high rate of blank responses observed in this study reflects the negative impact of classroom norms that treat mistakes as failures rather than as learning opportunities.

Finally, the limited use of technology restricted the range of instructional supports available. While learners in this study expressed curiosity about simulations and interactive tools, they had little access to them in practice. Prior research demonstrates that technology can make abstract probability structures more visible, support representational linking, and provide immediate feedback to reconcile intuition with formal models (Weigand, Trgalova, & Tabach, 2024; Sánchez, García-Ríos, & Sepúlveda, 2024). Integrating such tools could therefore enhance strategy use and conceptual understanding if access barriers are addressed. These findings show that teaching methods and classroom environments strongly shaped learners' experiences of probability. Constructivist learning theory reinforces that progress depends on instruction that deliberately confronts intuitive ideas, integrates scaffolds and representations, normalizes errors as part of learning, and uses mediating tools such as technology to support learners in reconstructing their understanding.

In Mthatha schools, these classroom and teaching factors are shaped by limited resources, large class sizes, and high syllabus pressure. Teachers often teach in English to learners whose home language is isiXhosa, which makes explaining and checking understanding harder. These realities help to explain why many learners showed surface learning and abandoned problems early. Recognizing these local teaching conditions highlights how systemic and language factors influence learners' performance in probability.

These findings reinforce the value of Constructivist Learning Theory as an interpretive lens for understanding learners' difficulties and classroom experiences. The analysis showed that learners approached new probability ideas through their prior knowledge and intuitive reasoning, and that without enough scaffolding or discussion, these initial ideas were rarely reconstructed. Viewing the data through this framework helped explain not only the patterns of misconception and weak strategy use but also why progress was limited in fast-paced, resource-constrained classrooms. Constructivism, therefore, guided the interpretation of both quantitative and qualitative results, emphasizing that improvement depends on supporting learners as they actively rebuild their understanding of probability.

Conclusion

This study examined the experiences of Grade 11 learners in solving probability problems, focusing on the challenges they faced, the strategies they employed, and the ways in which teaching methods and classroom environments shaped their learning. Results from a diagnostic test revealed low performance, with widespread misconceptions about independence, conditionality, and sample spaces. Learners also struggled to shift between verbal, diagrammatic, and symbolic forms, while common probability terms were often misinterpreted in everyday rather than technical ways. From a constructivist perspective, these findings highlight the difficulty learners experience when intuitive reasoning is not reorganized into formal probability concepts.

In terms of strategies, learners' approaches were limited and fragile. Many responses were blank or incomplete and attempts at diagrams or probability rules often broke down without teacher scaffolding. This indicates that learners were aware of potential strategies but lacked confidence and conceptual grounding to apply them effectively.

The study also showed that instructional practices and classroom conditions significantly influenced learner experiences. Fast-paced, procedural teaching, limited scaffolding, and classroom norms that discouraged error exploration contributed to weak engagement. At the same time, learners expressed interest in supportive environments and technology-based tools that could make abstract probability concepts more tangible. The findings underscore the need for instructional scaffolds that strengthen stepwise reasoning, promote representational linking, and clarify probability vocabulary. Professional development for teachers is also critical to building pedagogical content knowledge and anticipating common learner errors. In line with the theoretical framework, improving learner engagement with probability requires learning environments that actively support knowledge construction through scaffolding, representation, technology, and guided reflection. Addressing these gaps can enhance not only learners' mastery of probability but also their broader mathematical reasoning and problem-solving confidence.

Limitations

This study was limited to three secondary schools within a single district, which constrains the generalizability of the findings to other regions or school contexts. In addition, the reliance on quantitative data from a diagnostic test provided a useful profile of learner performance but did not capture the richer perspectives that classroom observations, teacher reflections, or learner interviews might have offered. As a result, the study offers only a partial view of the challenges and strategies surrounding probability learning.

Implications

The findings of this study carry important implications for the teaching and learning of probability in Grade 11. First, the persistent misconceptions and low performance observed in this study suggest that the time currently allocated to probability in the CAPS curriculum may be insufficient. When the topic is compressed into a short unit, learners have limited opportunities to engage meaningfully with concepts, work with multiple representations, and consolidate their understanding. The implication is that curriculum planners and schools need to consider pacing and sequencing in ways that allow for a gradual introduction of probability ideas, from basic sample spaces to more complex conditional contexts. At the policy level, CAPS planners could review term-length allocations to ensure that probability is introduced earlier in the year and reinforced through short spiral reviews. Schools could also embed probability concepts across related topics such as data handling, giving learners repeated exposure instead of a single rushed unit.

Second, the challenges learners experienced with representation, language, and problem-solving strategies highlight the importance of teacher PCK in probability. Professional development that emphasizes conceptual rather than procedural approaches, such as lesson study or peer collaboration, has the potential to strengthen teacher capacity and improve learner outcomes. District-level initiatives could include demonstration lessons and bilingual glossaries that help connect English and isiXhosa terminology, making probability instruction clearer and more inclusive.

Third, the study underscores the role of representational and technological support in making probability concepts more accessible. Venn diagrams, two-way tables, and tree diagrams are most effective when used as tools for reasoning rather than as procedural notations. Similarly, digital simulations and interactive visualizations can help learners reconcile intuitive judgments with formal probability rules, providing immediate feedback and reducing reliance on misconceptions such as equiprobability bias. Even in resource-limited schools, low-cost alternatives such as card-sorting tasks and paper-based simulations can serve as effective substitutes for digital tools when used thoughtfully.

At a broader level, the findings have important implications for education policy through the Curriculum and Assessment Policy Statement (CAPS). The ongoing difficulties and low performance found in this study suggest that the current CAPS pacing for probability, which is usually taught near the end of the Grade 11 syllabus, gives learners too little time to understand and practise key ideas. Adjusting the pacing so that probability topics are introduced and revisited earlier in the year could help learners build a stronger understanding. CAPS also highlights the importance of teacher professional development. Regular workshops and collaborative lesson planning can help teachers anticipate common mistakes, use visual tools more effectively, and make lessons more engaging. In addition, aligning classroom and national assessments with CAPS goals that focus on reasoning and explanation rather than only correct procedures can encourage deeper learning. Working together on these areas, such as curriculum pacing, teacher support, and assessment design, can improve how learners experience and understand probability in South African classrooms.

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