

## Inexpressible Number and the Secrets of the Erechtheion

By Michael R. Ytterberg\*

*In the Classical period, the Erechtheion and the Parthenon were constructed opposite each other on the Acropolis of Athens. If the Parthenon represents perfection, what can be said of the Erechtheion? We are familiar with the idea that the perceived perfection of monuments such as the Parthenon is due in large measure to their proportional systems. Can such a system possibly be at work in the Erechtheion, a structure that otherwise seems the antithesis of the compositionally simple, perfectly pure Parthenon? Asymmetrical buildings closely configured to elaborate programmatic requirements are the norm in contemporary architecture, yet the Erechtheion continues to seem idiosyncratic, and the intentions of its designers obscure. Yet the exquisite beauty of its carving has always promised more. This paper will show that the Erechtheion does indeed follow a rigorous proportional scheme that is consistent down to its smallest details and which is located firmly within a Greek tradition as passed on to us by Vitruvius. The proportional apparatus of ancient Greek architecture, most often interpreted as relationships of rational numbers, will be shown to control the design of the Erechtheion through the rational approximations of irrational numbers as described by ancient authors.*

**Keywords:** *Erechtheion, Vitruvius, Ionic order, irrational numbers, square root of two*

### The Buildings of the Acropolis

In ancient Athens under Pericles, during the height of its glory, four buildings were constructed on the Acropolis, each apparently by a different architect,<sup>1</sup> and each one of which has claimed a place in the history of western architecture: the Parthenon, the Temple of Athena Nike, the Propylaea and the Erechtheion. Three of these exemplary buildings adhered to known conventions of Greek temple design in the Doric Order. The fourth, however, the Erechtheion, is both Ionic and unique in the history of architecture (Figure 1). No other Greek temple building exhibits such striking idiosyncracies and peculiarities of construction. No other Greek temple integrated into one asymmetrical structure multiple shrines, each with its own distinct expression. No other Greek temple straddled a three and a quarter meter change of ground level, unchangeable because of the sanctity of the ground on which the temple stood. Vincent Scully, writing in his classic work on

---

\*Adjunct Professor, Drexel University, USA.

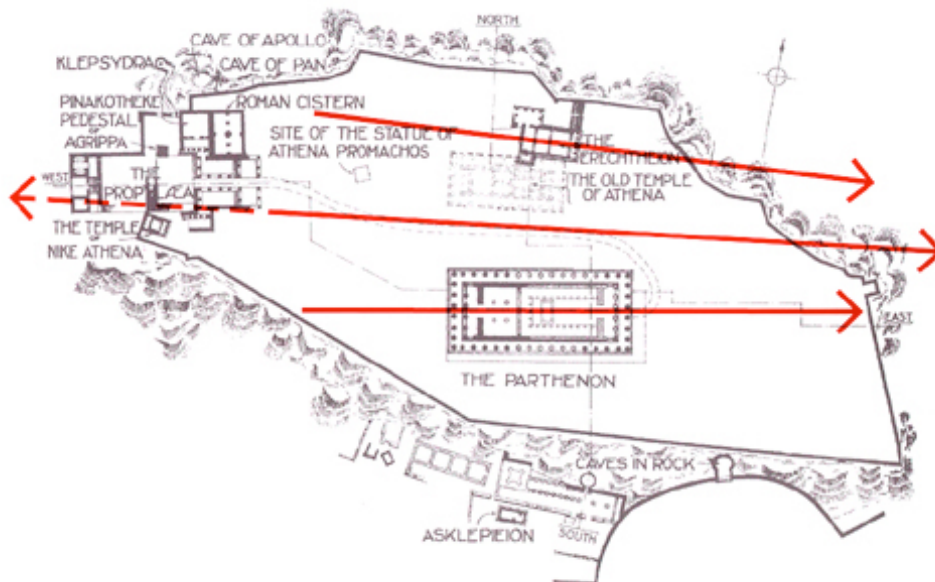
<sup>1</sup>Iktinos and Kallikrates were the designers of the Parthenon, constructed 448-432 B.C.E.; Mnesikles was the designer of the Propylaea, constructed 437-432 B.C.E.; Kallikrates designed the Temple of Athena Nike, but construction was delayed until 427-424 B.C.E.; and the designers of the Erechtheion, constructed c. 432 or 421-405 B.C.E., are unknown, though there are plausible reasons on stylistic grounds for considering both Mnesikles and Kallikrates as candidates.

the siting of Greek temples, *The Earth, the Temple, and the Gods*, went so far as to claim that “the Erechtheion, as a complex and elaborately scaled set of interlocking parts, is the only Greek temple which may be said to have been designed wholly in terms of existing conditions and wholly in response to other forms, those both of the landscape and of other buildings” (Scully 1979).

**Figure 1.** *The Erechtheion from the East*



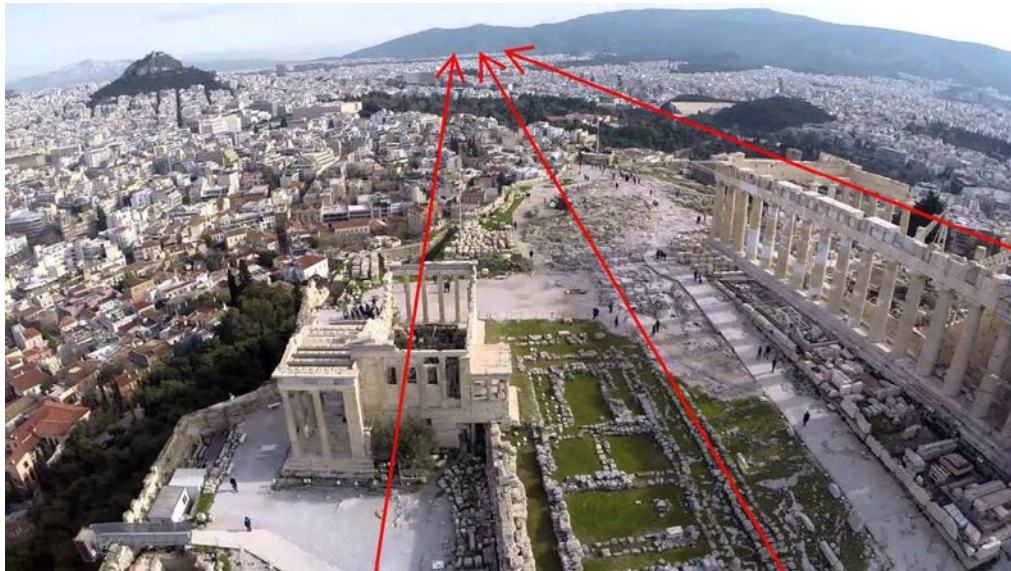
**Figure 2.** *Acropolis Site Plan with Axes, Drawing by Author*



As such it is the fitting antithesis to the Parthenon, with which it is paired on the Acropolis (Figure 2). The Parthenon’s relative simplicity contrasts with the Erechtheion’s compositional complexity. Though Vitruvius mentions the Parthenon only in passing, he singles out the Erechtheion as one of the first of an unusual

temple type where porticos are found on the sides of the cella.<sup>2</sup> The two temples roughly parallel each other across the open summit of the Acropolis. They form a cross axis with the primary thrust of the Acropolis *temenos* from Salamis in the distance to the west over the roof of the Propylaia and across the altar of Athena to the horizon to the east, where the rising sun emerges from the earth. The two temples are not precisely parallel but appear to be angled to focus our attention on the cleft in the Hymettos, a distant ridge. The shape of this cleft, according to Scully, suggests “horns,” which then may be identified with those of the bulls sacrificed on the great altar between the two buildings (Scully *supra* n. 2, p. 181) (Figure 3). Whereas the Parthenon celebrated the creative, god-like power of the men who conceived, crafted, and paid for its glory, the Erechtheion was redolent of the age old symbols of cthonic cults dedicated to the increase of the fertility of the earth, again, according to Scully: the womb of the cave, the serpent and its labyrinth, and the horns of the hoofed animals on which life depends.<sup>3</sup>

**Figure 3.** *Acropolis, Aerial View to the East between the Erechtheion and the Parthenon*



Source: Diagram by author.

Both temples were dedicated to differing aspects of a single deity, Athena. Athena Parthenos was the virgin, the pure, the symbol of all that was finest in Athens and its citizens. The Parthenon extolled civic virtues, celebrated Athens' power, and contained the treasury of the Ionic League. Athena Polias was guardian of the city, and all of the cults associated with the Erechtheion dealt with the origins of her city. Though on the mainland, Athens had never been conquered by the Dorians, and its power lay in its alliance with and dominance over the Ionian

<sup>2</sup>The Parthenon is mentioned in Vitruvius VII.1.2; the Erechtheion is described in IV.viii.4.

<sup>3</sup>The development of the symbolism of Greek architecture from these beginnings already present in prehistoric religious practice is traced in Scully (*supra* n. 2): 10 f. More recently, the importance of these themes in the development of the Ionic order specifically was treated in Rykwert (1996, pp. 236–315).

cities to the east. Suitably, the Erechtheion was the first truly monumental Ionic building on the Greek mainland.<sup>4</sup> In the Erechtheion the Ionic order reaches a perfection in its details that mirrors the perfection of the Doric in the Parthenon. The two buildings are joined, then, as heaven meets earth, as west meets east, and, in the gendered characterization of the orders as described by Vitruvius, as man and woman.

### The Cults of the Erechtheion

The travel writer Pausanias, writing in the 2nd century C.E., makes the first surviving reference to the building as the Erechtheion.<sup>5</sup> Previously, as in Vitruvius, reference to the building was as a temple of Athena. Directly to the south lay the site of an earlier temple, predating both the Erechtheion and the Parthenon, which had been built directly over the megaron of the Mycenaean palace which had originally occupied the Acropolis. Before there had been a temple on the Acropolis the gods had been welcomed in the megaron of the Mycenaean warlords. The subsequent temple, whether called the *hekatompedon* (“hundred footer”) or *archaios neos* (old temple), or both, may have housed the ancient wooden statue of Athena Polias prior to the Erechtheion as opposed to a predecessor of the Erechtheion itself.<sup>6</sup> In any case it affected the placement of the present Erechtheion, for its cella could not be demolished - if it was demolished - until the Erechtheion was completed (Figure 4). The reconstruction of the destroyed interior of the Erechtheion followed here is presumed to have mimicked the interior layout of the earlier temple, assuming that the Erechtheion was its replacement.

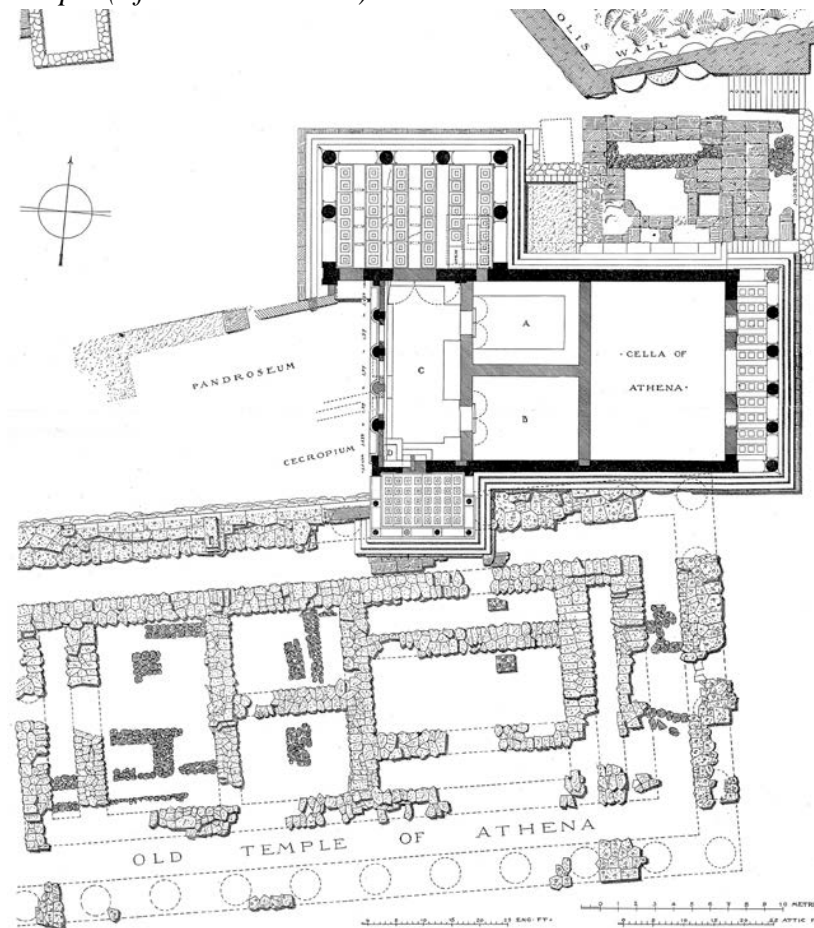
---

<sup>4</sup>The Siphnian Treasury at Delphi was first.

<sup>5</sup>Several paragraphs are devoted to the Erechtheion in the course of the description of a visit to the Acropolis in Pausanias, *Description of Greece* I.26.5-27.2. The following discussion of the cults of the Erechtheion is based on Scully (supra n. 2, p. 172 f), Rykwert (supra n. 5, pp. 133-138), Stevens et al. (1927, p. 452 f.), and Elderkin (1941, pp. 115-124).

<sup>6</sup>The existence of an earlier temple on the site of the Erechtheion, perhaps a temporary one, is mentioned in Herodotus, *Histories* V.55.

**Figure 4.** Site Plan of the Erechtheion with Adjacent Foundations of Previous Temple (After Stevens 1927)



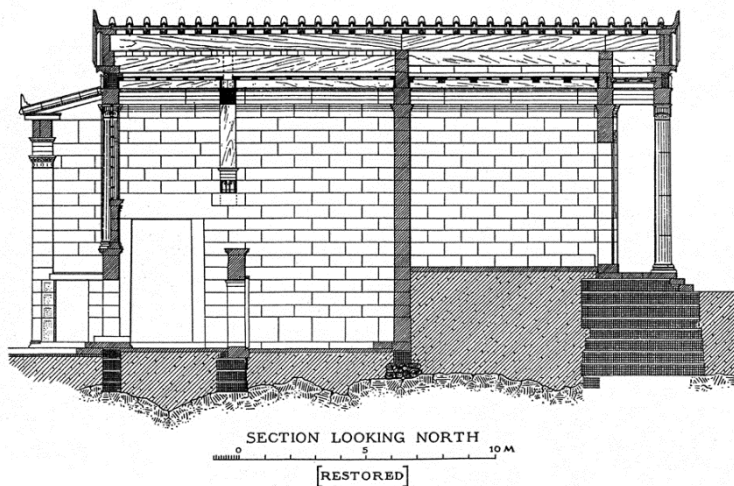
It was said that the ancient wooden image of Athena had not been made by men but had fallen to the earth from the heavens above. It had been saved from the Persians when the Acropolis was burned and afterwards was most likely housed in the upper cella of the Erechtheion behind the East Portico, explicitly paired with the great statue of the goddess by Phidias behind the East Portico of the Parthenon.<sup>7</sup> Athena had won the city in a contest with Poseidon. A sacred olive tree, the sign of her victory, was enclosed in the courtyard to the west of the Erechtheion and below the level of the East Portico, between the Kekropion and the Pandroseion. This lower level courtyard with its shrines was extant prior to the construction of the Erechtheion and could not be raised (Figure 5). Kekrops, “a son of the soil, with a body compounded of man and serpent,”<sup>8</sup> was the legendary first king of Athens. Pandrosus was one of his daughters who was entrusted by Athena with a

<sup>7</sup>The accepted location of the various artifacts and altars within the Erechtheion and the original configuration of the interior has recently been challenged in a doctoral dissertation of Lesk (2004). Her conclusions are not followed here. The complexity of the cults of the Erechtheion remains in any case.

<sup>8</sup>Apollodorus, *The Library* III.xiv.1.

chest containing Athena's illicit progeny by Hephaistos, Erichthonius. Erichthonius, protected by serpents or part serpent himself, was raised by Athena in the precinct of the Erechtheion. Upon becoming king of Athens, "he set up the wooden image of Athena in the acropolis, and instituted the festival of the Panathenaea."<sup>9</sup> He, like Kekrops before him, was identical with the sacred serpent which lived in the precinct of the Erechtheion and guarded the salt water pool, or *thalassa*, which Poseidon had created with a blow of his trident in the course of his contest with Athena. This "sea" lay within the western rooms of the Erechtheion, possibly in the southwestern corner, where it was seen by Pausanias.<sup>10</sup>

**Figure 5.** *Restored Section through the Erechtheion. After G.P. Stevens, 1927*



But what of Erechtheus the titular deity of the Erechtheion? Erechtheus, also a legendary king of Athens, was the grandson of Erichthonius, and may originally have been identical with him.<sup>11</sup> According to Homer, Erechtheus was a son of the earth, a foster child of Athena, and placed by her at his death in her own temple to be honored with annual sacrifices.<sup>12</sup> The citizens of Athens were known as Erechtheids, or descendants of Erechtheus. After winning the war with the Eleusinians through the sacrifice of his daughters, Erechtheus was struck down in revenge by Poseidon with his trident, or, alternatively, was struck down by a thunderbolt of Zeus at the request of Poseidon.<sup>13</sup> The mark of the trident or thunderbolt is visible still through the pavement of the North Portico. An opening

<sup>9</sup>Apollodorus III.xiv.6. See commentary by Sir James Frazer in note 3, Apollodorus, *The Library*, Cambridge, MA and London: Harvard and Heineman, 1921: vol. II, p. 93. Plutarch attributed the institution of the Panathenaea to Theseus in *Theseus* 24.

<sup>10</sup>Pausanias (supra n. 7). The unusual configuration of the southwest corner of the Erechtheion convinced G. P. Stevens that the *thalassa* was located there, marked "D" on Figure 4, though this is not an uncontested view. Most would probably prefer a centrally located position in the western part of the building, for which there is some evidence.

<sup>11</sup>Apollodorus III.xiv.6. See commentary by Sir James Frazer in note 2, (supra n. 9): 91.

<sup>12</sup>Homer, *Iliad* II.546-551.

<sup>13</sup>Apollodorus III.xv.4-5; Hyginus, *Fables* 46.

through the roof above made an altar here hypaethral, identifying it as that of Zeus Hypatos described by Pausanias.<sup>14</sup> Along with the *thalassa*, Pausanias names three altars, probably in the rooms of the lower, western half of the Erechtheion. They are those of Poseidon Erechtheus, identified as a single deity, of Boutes, the brother of Erechtheus who received the priesthood of Athena and of Poseidon Erechtheus when his brother was made king, and of Hephaistos, father of Erichthonius. The ancient Athenian festival of the Diipoleia with its sacrifice of an ox may have been the annual reënactment of the slaying of Erechtheus by Zeus, for it was officiated by a descendent of Boutes.<sup>15</sup> *Boutes* is the ancient Greek word for herdsman, derived from *bous*, an ox. Erechtheus as an ox seems to have succeeded the familiar identification of Poseidon as a bull in Athenian worship. And the *thalassa* came to be called the Erechtheis.

But of what use is salt water in an acropolis, a hilltop fortress, when in a siege the presence of a fresh water well is required? Pausanias was not surprised to find salt water in such a location for he knew of another example in Aphrodisias in Caria, in Asia Minor. Aphrodite, the patron goddess of Aphrodisias, was born of the sea, violated by the severed genitals of Ouranos. At Athens, a garden sanctuary of Aphrodite was on the north slope of the Acropolis, near where a sacred tillage was performed each spring.<sup>16</sup> The *arrhephoroi* were virgin quasi-priestesses lodged at state expense near the Erechtheion. Pausanias's account of their annual ceremony suggests that they may have descended to Aphrodite's garden sanctuary to receive their charge of a secret, wrapped object which they could have borne to the Erechtheion to cast into the *thalassa*,<sup>17</sup> afterwards returning to the garden sanctuary. Their point of entry was likely the portico which was borne by *korai*, or maidens, adjacent to the Kekropion, dedicated to the serpent which guarded the sacred pool (Figure 6). If the secret objects were phallic in nature, then the rite could have been a reënactment of the birth of Aphrodite, performed to assure the fertility of the earth. The same *arrhephoroi* wove the *peplos* offered to Athena Polias as bride during the Panathenaea. They began during the festival of the Chalkeia, i.e., of Hephaistos, her consort. According to Athenian tradition, previous to Athena the consort of Hephaistos had been Aphrodite.<sup>18</sup>

<sup>14</sup>This altar may be identical with another altar mentioned by Pausanias, that of the Thyechoüs, "the one who pours the sacrifice." Elderkin (supra n. 7): 114. There was also said to have been an altar to Zeus Herkeios in the Pandroseion.

<sup>15</sup>Elderkin (supra n. 7), p. 115.

<sup>16</sup>Elderkin (supra n. 6), p. 112.

<sup>17</sup>Pausanias I.27.3.

<sup>18</sup>The *korai* of the Erechtheion were preceded on the Acropolis by the *korai* which were found in a pit northwest of the Erechtheion, possibly buried after having been mutilated by the Persians. These statues further attest to the characteristics of the *arrhephoroi*. One carries a dove and two pomegranates, attributes of Aphrodite. On a final note, south of the East Portico of the Erechtheion was a preexisting altar of Dione, the mother of Aphrodite according to an Athenian tradition. Elderkin (supra n. 7), p. 120.

**Figure 6.** *The Erechtheion, Portico of the Maidens*

It was Vitruvius to whom we owe the name “caryatid” for load bearing female statues as a type.<sup>19</sup> But his explanation of the term makes no mention of the city of Aphrodite in the Ionian province of Caria or of the *korai* of the Erechtheion. For Vitruvius a caryatid is not a young maiden carrying sacred objects, but an enslaved matron of a disgraced town. Caria was a city in the Peloponnesus which sided with the Persians against their fellow Greeks. The victorious Greeks laid waste to the town, killed the men, and enslaved their women. This story parallels the one he tells about male column figures, which he names for defeated and enslaved Persians. But the maidens of the Erechtheion carry their load lightly and proudly, as do other examples of their type in the ancient world.<sup>20</sup> These Caryatids seem to make explicit the gendered characteristics of the Ionic order which Vitruvius describes elsewhere in his treatise,<sup>21</sup> though he does identify the Ionic order as specifically matronly, reserving for the Corinthian the aspects of the maiden. In any case, Vitruvius describes the transference of expressive characteristics of the human body to mute stone as a consequence of the search for dignity and propriety in monumental construction, not as an expression of domination, over man, woman, or nature.<sup>22</sup>

---

<sup>19</sup>Vitruvius I.i.5.

<sup>20</sup>In the Mourning Women’s Sarcophagus and the Neried monument, both of which feature female figures alternating with rather than replacing columns, the funereal function is served by the women being visibly distraught at the death of the fallen. Caryatids may have a special relationship with the dead. The Erechtheion, after all, is a *heroon*, the monumental tomb of a fallen hero, Erechtheus.

<sup>21</sup>Vitruvius IV.i.7.

<sup>22</sup>Vitruvius IV.i.6.



## Design Complexity and the Doric Foot

The stunning complexity of the cults of the Erechtheion and the meticulous accommodation of sacred needs which both constrained and inspired the designers of the temple support Vincent Scully's claim with which this paper began. It is specifically the intentions of the designers as regards the characterization of the Erechtheion as "a complex and elaborately scaled set of interlocking parts" that beg for further clarification. The design of the Erechtheion is notoriously puzzling, and its seeming arbitrary juxtapositions and violations of the standard practice of fifth century Greeks as they are currently understood has prompted various attempts to propose idealized, more regular initial designs that were frustrated in one way or another by costs, conservative religious sentiment, unseen site conditions, or mere miscalculation. Some of the greatest archaeologists of the early twentieth century attempted reconstructions of such a regularized, unrealized plan, and the attempts to do so have not abated.<sup>23</sup>

These reconstructions attest to a common need to understand the rationale for the Erechtheion's formal idiosyncracies. The American School of Classical Studies at Athens publication of 1927 is the definitive work on the Erechtheion. It gathered together and examined all of the evidence then amassed for the understanding of the Erechtheion: archaeological, epigraphical, and philological. In particular it contains the exhaustive (and beautiful) measurements, drawings, and restorations of Gorham Phillips Stevens, which have constituted one of the most basic resources for study of the Erechtheion ever since. The long history and ruinous condition of the building mean that substantial scholarship was required to ascertain its original condition, which is not without controversy. Roman repairs after a disastrous fire in the first century B.C.E., mainly to the western façade, did

---

<sup>23</sup>Dinsmoor presented his proposal in his survey of Greek architecture, *The Architecture of the Ancient Greeks*, while Harriet Boyd Hawes never advanced her theories to the point of being published. See Dinsmoor (1973). Harriet Boyd Hawes's reconstruction has been presented in a paper given at the annual meeting of the Archaeological Institute of America on January 4, 2002, by Alexandra L. Lesk of the University of Cincinnati.

The scholars to whom we otherwise owe our current understanding of the original state of the Erechtheion as constructed also believed in a more regular, unrealized initial design. Wilhelm Dörpfeld, director for many years of the Athenian branch of the German Archaeological Institute, first published his thoughts on an original plan for the Erechtheion just after the turn of the century. The final results of his efforts were not finished and published until 1942, after his death, by a former assistant. See Dörpfeld (1968). Dörpfeld believed that the centerlines of the North Portico and the Portico of the Maidens must have been intended to align. He therefore proposed an original plan for a symmetrical building that would have completely reconfigured the Pandrosion and Kekropion. His work was based on the exhaustive publication of the American School of Classical Studies at Athens of 1927, *The Erechtheum*, (see Stevens et al., supra n. 6) though he had many differences with it. The authors of that work had a more modest proposal for an original design. The existing doorways leading into the interior from the North Portico and the Portico of the Maidens are aligned with each other, but are off center in the room to which they lead. They proposed that this room was originally intended to be wider than as executed, with the west façade of the building further west, so that the doors would have been centered in this interior space. (See Stevens et al., supra n. 6, pp. 167–169). Alexandra Lesk, who challenged so much else in her dissertation (see Note. 9 above) still accepts the need for an ideal, regularized plan that was compromised in construction.

not follow the original design, and the use of the building as a church and then as a Turkish dwelling followed by over a century as an abandoned ruin (not to speak of the deprivations of Lord Elgin) all took their toll on the building. The construction of a cistern below the floor of the westernmost room of the building, which the authors of the 1927 work believed to have been called the *Prostomiaion*, destroyed all trace of the Erechtheis. No disagreement is advanced here with the conclusions made by Stevens and his coauthors regarding the original condition of the Erechtheion. The location of the cults in and around the building described above follows for the most part the 1927 work. And the proportional diagrams which follow use as a basis Stevens's glorious drawings, though it is his measurements which make possible any serious work.

But the task of uncovering original design intentions of the architects of the Erechtheion is doubly difficult, for even the simplest Greek buildings frequently defy the efforts of scholars to unravel the design process of their architects and discover the underlying proportions and corresponding generative geometrical structure. In spite of innumerable attempts, no solution has ever achieved scholarly consensus in the case of even such a building as the Parthenon. Vitruvius and other ancient sources have led scholars to believe that a modular approach using simple ratios was the standard procedure of Greek architects. Writers such as Plato emphasized the importance of exact measurements, for ethical as well as aesthetic reasons. But Greek buildings refuse to yield their secrets easily, and the Erechtheion is the most complex off all.

Several factors complicate matters. Where precision was required, there is evidence of the use of fractions in Greek practice down to  $1/448$  of a foot (Haselberger 1996, p. 410). Yet the rounding of measurements to the nearest whole measure may also have been a standard practice as well. The ponderous methods of numerical notation available to the Greeks and Romans (the Greeks used letters for numerical notation, as did the Romans before turning to Roman numerals) may have encouraged this. Then there is the question of refinements, of which there are two kinds. Vitruvius spoke of the need for optical corrections when exact measurements looked wrong to the eye, generally on account of distance, and needed to be distorted in order to look correct. However, some refinements, such as column entasis, are very noticeable and seem more likely to have been intended to produce an expressive effect. Both cases complicate retrieval of generative design dimensions. The Erechtheion, for the most part, lacks the curved lines which play such an important role in the Parthenon. The only curved lines are the entasis of the columns of the North Portico and those of the Roman restoration on the west façade. On the other hand, in contrast to what is often said about Ionic buildings, the majority of the vertical lines of the exterior of the building almost imperceptibly incline back towards the building. The columns of the North and East Porticos lean in between 0.0018 and 0.02 meter in their total height, the corner columns leaning in two directions, and the north and south walls lean in at a rate of 0.0115 meter in ten courses. In the Porch of the Maidens the axis of the architrave is placed 0.008 meter behind the axis of the podium. The

jambs of the exterior doors and windows incline inward as well.<sup>24</sup>

Nothing learned thus far makes the present task any easier. In one respect, however, much more is known about the Erechtheion than any other Greek structure. The Erechtheion was begun in either the last years of the Periclean era before the beginning of the Peloponnesian war in 432 B.C.E., which seems likely on stylistic grounds, or during the brief period of Athenian prosperity following the Peace of Nicias in 421 B.C.E. In either case war interrupted construction. In the summer of 409 B.C.E., a commission which had been appointed the previous year made a detailed report on the condition of the unfinished building. The building was close to completion, and the commission documented all of the prepared stones lying on the site ready for installation prior to the resumption of construction that same year. The report was inscribed on a marble stele and set up on the site, to be discovered on the Acropolis in 1765. Subsequent inscriptions have come to light which record expenditures on the work through the Athenian year of 405-404 B.C.E., when work on the building finally ceased.<sup>25</sup> A wealth of information about the Erechtheion is provided in these inscriptions, but inscriptions recording expenditures on and/or details of construction projects are not unusual in ancient Greece. What is unusual in this case is that dimensions of building components are given which can be precisely identified, and therefore measured, in the completed building. It was Wilhelm Dörpfeld who first performed the necessary analysis and published his results in 1890 (Dörpfeld 1890, pp. 167–187). He determined that the length of the foot measure that was used in the Erechtheion was 0.328 meter, making the cubit 0.492 meter, the palm 0.082 meter, and the dactyl 0.0205 meter. The authors of the 1927 American publication found only support for Dörpfeld's conclusions. Both of these publications called this an "Attic" foot, but subsequent scholarship has come to identify this measure as a "Doric" foot, reserving the name "Attic" for a foot of 0.293-0.296 meter which later came to dominate the Greek world, becoming the Roman foot as well.

The dimensions given in the inscriptions might be regarded as nominal dimensions. Most of them are even foot lengths and only occasionally involve one half or one quarter of a foot. Given that the Erechtheion is renowned for the delicacy and precision of its carving, the degree of variation in the actual measurements of building components is somewhat surprising. A standard wall block is given in the inscriptions as four feet long, two feet wide, and one and a half feet high. If a foot is 0.328 meter, then the standard wall block in meters would be 1.312 m. long, 0.656 m. wide, and 0.492 m. high. However, the actual blocks in the walls vary from 1.29 to 1.31 m. in length, 0.65 to 0.675 m. in width, and 0.478 to 0.504 m. in height.<sup>26</sup> This a variation of up to -1.7% in length, of 1.0% to +2.9% in width, and -2.8% to +2.4% in height, which, though greater than

---

<sup>24</sup>A complete listing of the refinements of the Erechtheion is in Stevens et al. (supra n. 7, pp. 214–216).

<sup>25</sup>The texts of the inscriptions along with a translation are provided in Stevens et al. (supra n. 7, p. 277 f). It is interesting to note that some of the details of the building were never finished. For example, cylinders of stone on the entablature of the Portico of the Maidens were never carved into rosettes, and some wall blocks never had their handling bosses removed.

<sup>26</sup>The dimensions given here are slightly different from Dörpfeld's measurements. Dimensions given by Stevens have been interpolated into the list. See Stevens et al. (supra n. 6, pp. 222–223).

modern construction tolerances, might have been accurate for the ancient world. There is greater variation in the frieze blocks. The inscriptions give the size of the standard frieze block as four feet long, one foot wide, and two feet high. In meters this would be 1.312 m. long, 0.328 m. wide, and 0.656 m. high. Dörpfeld's measurements for these blocks varied from 1.31 to 1.37 m. in length, 0.28 to 0.33 m. in width, and 0.62 to 0.68 m. in height. This is a variation of up to +4.4% in length, from -14.6% to +0.6% in width, and from -5.5% to +3.6% in height. These values continue to hover about dimensions based on the .328 meter foot but with much greater variation than in the case of the wall blocks. This suggests that the builders were much more concerned with a conceptual rigor as expressed by the nominal dimension given in the inscriptions than with extreme accuracy in the actual construction itself.

Though the correct values for ancient Greek foot measures have continued to occupy scholars in the years since these publications, the results for the Erechtheion have continued to hold.<sup>27</sup> Dörpfeld and the American authors of the 1927 publication agreed that many of the key dimensions of the Erechtheion do work out to even foot measures. For example, the width of the interior above the base of the walls is 30 "Doric" feet and therefore the overall width of the main block of the building is 34 feet. The width of the *Prostomaion* is 13 feet, the interior rooms are 18 feet deep, etc. A lack of interest in the precise accuracy of actual measurements may not be the same as rounding to the nearest whole foot, but the effect on the ability to gauge the intentions of the designers may be the same. But in spite of potential pitfalls, two important bits of information are now in hand: the exact size of the standard measure used in the construction of the Erechtheion and a sense of the order of magnitude of accuracy that might be expected in the work. It is time to examine the Erechtheion for evidence of an underlying proportional system.

### Similar Rectangles in Plan

For many observers, close study of the Erechtheion eventually reveals that behind the Caryatids and the exquisite carving of architectural detail lies an extraordinarily pleasing and sophisticated three dimensional composition of similar rectangles<sup>28</sup> (see Figure 8 in Appendix). Staring at the plan, it gradually becomes apparent that the main cella area, the North Portico, and the Portico of the Maidens all appear to be based on the same rectangle. The first goal is to identify this generating rectangle. A trial of all the likely possibilities leads to the result that the *exterior* of the two porticos and the *interior* of the cella area between the East

<sup>27</sup>Controversy over the correct length of ancient measures is a topic that never goes out of fashion. Jari Pakkanen has recently used sophisticated mathematical analysis to propose a 0.324 meter dimension for the Doric foot at the Erechtheion. Pakkanen, J. The Erechtheion and the Length of the 'Doric-Pheidonic' Foot," *Talanta* XXXVII-XXXIX (2006-2007) and *Classical Greek Architectural Design: A Quantitative Approach*, Helsinki: Foundation of the Finnish Institute at Athens, 2013.

<sup>28</sup>Such was the conclusion of an author of a late 19<sup>th</sup> century German architectural handbook, A. Thiersch. See Thiersch (1889, Bk. VI, Ch. 1). The plan of the Erechtheion used in this handbook is that of the existing remains prior to reconstruction of its original condition.

Portico and the *Prostomiaion* all conform to the ratio of a rectangle whose long side is equal to the diagonal of a square whose sides are equal to the short side of the rectangle, i.e., the square root of two times the length of the short side. These are the rectangles indicated as P3, P4, and P7 on the plan diagram. When the dimensions for these rectangles in meters are calculated from the work of Stevens, and then converted to Doric feet, the results are very encouraging: all three rectangles fall plausibly into even measures of Doric feet. Moreover, the variations between the proportion produced by the actual dimensions and the ideal all fall within a margin of error of less than 2.0%, well within the margin of error that was found on the standard wall blocks of the inscriptions. All of the results for these rectangles and the following analysis are to be found in the accompanying table.

**Table 1.** *The Erechtheion Plan and Elevation Proportional Analysis*

Rectangle	Dimensions		Ratio			Percentage Variance from Ideal*
	Meters	Doric Feet	Meters	Doric Feet	Ideal	
<b>Plan</b>						
<b>P1</b>	22.304/11.187	68/34	1.9937/1	2/1	2/1	00.32%
<b>P2</b>	15.706/11.187	48/34	1.4040/1	1.4118/1	1.4142/1	00.72%
<b>P3</b>	10.124/7.199	31/22	1.4063/1	1.4091/1	1.4142/1	00.56%
<b>P4</b>	5.535/3.992	17/12	1.3865/1	1.4167/1	1.4142/1	01.96%
<b>P5</b>	6.500/4.594	20/14	1.4149/1	1.4285/1	1.4142/1	00.05%
<b>P6</b>	11.187/7.968	34/24	1.4040/1	1.4167/1	1.4142/1	00.72%
<b>P7</b>	13.818/9.837	42/30	1.4047/1	1.4000/1	1.4142/1	00.67%
<b>Elevation</b>						
<b>E1</b>	11.187/7.999	34/24.375	1.3985/1	1.3949/1	1.4142/1	01.11%
<b>E2</b>	4.946/3.541	15/10.75	1.3969/1	1.3953/1	1.4142/1	01.23%
<b>E3</b>	9.232/6.524	28/20	1.4151/1	1.4000/1	1.4142/1	00.06%
<b>E4</b>	22.304/7.999	68/24.375	2.7883/1	2.7897/1	2.8284/1	01.41%
<b>E5</b>	5.535/4.946	17/15	1.1190/1	1.1333/1	1.125/1	00.58%
<b>E6</b>	14.617/6.097	44.5/18.5	2.3974/1	2.4054/1	2.4142/1	00.67%
<b>E7</b>	9.837/7.028	30/21.375	1.3998/1	1.4035/1	1.4142/1	01.02%
<b>E8</b>	22.304/11.004	68/33.5	2.0269/1	2.0299/1	2/1	01.35%
<b>E9</b>	10.124/9.232	31/28.125	1.0966/1	1.1022/1	1.125/1	02.52%
<b>E10</b>	12.808/9.102	39/27.75	1.4072/1	1.4054/1	1.4142/1	00.50%

Note: \* This column lists the degree of variance between the ratio in the metric column and the ideal ratio.

The production of consistent results requires addressing the question of methodology. Where is the rectangle to be measured that bounds the North Portico: at the base of the columns or at the bottom of the architrave, or perhaps at the centerline of both? The answer is to be found by examining the East Portico where there is a clear relationship between wall and column. As mentioned above, both walls and columns incline backwards, but at a different rate. The epistyle of the wall runs continuously into the architrave of the portico: one stone bridges between wall and entablature. At the level of the top of the capital/bottom of architrave, the center of the corner column aligns with the centerline of the anta and architrave/epistyle. At the base level the center of the corner column lies beyond the centerline of anta and wall. This compellingly suggests that the building should be measured where wall and portico align: at the bottom of the architrave. That this constitutes the clearest conceptual framework for the building receives

verification in the restored plan as drawn by Stevens. He juxtaposed plans of the ceilings in the porticos with the floor plan in the interior of the building as the clearest way of explaining the building. It is plausible that the same was true for its designers.<sup>29</sup> Additional evidence for this point of view will be discussed when considering the layout of the East Portico.<sup>30</sup> Mark Wilson Jones supports the idea that the architects of Greek temples began their layouts at the level of the architrave with an analysis of Doric temples (Wilson 2001, pp. 675–713, see also Wilson 2014).<sup>31</sup>

Thus far, the exterior dimensions of the North and South Porticos have been compared to an interior space. To be compelling, the analysis must account for the dimensions of the building in a more systematic way. Without the two side porticos the exterior of the main block of the Erechtheion, including the East Portico, is a double square 68 Doric feet long by 34 Doric feet wide (P1 in Figure 8). This echoes, without precisely duplicating, Vitruvius's observation of the Erechtheion that "the length of [the] cella is twice the width."<sup>32</sup> The two side porticos are clearly additive elements to the central part of the building: both porticos end below the cornice line of the main block which thus reads as an unbroken volume from all sides. Establishing the double square plan for the central block is possibly the starting point of the overall design. If the diagonal of the easternmost square is pivoted up to the north, it marks the spot along the northern wall where the southwestern corner of the eastern anta of the North Portico is located (P2). This length is 48 Doric feet. The rectangle of the plan of the North Portico, 31 feet by 22 feet (P3), has been scaled so that the 22 foot generative square of this  $\sqrt{2}$  rectangle reaches just shy of the northwestern corner of the main block. On the south side, the 17 foot by 12 foot rectangle of the Portico of the Maidens (P4) sets the southeastern corner of the cross wall that divides the *Prostomiaion* from the inner rooms. With its western edge on the west side of the 2 foot thick cross wall, and its width set by the width of the inner room created by the central (hypothetical) 2 foot thick longitudinal wall, a fifth rectangle (P5), 20 feet by 14 feet, sets the location of the second cross wall at the western side of the East Cella. The western edge of its generative square aligns with the western edge of the first of the  $\sqrt{2}$  rectangles (P2). Starting from the eastern side of this 2 foot thick cross wall, a sixth rectangle (P6), 34 feet by 24 feet, sets the

<sup>29</sup>To conceive the building in this way may or may not have required the making of scale drawings, of which fifth century Greeks were certainly capable in any case. For the evidence for full size drawings in Greek practice, see Haselberger (1985, pp. 126–132). For general discussion of Greek practice see Coulton (1977, pp. 51–73).

<sup>30</sup>The case of the Portico of the Maidens is somewhat ambiguous because of the presence of the podium below the maidens. However, in the interest of consistency, this rule of measuring at the architrave has been followed here as well. Another ambiguity is that the northern edge of the circumscribing rectangle for this Portico (P4) falls within the thickness of the wall of the central block and therefore has a purely theoretical value. This point was established by simply assuming a conceptual thickness inside the thickness of the wall equal to the width of the architrave of the Portico of the Maidens. This measures .458 m. in reality and one foot and six dactyls or .451 m. ideally (1.375 of one foot of .328 m.).

<sup>31</sup>Jari Pakkanen's disagrees with Wilson Jones' conclusions. See *Supra* no.11, Pakkanen 2013, pp. 28–31.

<sup>32</sup>Vitruvius IV.ix.4.

eastern edge of the 2 foot thick exterior wall at the eastern side of the East Cella. A final  $\sqrt{2}$  rectangle (P7), 42 feet by 30 feet, reaffirms the size of the interior rooms.<sup>33</sup> Thus is the plan of the Erechtheion the result of a simple procedure that produces a “complex and elaborately scaled set of interlocking parts.”

As perfect as this operation may seem, it is possible to imagine that its first approximation might have been even tighter. Figure 9 in Appendix suggests an early stage in the process of design. The two foot wall thickness called out in the inscriptions and the 30 foot interior width suggest a grid of two foot squares. That the plans of Ionic temples had always tended to have a more regular gridded character than Doric temples, and tended to be rigorously planned on a square grid from the fourth century onward, is well known.<sup>34</sup> In this initial design stage, before much had been decided about the vertical dimension of the temple, the architect may have assumed a nominal two foot wall thickness for the entire building with the exception of the Portico of the Maidens. The decision to replace the columns of this portico with figures would have made it unique from the start. If the Portico of the Maidens had originally been proposed to be 16 instead of 17 feet long, then all of the rest of the proportional rectangles of the plan as given above would fit together absolutely perfectly as shown.

Given the apparent desire for dimensions in whole foot values (i.e., the use of the foot as a module), the problem with this scheme would have been that a 16 foot long  $\sqrt{2}$  rectangle would have a width that was too far from a whole number of feet ( $16 \div 1.4142 = 11.3138$ ).<sup>35</sup> This appears to have been important, even though there was no way to experience this in the final building since the northern most edge of the rectangle falls within the thickness of the wall of the main block. Enlarging the length one foot produces a width that is very close to an even 12 feet ( $17 \div 1.4142 = 12.020$ ).<sup>36</sup> Doing this would drag the interior walls of the temple one foot to the east exactly as they now are believed to have stood.<sup>37</sup> In addition the decision was made to make the columns of the North Portico taller than those of the East, necessitating a wider architrave. This pulled the southwest corner of

<sup>33</sup>Figure 13 shows one more  $\sqrt{2}$  rectangle over the location of the Erechtheis proposed by G.P. Stevens. His reconstruction of a stone canopy at this point remains more hypothetical than other aspects of the reconstruction. As such its dimensions have not been included for analysis in Figure 10.

<sup>34</sup>See Coulton (supra n. 33, pp. 70–71) for a discussion of the grid based plans of Ionic temples and the rules for Ionic temples devised by Hermogenes.

<sup>35</sup>Fifth century B.C.E. architects would not have achieved this result by long division, but by measuring the diagonal of a square.

<sup>36</sup>In *The Mathematics Useful for Understanding Plato*, written in the 2<sup>nd</sup> century C. E., Theon of Smyrna demonstrated an algorithm for approximating the value of  $\sqrt{2}:1$  that produced a series of rational ratios that converged on the precise value. He stopped his demonstration with 17:12 (1.4167:1), the first “rational convergent” approximating  $\sqrt{2}:1$  within 1% of the actual value. The previous convergents in the series are 3:2 (1.5:1) and 7:5 (1.4:1). See Lawlor and Lawlor (1979, pp. 29–30).

<sup>37</sup>Interestingly, there is evidence that a cross beam was constructed one Doric foot west of the east wall of the *Prostomiaion*, that is, in line with the originally proposed location of the wall according to this proposed original design scheme. The misalignment of the wall and beam can be seen in Figure 5. After the fire of the first century B.C.E. the east wall of the *Prostomiaion* was replaced directly underneath this beam.

the generative square of the North Portico rectangle off the corner of the main block by the amount of increase in the width of the architrave (0.766 m. - 0.656 m = 0.11 m. = 1/3 foot). Thus, in order to accommodate detailed requirements, two small changes to an initially perfect proportional layout were made. Vitruvius seems to have described this process precisely when he wrote, "I put it beyond doubt that something must be added or taken away according to the requirements and nature of their situation... First therefore the measure of the symmetries must be established from which surely the modifications may be deduced."<sup>38</sup>

Some scholars feel that any proportional scheme for a Greek temple must start with the stylobate. The steps of the stylobate of the Erechtheion form a regular border around the building, outside of the preexisting Pandroseion and Kekropion, but never complete a regular rectangle comparable to a typical temple plan that could become the basis for the laying out of a proportional system on the site. The distance from the lowest step to the edge of the euthynteria, or leveling course, where one can be found, is very irregular. It seems much more likely that the most basic element in the Erechtheion is the 34 foot square that is doubled to make the plan of the main block. Given a two foot plan module, the north and south walls established by this square are sixteen modules on center, the most perfect number of all according to Vitruvius and the Pythagoreans.<sup>39</sup> The point of beginning for the Erechtheion is most likely this nominal 16 module dimension across the East Portico, in front of the cella with the ancient image of Athena Polias, paired with the Parthenon across the open summit of the Acropolis and facing the eastern horizon.

This analysis of the evidence suggests that the Erechtheion has a rigorously planned order exactly as it stands. There is less motivation for imagining that the resulting building is the product of some set of compromises that prevented the construction of a more symmetrical scheme. Even the modest "original plan" from the 1927 American publication is based on finding an axial north/south relationship for the western half of the building. But the off center north and south doors in the *Prostomiaion* do not violate any sensibilities, because the room was never intended to be symmetrical. The interior walls of the lower, western half of the building were only partial height partitions (Figure 5). The doors are actually in the corners of a rather large, asymmetrical space. The authors of the 1927 publication also felt that the location of the southwest corner of the main block over the edge of the Kekropion and the subsequent inability to place a foundation there was an accident, that the builders did not have exact knowledge of the location of the tomb of Kekrops even though they knew the precise location of the Erechtheis, which the same authors believed was located only inches, or dactyls, away. If the tomb of Kekrops and the Erechtheis were located, literally, back to back, then the corner of the building must have had to have been placed exactly where it is now in order to keep the tomb of Kekrops outside of the tomb of Erechtheus and the Erechtheis within. The great lintel at the corner would have been planned from the beginning (Figure 11).

The placement of the North Portico against the building has always been the

<sup>38</sup>Vitruvius VI.ii.4-5. Transcribed here as translated in Rykwert (supra n. 5, p. 227).

<sup>39</sup>Vitruvius III.i.8.



single most puzzling aspect of the Erechtheion. The North Portico contains the mark of Zeus's thunderbolt, the entry into the *Prostomiaion*, and an entry into the Pandroseion. Yet the geometry of the plan does not correspond to the functional division into thirds. A regular organization according to the three intercolumniations of the Portico would have been an obvious resolution of the plan. The  $\sqrt{2}$  proportion provides the first plausible rationale for the amount of the extension of the Portico to the west beyond the main block of the building.

### The Square Root of Two

The proportion of the diagonal of the square is normally associated with a procedure of rotating squares, called *ad quadratum* in the Middle Ages. The asymmetrical design of the Erechtheion has little to do with the kind of plans normally generated from the *ad quadratum* method. Particularly interesting is that a plan using this proportion has been implemented without the need for geometric procedures, for though  $\sqrt{2}$  is an irrational number, all of the dimensions have been reduced to the rational numbers found in the third column of Table 1. Would the choice of the  $\sqrt{2}$  proportion as an organizing principle have been arbitrary because it was a commonplace procedure? Would it have been made on the basis of what worked best, following a trial and error process? Or could there have been another reason for its use?

Vitruvius mentions this proportion three times in his treatise. The first is in his rules for a Corinthian capital, where the diagonal of the abacus is set as equal to twice the height of the capital, i.e., the side of the abacus is the  $\sqrt{2}$  times the capital height.<sup>40</sup> The second occurrence is as an allowable proportion for the atrium of a private home.<sup>41</sup> The last is in a discussion of the contributions of famous men.<sup>42</sup> There Plato's theorem for the doubling of the area of a square is introduced as indispensable for the measuring of the earth's surface. Plato had named the cube of the square as the regular solid identified with the earth (and in this agreed with the Pythagoreans).<sup>43</sup> It would seem that for Plato at least, writing in the years following completion of the Erechtheion, the square and its powers were uniquely cthonic in nature. The character of this proportion, the only irrational proportion mentioned by Vitruvius, may have been uniquely suited to the character of the cults of the Erechtheion: a pre-rational world of the sea and the earth, of the goddess, and of beasts and fertility.

After all, a rational number is one that can be expressed as a ratio of integers. An irrational number is one that cannot. The  $\sqrt{2}$  is the length of a diagonal across a square with sides of one unit of length. This follows from the Pythagorean theorem and is sometimes called the Pythagorean constant. It was probably the first number known to be irrational. Pythagoras (c. 570-495 BCE) preached that all numbers could be expressed as the ratio of integers, and the discovery of irrational

<sup>40</sup>Vitruvius IV.i.11.

<sup>41</sup>Vitruvius VI.iii.3.

<sup>42</sup>Vitruvius IX.pref.4-5.

<sup>43</sup>Plato discusses the five regular solids in the *Timaeus* 53c-55c.

numbers is said to have shocked his followers. A certain Hippasus of Metapontum (c. 530- 450 BCE) is sometimes credited with the discovery of irrational numbers by developing a proof that the  $\sqrt{2}$  was one, notwithstanding that the Babylonians of a thousand years earlier already had approximations for its value. Hippasus may have been drowned at sea by his fellow Pythagoreans for divulging knowledge of irrational numbers, behavior which they believed the gods considered impious.<sup>44</sup>

But ultimately there is a practical need to estimate irrational numbers with rational approximations. It was not until the first century of the Christian era that mathematicians Theon of Smyrna and Heron of Alexandria independently published algorithms for creating arithmetical estimates, or rational convergents, to the irrational number of the  $\sqrt{2}$ .<sup>45</sup> Yet building practice requires such estimates, for only rational numbers can be transferred by the use of a dividers from a building module to the building site. There can be no irrational numbers on a measuring rod in premodern times any more than there are on a contemporary tape or laser measure. Measuring systems are perforce modular ones. Building construction requires arithmetic procedures. The inexpressible must be expressed. And so the approximation of the irrational  $\sqrt{2}$  by rational ratios of integers in the plan of the Erechtheion appears to be the demonstration of practical mathematics developed by craftsmen in response to needs encountered in the practice of their craft, centuries before the professional mathematicians provided elegant methods to accomplish the same.

## Elevations

If the plan responds to a gridded treatment, the elevations do not. The vertical dimensions of the building do not fall as readily into whole number values. This is in keeping with the rules for the Ionic order given by Vitruvius following the second century B.C.E. architect/theorist Hermogenes, who outlined a successive proportional design procedure where each element in an Ionic building is proportioned on the basis of the one below it.<sup>46</sup> Yet following the intuition that the Erechtheion is a three dimensional composition of similar rectangles, it will be demonstrated that the square and  $\sqrt{2}$  proportions of the plan organize the elevations as well.

If the East Portico is the point of beginning for the design of the plan, the explorations of the elevations should begin there as well (see Figure 10 in Appendix). If sloping roof surfaces and steps are eliminated from consideration and attention is concentrated on the visible rectangles with which the temple is composed and which attracted attention from the start, the critical height is the dimension from the bottom of the base to the top of the molding of the corona at the bottom of the cyma, or gutter. This accords with the Portico of the Maidens,

<sup>44</sup>Wikipedia, "The Square root of 2," [https://en.wikipedia.org/wiki/Square\\_root\\_of\\_2](https://en.wikipedia.org/wiki/Square_root_of_2), accessed June 2, 2023 and Wikipedia, "Hippasus," <https://en.wikipedia.org/wiki/Hippasus>, accessed June 2, 2023.

<sup>45</sup>See Note. 40 above. Also see "Inexpressible Proportion" in March (1998, pp. 65-69) and Filep (1999, pp. 1-7).

<sup>46</sup>Vitruvius III.v.1-15.

where the roof is flat, and there is no cyma on top of the entablature at all. Taking this as a rule, the east sides of the three visible porticos do indeed all conform closely to the proportion of a  $\sqrt{2}$  rectangle (E1, E2, E3). Moreover, the tops of the generative squares of the bounding rectangles of the two side porticos align with each other and with the central rectangle in the intriguing manner shown.

The measurements are most precise if the height is calculated to the bottom of the moldings that crown the corona as opposed to the top. Perhaps this shadow line is just the point that was considered most expressive. Another possibility is that this represents a visual refinement in the design of the Erechtheion. Vitruvius echoed numerous ancient writers in his concern for the need to correct for optical distortions due to height. “For the higher that the eye has to climb... there must always be a corresponding increase in the symmetrical proportions of the members...” he states in his account of the Ionic order that he adapted from Hermogenes.<sup>47</sup> Of the three  $\sqrt{2}$  rectangles that underlie the east elevation of the Erechtheion, the central one (E1) lies on its long side while the others (E2 and E3) lie on the short side. As an initial design, the height of the coronas of the three porticos would have been set precisely by the three rectangles. In the end, the vertical dimension was lengthened in each case by placing the top moldings above rather than below the line of the bounding rectangle, regardless of the orientation of the rectangle.

The analysis continues moving clockwise about the building. The main block is a double square in plan, and its south elevation resolves into two  $\sqrt{2}$  rectangles in elevation (E4). The Portico of the Maidens appears to have no proportion conforming to the  $\sqrt{2}$ , but the portico above the stylobate does fit neatly within a 8:9 rectangle, continuing the theme of the square (E5).<sup>48</sup> The flat expanse of wall of the main block not covered by the Porch of the Maidens, measured from the bottom of the base to the bottom of the wall capital, or epikranitis, does conform to a rectangle composed of a  $\sqrt{2}$  rectangle plus a square (E6).<sup>49</sup>

Moving to the west elevation, obviously there is a relationship to the east elevation, particularly considering the wall which contained the garden and concealed the lower portion of the façade (see Figure 11 in Appendix). The west facade of the main block resembles a portico in antis, and columns, antae, and entablature also fit within a  $\sqrt{2}$  rectangle (E7). On either side the North Portico and Portico of the Maidens present the same elevation as to the east, with the same  $\sqrt{2}$  proportion present (E2 and E3). The alignment of the side bounding rectangles with each other and the central one is similar to that of the east elevation but with a square as well as a  $\sqrt{2}$  rectangle as the interlocking agents.

The north elevation repeats the relationships of the south elevation with variations to account for the change in height. The north elevation of the main

<sup>47</sup>Vitruvius III.v.9.

<sup>48</sup>This 8:9 proportion, here and on the North Portico, are the only examples in this analysis of the Erechtheion that correspond to the Pythagorean musical proportional system. 8:9 is the tone.

<sup>49</sup>Euclid defined four variations of a ratio: inverse, composition, separation, and conversion (*Elements* V, definitions 13-16). If the ratio is  $p:q$ , then the composition is  $p+q:q$ , which, in geometric terms, is the base rectangle plus a square. In this case,  $\sqrt{2}:1 = p:q$ , so  $p+q:q = \sqrt{2}+1:1 = 2.4142:1$ . Euclid's terms are explained in March (1998, pp. 10–11).

block repeats the proportion of its plan, the double square (E8). The North Portico, echoing that of the Maidens on the other side, fits within the same 8:9 rectangle (E9). The blank wall surface of the main block not obscured by the North Portico conforms neatly to a  $\sqrt{2}$  rectangle (E10).

## Columns

There remains to be considered the most glorious aspect of the Erechtheion of all, the Ionic columns and their human counterparts (see Figure 12 in Appendix). Even if Vitruvius had not explained that the Ionic column was the *mimesis* of the female human figure, the Maidens of the Erechtheion would have given the secret away. Vitruvius describes a simple metaphor: volutes as ringlets of hair, flutes on columns as folds in robes, and bases as shoes.<sup>50</sup> Yet just as the form of the Erechtheion as a whole responds to deep seated religious beliefs in a world of half men/half serpents, heroes identified with sacrificial oxen, and secret rites invoking the impregnation of the sea to ensure the fertility of the earth, so these same archaic symbols are intertwined with the form of the Ionic order in a manner at once less obvious and more profound than Vitruvius so glibly suggests. Predecessors to the Ionic capital in the ancient eastern Mediterranean world include many examples that link archaic religious symbols with proto-Ionic forms more literally than the Ionic capital itself. The Aeolic capitals of ancient Lycia seem to more obviously represent simultaneously spiraling vegetation and the horns of a ram. The Hathor capitals of ancient Egypt already combined the head of a goddess with bovine ears with the curls of a human wig. Steeped in the religious heritage of prehistoric fertility rites, as were these ancient capitals, the forms of the Ionic order emerged on the Acropolis into the clear hard light of Classical Greece.<sup>51</sup>

If the point of beginning for the building is the East Portico, then these are the columns that should be considered first. An immediate expectation might be that the module of 2 Doric feet that controls the plan would also be the lower diameter of these columns, but it is not. The lower column diameter of the East Portico columns is 0.692 m., or close to 2.125 Doric feet. On the other hand, the height of these columns, 6.586 m., seems to be exactly 20 Doric feet, a multiple of the plan module. The intercolumniation at the base is 1.422 m., which is 2.055 times the lower column diameter. This is close to Vitruvius's, or rather Hermogenes's, rule for a systyle intercolumniation: two column diameters. Vitruvius gives the complete rules only for the most perfect spacing, the eustyle,<sup>52</sup> but these rules can be extrapolated for the systyle. The comparable method for a six column systyle portico, which requires 6 column diameters for the columns and 10 for the intercolumniations, would be to divide the overall portico width of 34 feet into sixteen parts (again the perfect number) and let one part be the lower column

<sup>50</sup>Vitruvius IV.i.7.

<sup>51</sup>See Rykwert (supra n. 7, pp. 236–315) for a discussion of the background of symbols implicit in the Ionic order. Other interpretations are to be found in Hersey (1988).

<sup>52</sup>Vitruvius III.iii.1, 6–7.

diameter. 34 feet divided by 16 equals 2.125 feet, which is what has already been shown to be the column diameter. In meters, the actual width of the building, 11.187 m., divided by 16 equals 0.699 m., only 1.0% over the actual measured value of the column diameters, 0.692 m. However, the intercolumniations should then be 4.25 feet or 1.398 m., which is still short of the actual value. The incline of the corner columns adds 0.04 m. (2 dactyls) to the overall width at the level of the column bases (which are equally spaced), for the controlling measurements of the design have been established to occur at the bottom of the architrave. That is, the bases are moved out from the controlling dimension, not the tops in. Adding 0.04 m. to the width of 11.187 m. gives 11.227 m. at the bottom of the column shafts. Subtracting the total actual column width (0.692 m. multiplied by 6) of 4.152 m., gives 7.075 m., divided by 5 intercolumniations equals 1.415 m. This is only 0.5% less than the measured value of 1.422 m.

For the relationship of height to column diameter, Vitruvius/Hermogenes has these directions: “in the systyle, let the height be divided into nine and a half parts, and one of these given to the thickness of the column.”<sup>53</sup> In the case of the East Portico, 20 feet divided by 9.5 equals 2.105 feet, approximately equal to our previous result of 2.125 feet. In meters, 6.586 m. divided by 9.5 equals 0.693 m., only 0.001 m. or 0.01% greater than the actual measured value. Finally, the diminution of the top of the column is to vary with the height. The top of a twenty to thirty foot tall column<sup>54</sup> is to be 6/7ths of the bottom, and that is exactly the case in the East Portico (see Figure 13 in Appendix). Two centuries after the construction of the Erechtheion, Vitruvius’s source, Hermogenes, wrote down rules for the Ionic order that codified precisely what had been done in the East Portico of the Erechtheion, and Vitruvius transcribed them yet another two centuries later.

Given the complexity of the design of the Erechtheion, the other three porticos are constrained by their particular circumstances within the overall controlling proportional system and do not conform so exactly to the proportions given by Vitruvius for the Ionic order (Figure 12). The height of the North Portico is presumably fixed by the desire to bring the ridge line of the roof under the cornice of the main block. Four columns across the northern face of this portico are the only viable solution to the number of columns on the given plan. The resulting intercolumniation of approximately 2.75 diameters does not conform to one of the Vitruvian categories. But the designers of the Erechtheion agreed with Hermogenes’s principle that the farther columns are apart, the thicker they need to be in relation to their height, and so the ratio of column diameter to column height for the North Portico is decreased from 9.5 to 9.25 diameters. The column diameter is approximately 2.5 feet. On the west elevation the colonnade fits into the 30 foot internal dimension of the main block, and the six columns which mirror the arrangement of the East Portico simply divide the width into five equal spaces of 6 feet each. These columns could not meet the standard formula because they share the same entablature as the East Portico while being shorter to fit within the  $\sqrt{2}$  bounding rectangle. Since the architrave rules in this case, the column diameter was simply made equal to the architrave height (0.62 m. and 0.63 m.

<sup>53</sup>Vitruvius III.iii.10.

<sup>54</sup>Vitruvius III.iii.12.

respectively), or approximately 1.875 feet. The Portico of the Maidens duplicates the number and spacing of the columns of the North Portico as it does the overall proportional system. The Maidens themselves are slightly larger than life size, measuring 7 feet tall from the soles of their sandals to the underside of the architrave. Thus the podium wall is required underneath them to fill out the predetermined proportions of the portico. In this case there is no column diameter to relate to the proportional system.

Hermogenes's rules relate the height and details of the entablature to the height of the column through the height of the architrave, creating a reciprocal relationship between column diameter and architrave height.<sup>55</sup> For a twenty to twenty five foot column the column should be divided into twelve and a half parts and one part taken for the height of the architrave. In the East Portico of the Erechtheion, however, the architrave is relatively taller, for the twenty foot tall columns are divided into ten and a half parts and one part is taken as the height of the architrave. In the North Portico, as Hermogenes directs, the absolute height of the architrave is greater with taller columns, but the relative height decreases instead of increasing. As mentioned above, on the west elevation the column diameter and the architrave height are equal, and the column height is an exact multiple of this value. And in the Porch of the Maidens, where there are no columns, the height of the Maidens plus their podium equals nine and a half times the architrave height, the ratio of height to column diameter of the East Portico.

## Details

The exquisite carving of the details of the Ionic orders of the Erechtheion depart further from the rules later set down by Hermogenes and copied by Vitruvius (Figure 13). The column bases lack a plinth, though this was not uncommon in early Ionic temples. As regards the entablatures, just as in the Parthenon Ionic details are integrated into a predominately Doric structure, so the Doric has influenced the details of the Erechtheion. The cornices of the main block and the North Portico lack the normal Ionic dentils. Instead, the corona juts forward directly from the top of the frieze just as in the Doric order (though this is not unique to the Erechtheion).

The entablature of the Porch of the Maidens differs from that of the rest of the building (see Figure 14 in Appendix). Dentils are present, but the frieze is not. This is the original form of the Ionic entablature - not mentioned by Vitruvius - as found in archaic temples in Ionic lands. This archaism lends support to the theory that the Caria which these "Caryatids" reference is the Ionic province of Caria and not the Peloponnesian town mentioned by Vitruvius. If the ritual of the *arrephorai* was dedicated to the Carian Aphrodite, then conservative religious belief could not allow the sacred forms of the Portico of the Maidens to be altered as in the rest of the building dedicated to the protectress of Athens and her chosen heroes. On the other hand, the capitals which the Maidens carry bear a surprising likeness to the

---

<sup>55</sup>Vitruvius III.v.8.

later Roman version of the Doric capital (Dinsmoor supra n. 25, p. 193).

In Vitruvius the dimensions of the parts of the Ionic entablature are related to the height of the architrave and the dimensions of the parts of the column are related to the lower column diameter. In the Erechtheion the vertical dimensions of the parts of all of the order seem to relate to the architrave height, leaving the plan dimensions to relate to the column diameter. In the East Portico the ratio of these two key dimensions (0.63 m. for the architrave height and 0.692 m. for the lower column diameter) is within 0.05% of the proportion of 19/21 suggested by the ratios which each makes separately with the column height.

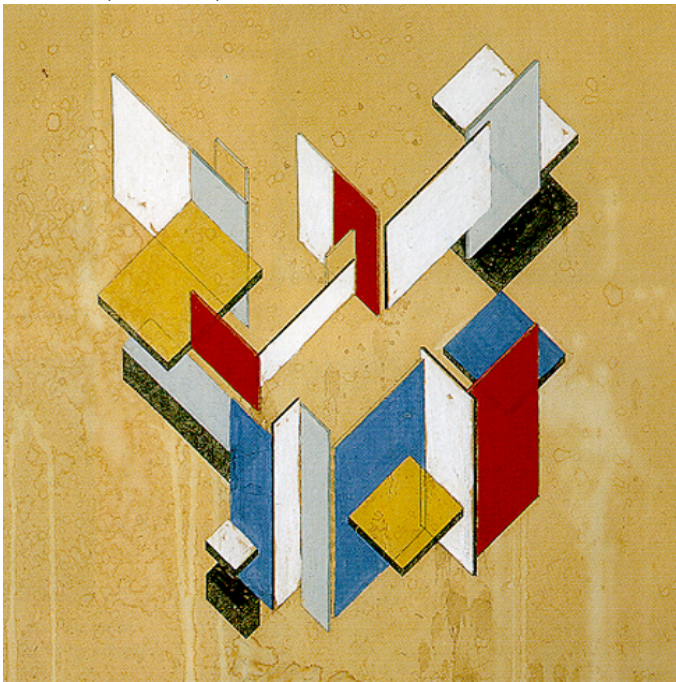
The eyes of the volutes of the capitals of the East Portico appear to be equal to one seventh of the height of the volutes instead of the one eighth given in Vitruvius. G. P. Stevens painstakingly reconstructed the centers of the arcs described by the volutes of these capitals (Stevens et al. supra n. 7), p. 21). His reconstruction suggests a different method for the construction of an Ionic volute than that described by Vitruvius. Here at the scale of one of the smallest details of the Erechtheion the proportional basis of the whole reasserts itself, binding the entire structure into one coherent system. The centers of the arcs were ascertained by Stevens by trial and error, but they are revealed here to occur at the corners of a rectangle and two squares whose proportions are controlled by the diagonal of the square:  $\sqrt{2}:1$  (see Figure 15 in Appendix).

## Conclusion

Asymmetrical buildings closely configured to elaborate programmatic requirements are the norm in contemporary architecture, yet even to 21<sup>st</sup> century eyes the Erechtheion has seemed idiosyncratic and peculiar, and the intentions of its designers obscure. However, the exquisite beauty of its carving and the awesome reputation of fifth century Athens have always promised more. Nothing presented in this paper proves that at some point a more symmetrical building had not been envisioned by the designers. But if the arguments advanced here have any validity, the motivations for the unusual configuration of the Erechtheion as it has come down to us can no longer be considered obscure. Vincent Scully argued that the archaic and classical period Greek temple was intended as an embodiment of a sacred spirit understood by its builders to have been manifest in a given landscape. If so, the design of the Erechtheion takes this principle to its logical conclusion. The extraordinary sensitivity to the landscape which produced the asymmetrical organization of the typical temple site such as the Acropolis, in the case of the Erechtheion informed the design of the building itself. The proportional apparatus of ancient Greek architecture, developed for traditional, bilaterally symmetrical temple structures, proved capable of binding a set of specific responses to an elaborate set of programmatic requirements and site conditions into a conceptually rigorous whole. In service to the cults which sanctified the ground on which it rose, the Erechtheion is a fetish object of unusual sophistication and brilliance. This is the implication of the quote from Scully with which this paper began.

Though the religious cults which gave rise to the Erechtheion long ago lost their hold on western populations, the cult of number and geometry has not lost all of its power. The Erechtheion taps into a continuous thread of the western tradition that still inspires: behind the physical reality of sensuous surface lies a Platonic conception of virtual order. Evidence suggests that this ideal has appealed to architects of every century, including the last one. Without claiming that the Erechtheion in any way anticipated the architecture of the twentieth century, a look at more contemporary examples may promote the appreciation of principles shared by both. The creators of modern architecture in Europe at the beginning of the last century appealed to these time tested ideals. Members of the movement called De Stijl explicitly invoked a mathematical/Platonic conception of the world and believed that art and harmony should extend beyond the frame of the artwork into the total human environment. Two works by its chief theorist, Theo van Doesburg, serve to demonstrate the continued interest in compositions of mathematically controlled planes arranged in three dimensional space. His *Study for an Arithmetic Composition* of 1929/30 explicitly invokes mathematical ordering processes and specifically  $\sqrt{2}$  relationships. The painting *Contra Construction* of 1923 (Figure 7) describes the organization of an environment by a virtual three dimensional assemblage of rectangular planes, a now commonplace paradigm of modern architecture, but one that seems curiously close to the conceptual framework of the Erechtheion, designed by an unknown Greek architect over two thousand years ago.

**Figure 7.** Theo van Doesburg, "*Contra Construction*," 1923, Kröller-Möller Museum, Otterlo, Netherlands





## References

- Coulton JJ (1977) *Ancient Greek architects at work*. Ithaca: Cornell University Press.
- Dinsmoor WB (1973) *The architecture of Ancient Greece: an account of its historical development*. New York: Biblo and Tannen.
- Dörpfeld W (1890) Metrologische Beiträge. (Metrological contributions). *Mitteilungen des Kaiserlich Deutschen Archaeologischen Instituts Athenische Abteilung* XV(1890): 167–187.
- Dörpfeld W (1968) *Erechtheion*, drawings and revisions by Hans Schief. Osnabrück: Otto Zeller.
- Elderkin GW (1941) The Cults of the Erechtheion. *Hesperia* X(2): 115–124.
- Filep L (1999) Pythagorean side and diagonal numbers. *Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis* 15(1999): 1–7.
- Haselberger L (1985) The construction plans for the Temple of Apollo at Didyma. *Scientific American* ccliii(6): 126–132.
- Haselberger L (1996) Greece, ancient, §II.3.iii: architecture: theory and design: measurement. In J Turner (ed.), *The Grove Dictionary of Art*, volume 13. New York: Grove Dictionaries, 410.
- Hersey G (1988) *The lost meaning of classical architecture*. Cambridge, MA: M.I.T. Press.
- Lawlor R, Lawlor D (1979) *Theon of Smyrna: mathematics useful for understanding Plato*. San Diego: Wizard's Bookshelf.
- Lesk A (2004) *A diachronic examination of the Erechtheion and its reception*. Doctoral Dissertation.
- March L (1998) *The architectonics of humanism*. Chichester: Academy Editions.
- Pakkanen J (2006-2007) The Erechtheion and the length of the 'Doric-Pheidonic' foot. *Talanta* XXXVII-XXXIX(2006-2007): 97–122.
- Pakkanen J (2013) *Classical Greek architectural design: a quantitative approach*. Helsinki: Foundation of the Finnish Institute at Athens.
- Rykwert J (1996) *The Dancing Column*. Cambridge, MA, and London: M. I. T. Press.
- Scully V (1979) *The earth, the temple, and the Gods*. New Haven and London: Yale University Press.
- Stevens GP, Paton JM, et al. (1927) *The Erechtheum*. Cambridge, MA: Harvard University Press.
- Thiersch A (1889) Die Proportionen in der Architektur. (The proportions in architecture). In *Handbuch der Architektur*. Berlin.
- Wikipedia. *The square root of 2*. Retrieved from: [https://en.wikipedia.org/wiki/Square\\_root\\_of\\_2](https://en.wikipedia.org/wiki/Square_root_of_2). [Accessed 2 June 2023]
- Wikipedia. *Hippasus*. Retrieved from: <https://en.wikipedia.org/wiki/Hippasus>. [Accessed 2 June 2023]
- Wilson Jones M (2001) Doric measure and architectural design 2: a modular reading of the classical temple. *American Journal of Archaeology* 105(4): 675–713.
- Wilson Jones M (2014) *Origins of classical architecture: temples, orders and gifts to the gods in Ancient Greece*. New Haven and London: Yale University Press.

## Classical Texts

- Apollodorus. *The library*.  
 Euclid. *Elements*.  
 Herodotus. *Histories*.

Homer. *Iliad*.

Pausanias. *Description of Greece*.

Plato. *Timaeus*.

Theon of Smyrna. *The mathematics useful for understanding Plato*.

Vitruvius. *Ten books on architecture*.

Appendix

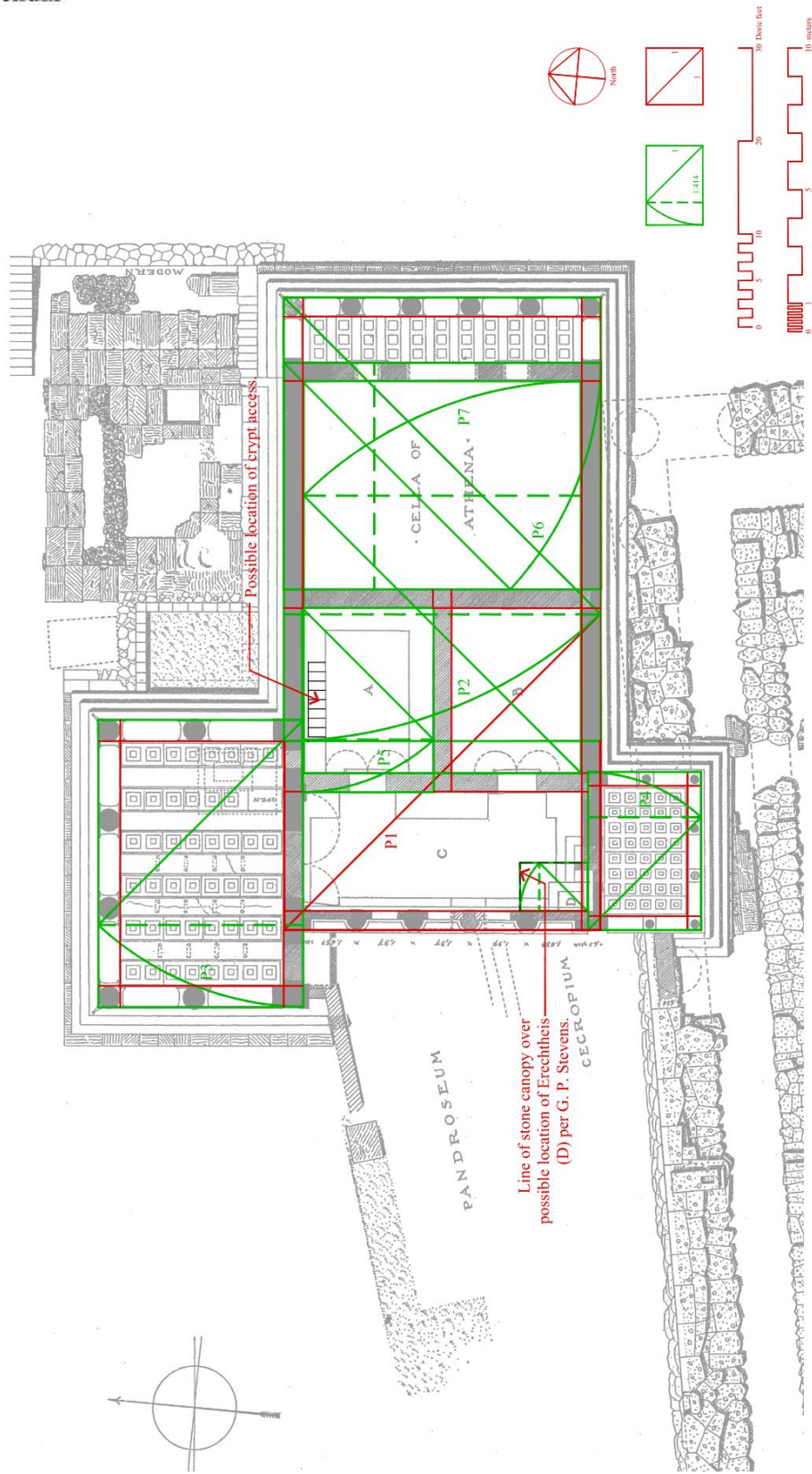


Figure 8  
PROPORTIONAL SCHEME: COMPOSITE FLOOR PLAN / CEILING PLAN  
ATHENS | ERECHTHEION

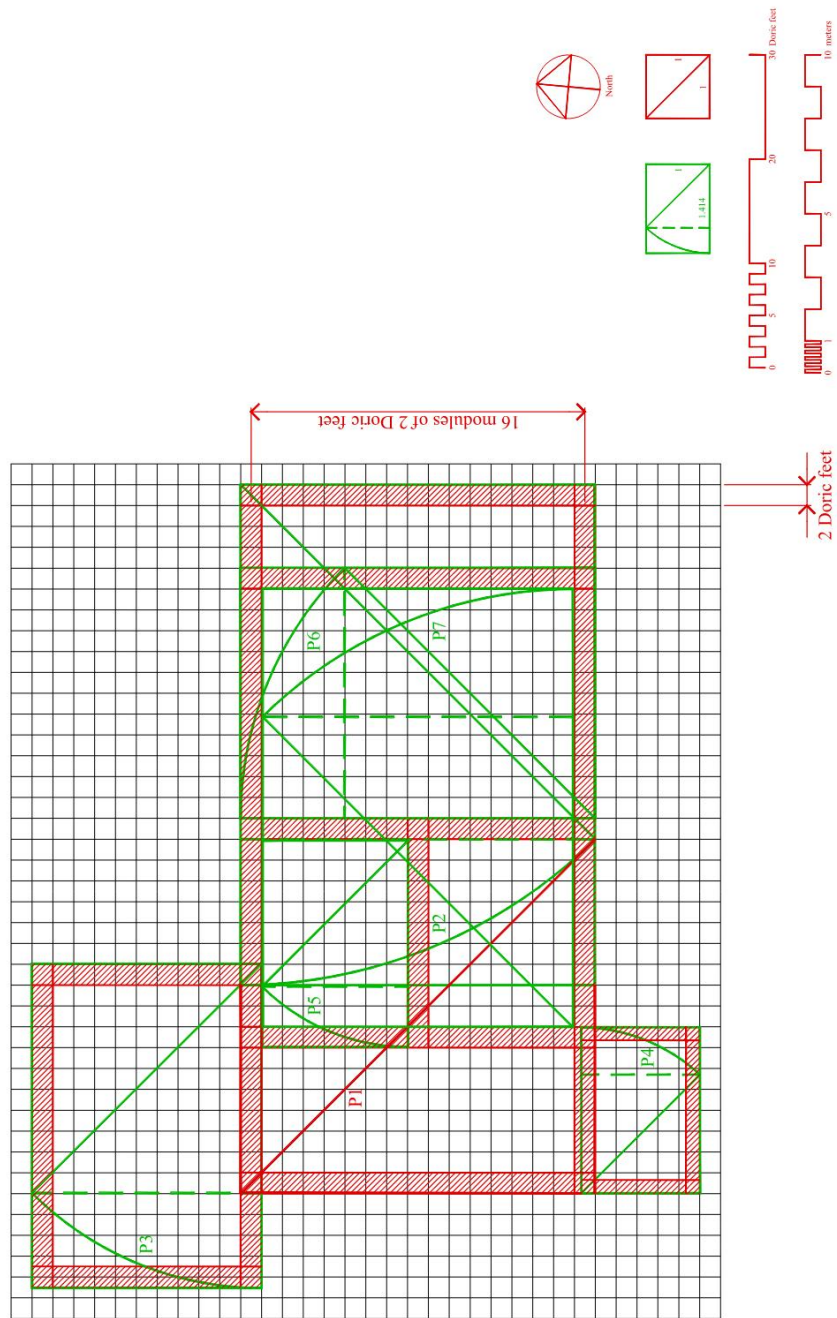


Figure 9

PROPORTIONAL SCHEME: IDEALIZED PLAN  
ATHENS | ERECHTHEION

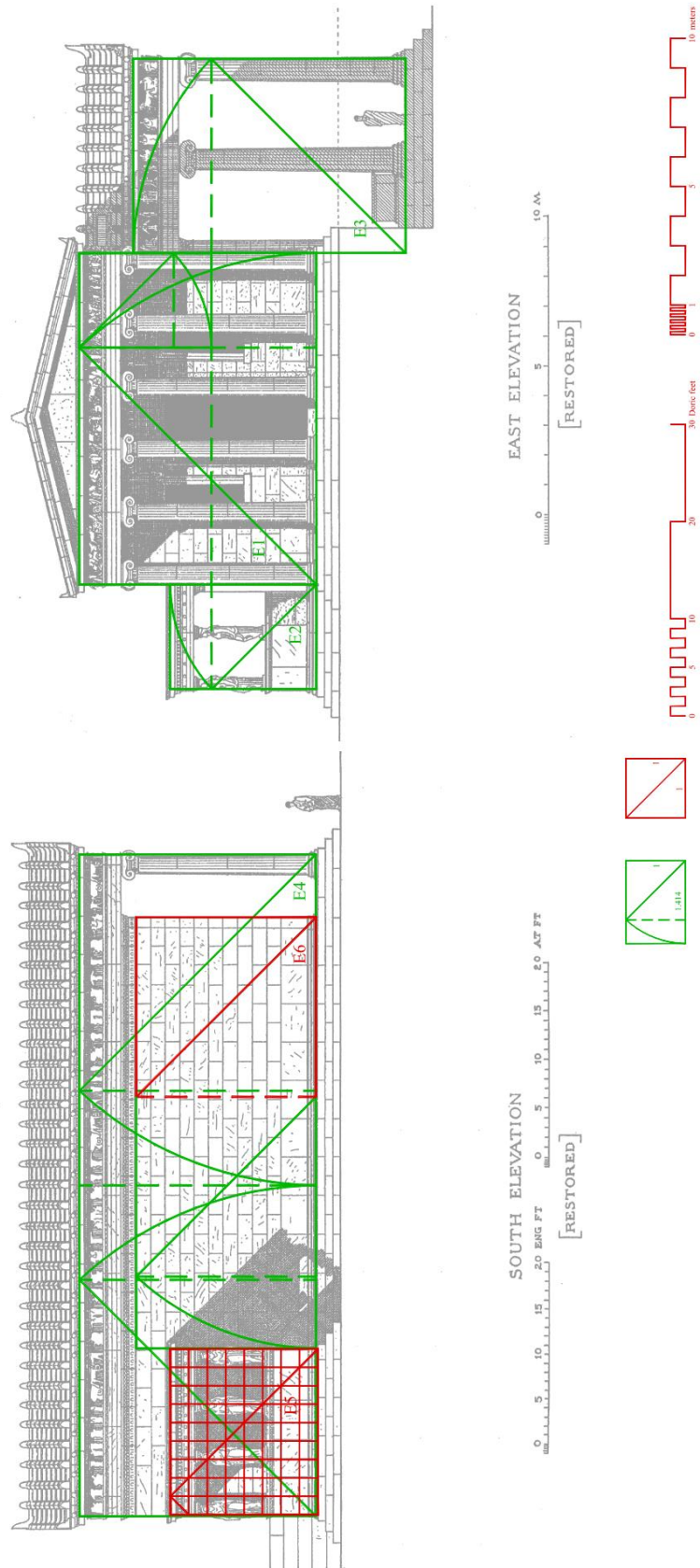


Figure 10  
PROPORTIONAL SCHEME: ELEVATIONS  
ATHENS | ERECHTHEION

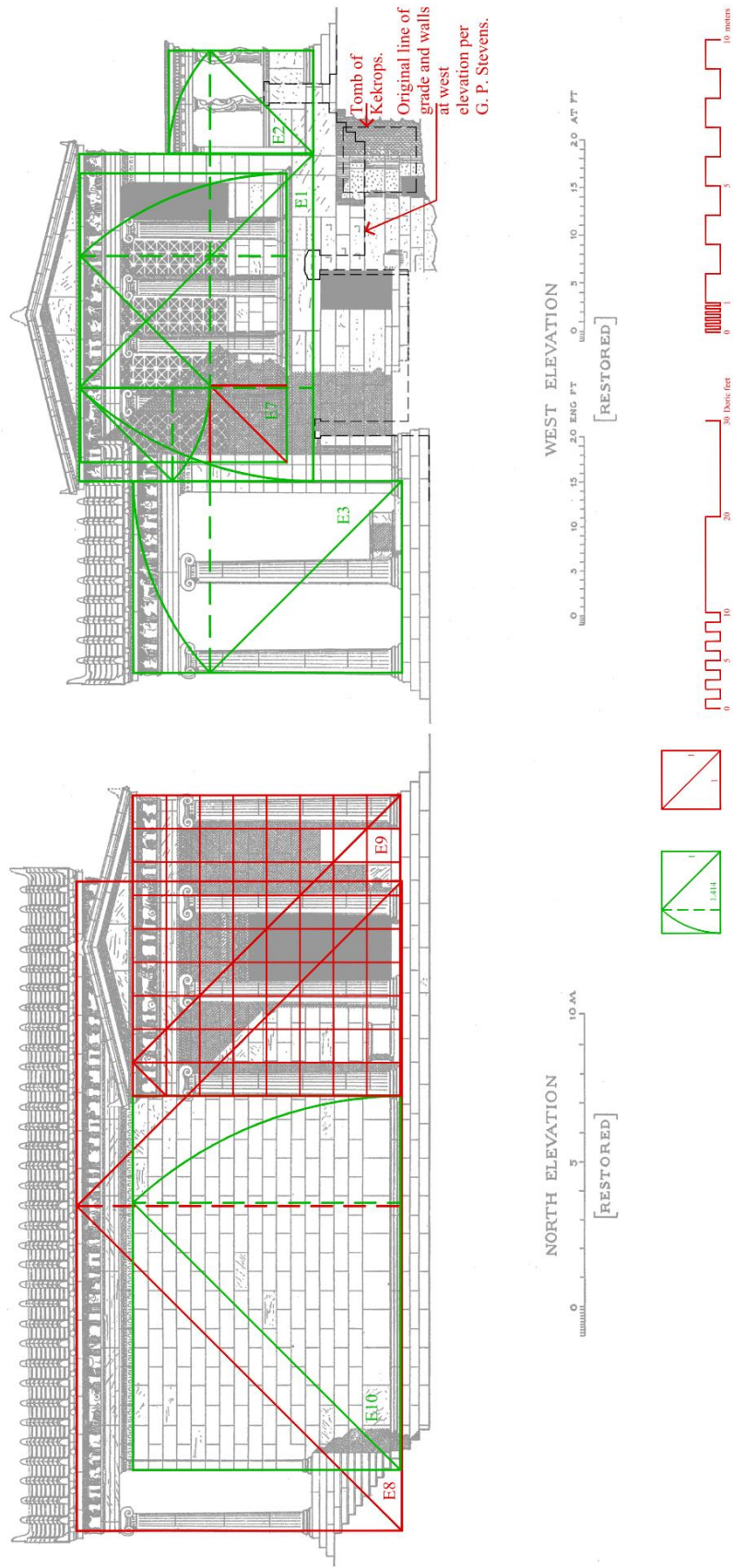


Figure 11  
PROPORTIONAL SCHEME: ELEVATIONS  
ATHENS | ERECHTHEION

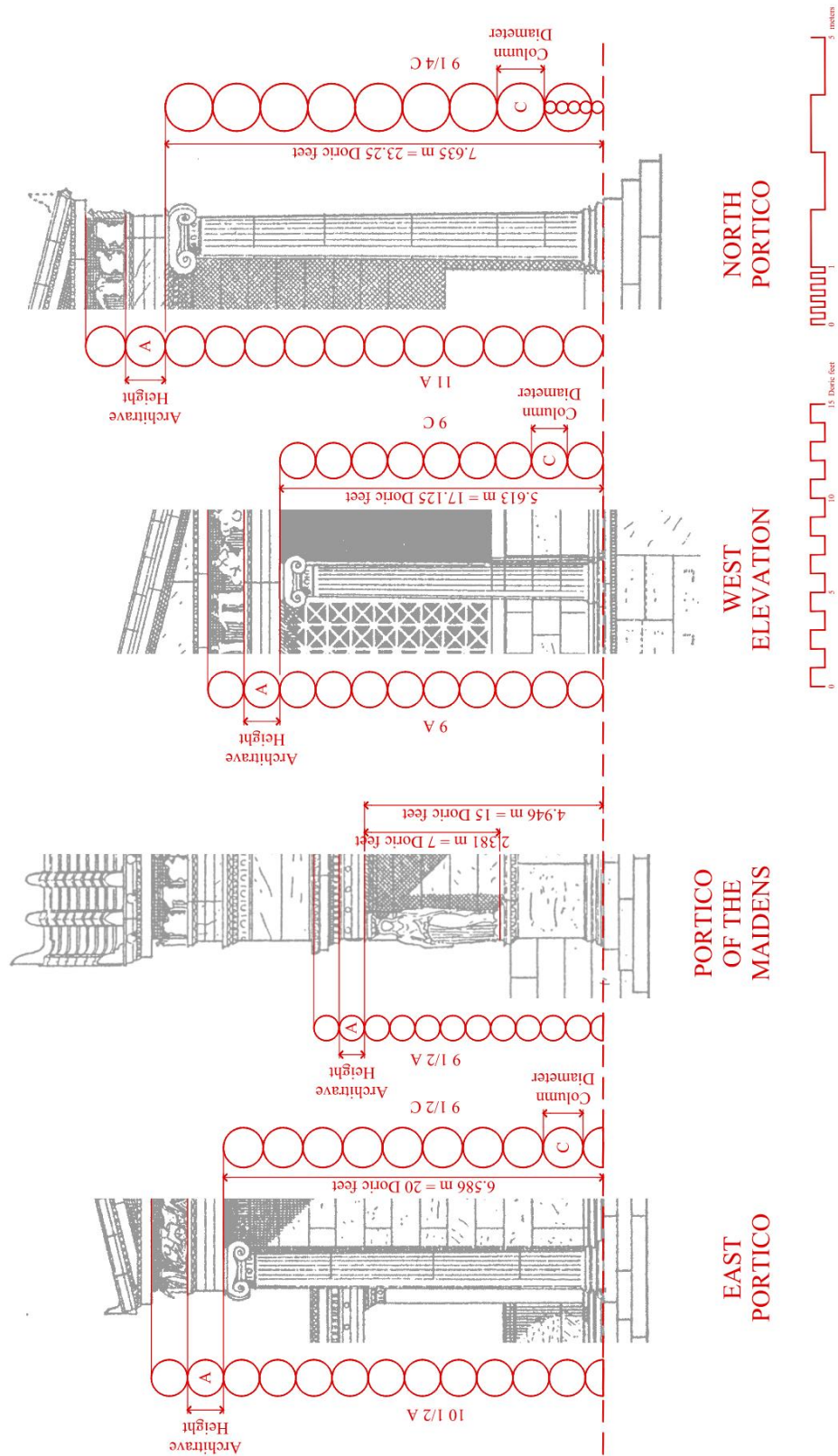
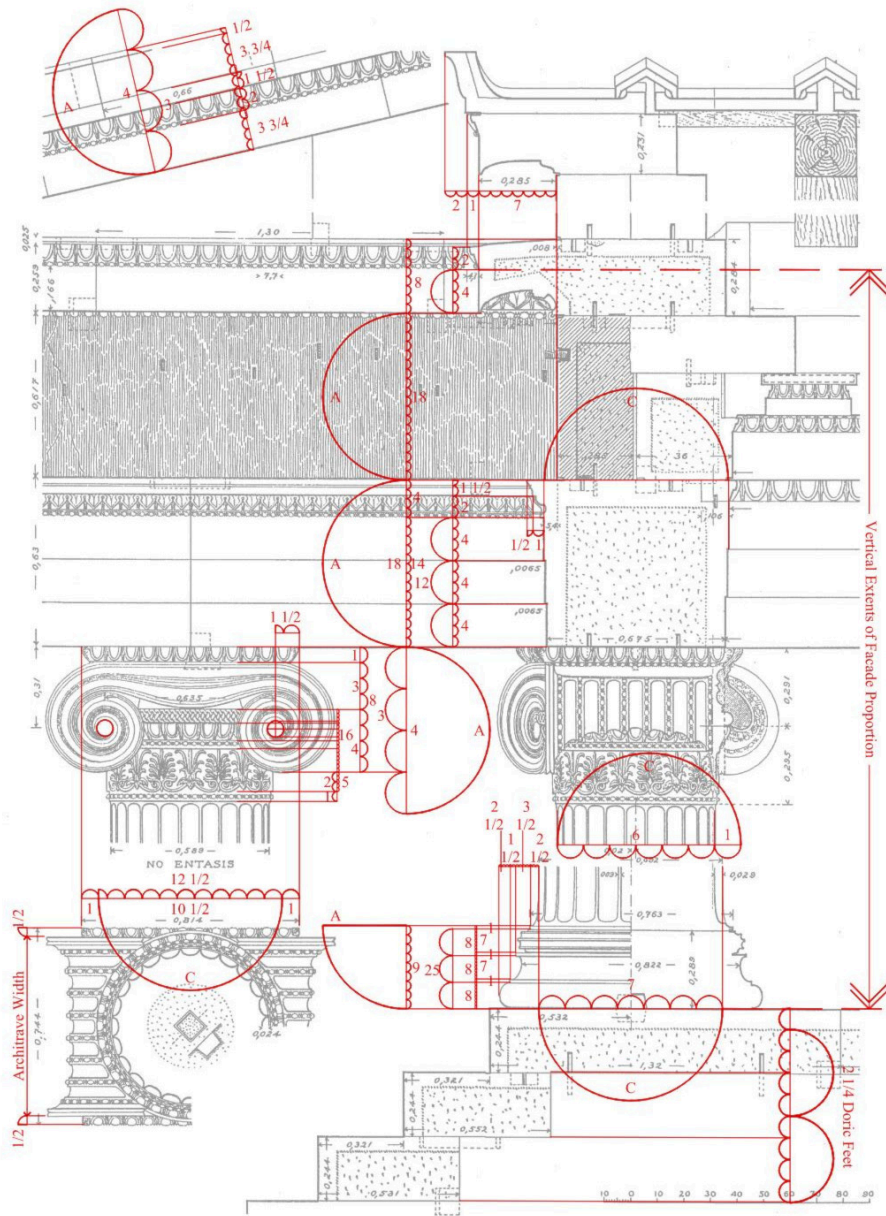


Figure 12

PROPORTIONAL SCHEME: COMPARATIVE ORDERS  
ATHENS | ERECHTHEION



DETAILS OF THE EAST PORTICO · I

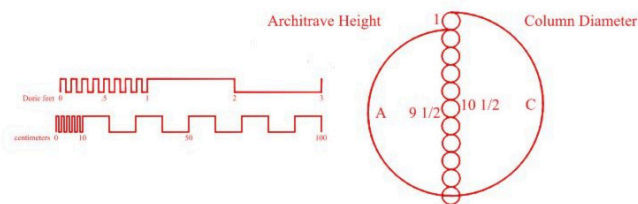


Figure 13

PROPORTIONAL SCHEME: EAST PORTICO ORDER  
ATHENS | ERECHTHEION



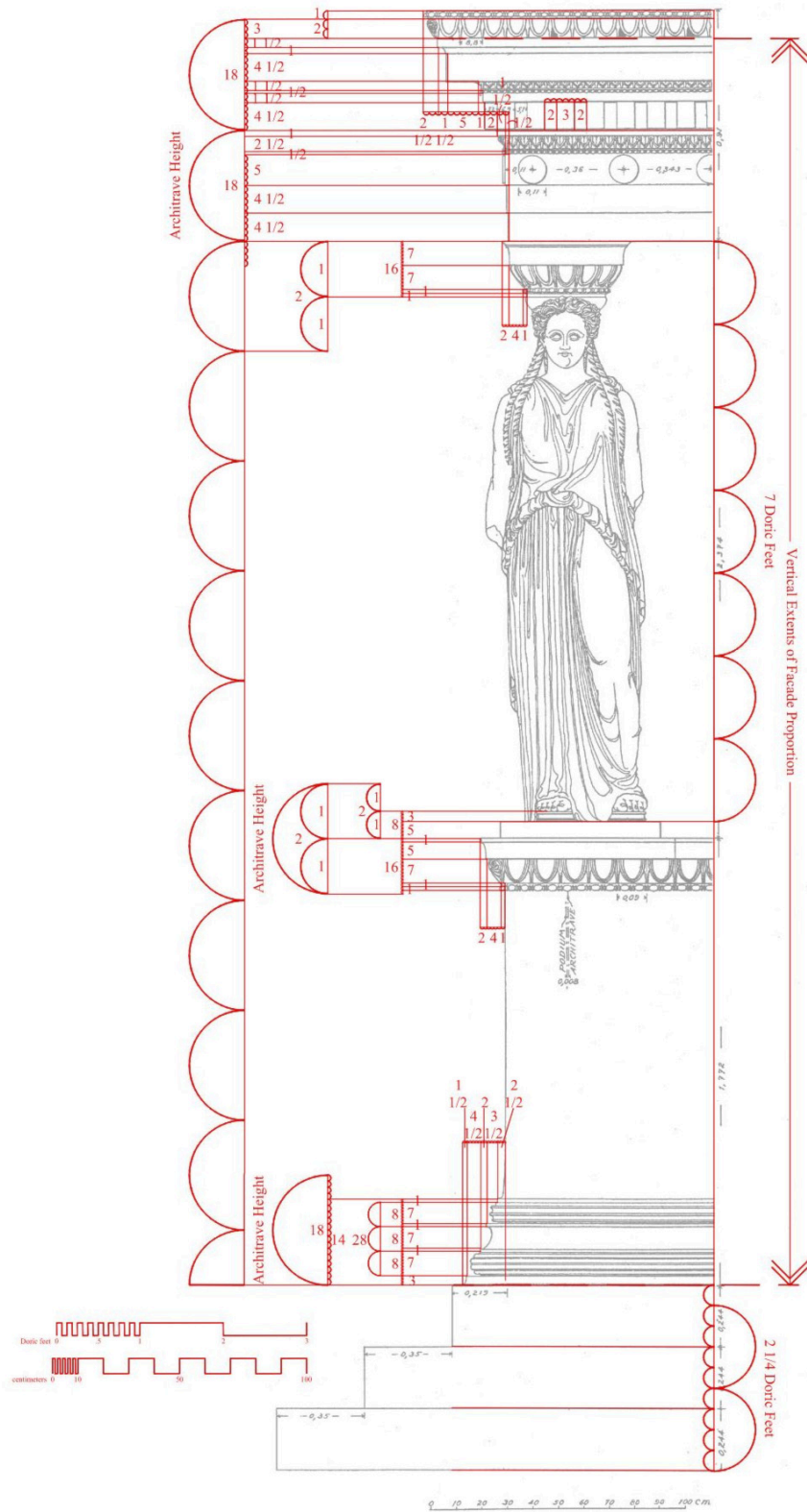


Figure 14  
**PROPORTIONAL SCHEME: CARYATID  
 ATHENS | ERECHTHEION**

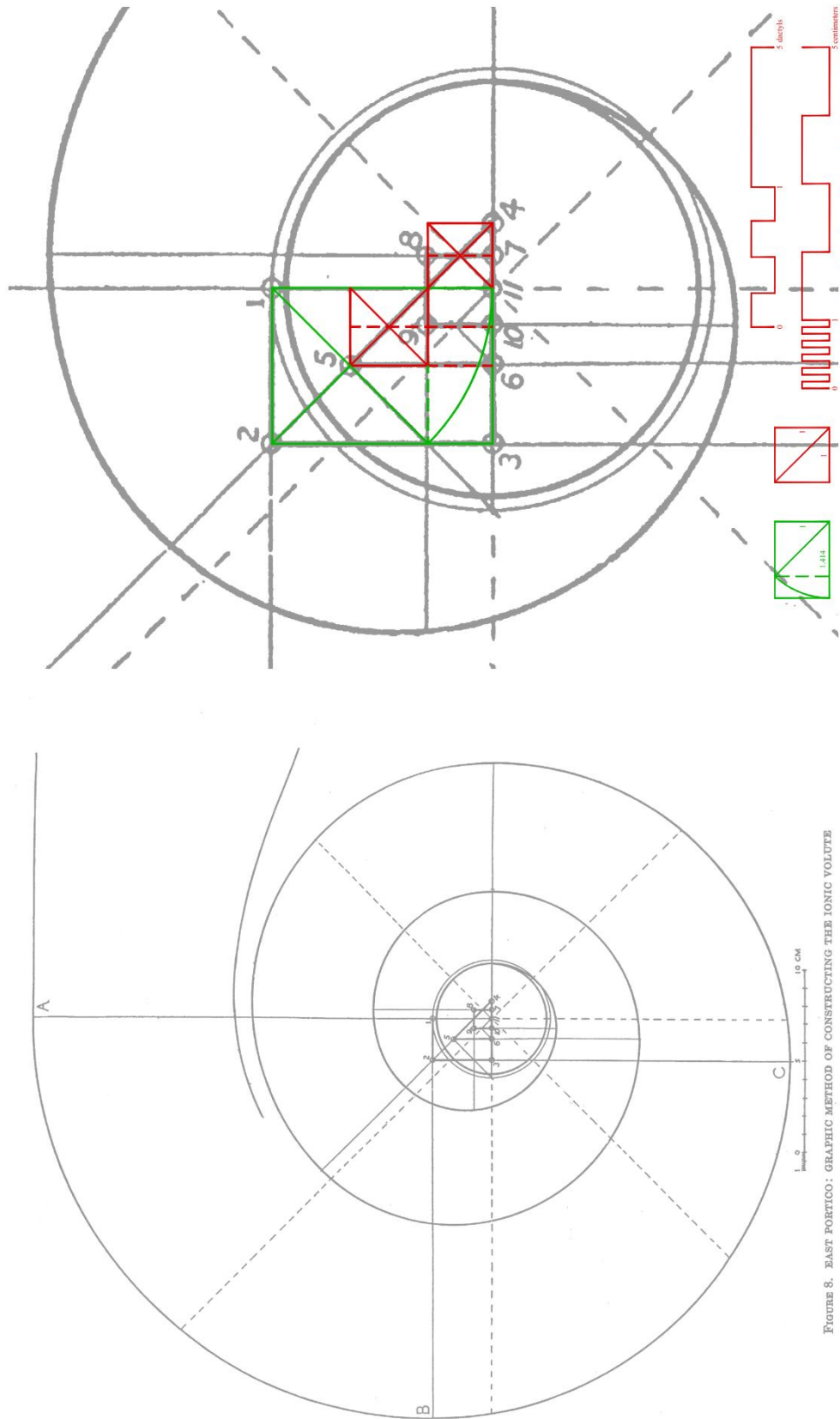


FIGURE 8. EAST PORTICO: GRAPHIC METHOD OF CONSTRUCTING THE IONIC VOLUTE

Construction of East Portico Volute According to G. P. Stevens.

Figure 15

PROPORTIONAL SCHEME: VOLUTE  
ATHENS | ERECHTHEION