



# *Athens Journal of Philosophy*

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# Athens Journal of Philosophy

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- **Dr. William O'Meara**, Head, Philosophy Unit, ATINER & Professor, Department of Philosophy and Religion, James Madison University, USA.

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## **Interpretative *Phronesis* (Practical Wisdom) Analysis: A Hermeneutic Narrative of Research Participant Caring**

*By Tony Wilson\**

*Aristotle's distinction between phronesis (practical wisdom) and episteme (theory) has been centrally influential in the development of hermeneutics. Heidegger, initiating hermeneutic phenomenology, foregrounded practical understanding as foundational (or 'ready-to-hand'): scientific theory was but secondary ('presented-at-hand'). Gadamer subsequently emphasised understanding as primarily practical, as an applicative achievement, within broad assumptions, 'horizons of understanding', a metaphor signalling explicitly/implicitly represented surroundings. How should Aristotle's idea of practical wisdom in human affairs articulated in phenomenology's hermeneutic thought - principally Gadamer's scholarship - inform researcher analyses? Here an account of hermeneutic philosophy, with its core conceptual formations, is presented as concerning situated understanding in practice, phronesis. Multiple instances of this behavioural research focus from psychology's Interpretative Phenomenological Analysis then receive reflection. Interviewing proceeds from 'horizons of expectation' (Jauss 1982). Themes are viewed as 'horizons of understanding' (Gadamer 1975), interviewees' perspectives on practices. A researcher may engage in resolving 'indeterminacy' (Iser 1978). Participants' reflectively recounted meaning-making phronesis practices can be structured in their analyses by locating their a priori, universally discernible aspects. Thus phronesis is constituted by generic, care (Heidegger's *Sorge*) embodying activity, 'emplaced' or understood from tacit representational affective 'horizons of understanding': participant bodies can become denoted 'equipment' (Heidegger's *Zeug*).*

**Keywords:** *caring, hermeneutic phronesis, interpretative horizons of understanding, phenomenology*

A person who 'is truly good and wise bears all the accidents of life with proper dignity and always makes the best of present circumstances' (Warrington 1963, p. 19).

### **Introduction: Research Participant *Phronesis* as Interpretative Practical Wisdom**

How a subject matter is conceptually conceived, explicitly or implicitly, shapes its further study. Disciplinary philosophies, the philosophy of natural or social science, reflexively consider such connection. This paper reflects on practice underwritten by philosophy in qualitative psychology, and particularly discusses *phronesis* as a perspective in hermeneutic psychology research.

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The 'philosophical stance one assumes in relation to the nature of reality and the nature of knowledge largely determines how the researcher conducts research, what he or she considers as legitimate research evidence' (Seamon and Gill 2016, p. 117).

Svenaesus (2003) argues that the 'development of Aristotle's practical philosophy plays a key role' in 'the philosophical hermeneutics of Hans-Georg Gadamer' (Svenaesus 2003, p. 407). Gadamer's perception of understanding is practical, 'hermeneutics in its purest form is found in the living dialogues carried out between people of real flesh and blood' (Svenaesus 2003, p. 415). 'Gadamer's philosophical hermeneutics is deeply influenced by the concept of practical wisdom (*phronesis*)' (Duvenage 2015, p. 77). *Phronesis* concerns ethics in practice, a central instance of requiring interpretative judgement with respect to which a hermeneutics study is appropriate rather than the application of laws as in science.

'if we relate Aristotle's description of the ethical phenomenon and especially the virtue of moral knowledge to our own investigation, we find that his analysis in fact offers a kind of *model of the problems of hermeneutics*' (Gadamer 1975, pp. 320, 321, emphasis in original).

Acknowledging such influence one may ask can such a Gadamerian hermeneutics informed by Aristotle's *phronesis* supply concepts applicable to analysing research participant account? This article argues for a positive response, that there is a locational framework of characteristic features, aspects or (using phenomenology's term) 'moments' which can then be perceived in these research narratives. Methodologically, an investigative conclusion can be structurally accommodated, albeit psychologically and culturally diverse, within globally applicable concepts.

Norlyk and Harder (2010) argue in their review of nursing research that 'there is a need for clarifying how the principles of the phenomenological philosophy are implemented in a particular study'. Integrating Aristotelian and Gadamerian perspectives, 'research could be strengthened by greater attention to its philosophical underpinnings' (Norlyk and Harder 2010, pp. 420, 427). Their viewpoint is shared within this paper reviewing Interpretative Phenomenological Analyses that are 'strongly connected to the interpretive or hermeneutic tradition' (Smith 2004, p. 40) through furthering these analyses as being the hermeneutic narrative of *phronesis* or practical wisdom instantiated.

Embodied, enlacing tacit material and metaphorical 'horizons of practical understanding', representational frameworks of participant perception, *phronesis* as considered hermeneutically, is posited here as focus of Interpretative Phenomenological Analysis. Many of the empirical studies illustrating *phronesis* cited in sections below are chosen to explicate and further IPA research.

Moreover, reflecting on research participant thematic account considered as practice of self-understanding, IPA analyses can be conceptually furthered by employing hermeneutic philosophy's spatio-temporal metaphors, models of understanding. Through knowing this 'primary hermeneutic technique, the metaphor' (Papaloukas et al. 2017, p. 427), one can follow structurally informed routes in closely reflecting upon research participant account. Notably, 'the term

*horizon* has a long history as a metaphor in philosophy': being the 'framework of our experience, it is both limit and condition of possibility' (Evink 2013, pp. 297, 298 emphasis in original) in representing understanding's context.

### **Interpretative Hermeneutic Reflecting on *Phronesis* Practices: Caring's Perspectival Representing Horizons of Understanding**

Hermeneutics engages with our embodied understanding as fundamentally informing being in the world, our location on the cultural perimeters or horizon from which we interpret experience. With analysis of positioned *phronesis* we further establish our identity. Being-in-the-world *matters*.

We care about the goals embedded with our activities, re-presenting them perspectively to ourselves, understanding our purposes in our practices as important, trivial... 'Being-in-the-world is essentially care (*Sorge*)' (Heidegger 1962, p. 237). People are concerned about things (*Besorgen*), themselves (*Selbstsorge*) and feel solicitude for other persons (*Fursorge*) (Heidegger 1962).

Caring affectively interprets a situation. Spatio-temporal images or metaphors are available through hermeneutic philosophy to 'picture' the process of human sense-making, the 'event-structure of all understanding' (Gadamer 1975, p. 481). Caring represents from a person's valuing perspective.

Hermeneutic *phronesis* offers integrating concepts resourcing an experiential analysis. This philosophically generated theory is applied in discussion below, presenting a research 'horizon of understanding' (Gadamer 1989) of studies such as psychology's Interpretative Phenomenological Analysis. Interviewees are considered re-presenting their pre-reflective meaning-making 'practical consciousness' (Giddens 1979, p. 2) as a recurring hermeneutic *phronesis*. How, for instance, did they interpret engaging, continuously or intermittently in behaviour shaped by chronic pain? What was the nature of their 'knowing how' to deal through practical wisdom with this chronic issue?

Hermeneutically, discussion of research interviewee topics are viewed in appropriate terms as exploring through talking, how they represent, their affective horizon of understanding recurrent *phronesis*. 'Horizons' may take material as well as a metaphorical shape encompassing a person's lifeworld, 'our lifeworld-our horizon of meaning' (Svenaeus 2003, p. 415). Thus Ahmad and Talaei's (2012) article on 'understanding chronic pain' refers to a research participant's regular practice in response to chronic pain where a space (a room) becomes an enclosing protective place, a domestic as well as discursively referenced horizon of surviving physical issues.

Positioning her practical wisdom, *phronesis*, she narrates 'I'll avoid light ... noise, too many people around get me upset ... I keeping myself alone, control myself in the room'. Enabled by this sanctuary, materially understanding its use in her embodied perspective on her habitual practice as a horizontal refuge, she establishes in self-caring an equilibrium beyond the 'light', 'noise', 'too many people' in 'liking to be alone'. Her account reflects on, representing her 'attuned' (Heidegger 1962) anticipated practice, a realisation of interpretative practical

wisdom. 'What *phronesis* does is that it enables one to hit the adequate thing to do in a certain situation' (Bobb 2020, p. 33).

Horizons of understanding in representational narratives of the world have shaped practices.

Essential to the concept of situation is the concept of 'horizon.' The horizon is the range of vision that includes everything that can be seen from a particular vantage point. Applying this to the thinking mind, we speak of narrowness of horizon, of the possible expansion of horizon, of the opening up of new horizons, and so forth (Gadamer 1975, p. 301).

### **Using Hermeneutic Spatio-temporal Metaphors in Analysis: Locating Perspectival Horizons Positioning Practical Understanding of *Phronesis***

'For Gadamer' participation in language acknowledges that an individual is located within a substantive horizon of meanings which transcends subjective consciousness', 'horizons of meaning which implicitly sustain reflection and which can, when made explicit, bring us to think quite differently of ourselves (Davey N. 2016).

Exemplifying an 'IPA in practice' Smith (1996) considers hospital haemodialysis where machines replace functioning of damaged kidneys (Smith 1996, pp. 267-270). A participant reflects on being 'really fed up with the repetition' of such recurring 'passive' participating in the ongoing hospital practice. During this repeated practice in 'becoming part of this machine' her embodiment and equipment unhappily merge. Haemodialysis emplaces her affective horizon of awareness: - 'dialysing at home would be - I'm still being myself', refiguring a separate human identity. Her routine, equipped engaging with this practice - her 'passive' engaging 'tethered to one place' - establishes tacit thinking, a horizon of understanding 'place' discerned in emerging discussion. If she was dialysing at home, 'I've still got my identity' with equipment 'under my control and I use it', materialising *Selbstsorge* (self-caring) attaining, instituting, realising practical wisdom, *phronesis*, a facility perhaps economically challenged.

Hermeneutic practices can be continuously challenging. In Eatough and Shaw's presentation (2019) of a research participant with Parkinson's Disease, personal *phronesis* and challenge emerge. Embodiment becomes 'conspicuous' (54). Barbara's affective 'coping style' (ibid) is care -directed in 'battling all the time' to 'deal with' her issues as generated by the 'evil twin'. Habituated practice, notably her morning routine, is 'like a tortoise' (55) seen from friends' horizons of understanding as 'a drag on them' (54). Subject (the human agent) and structure (the social) are hereby 'constituted in and through recurrent practices' (Giddens 1982, p. 8) emplacing a caring (*Selbstsorge*) perspective.

Practices, as earlier considered, establish identities, likewise open to challenging. Behaviour can constitute the 'boundary object' (Star 2010) of contestation. In 'Stigma and the Delegitimation Experience' (Dickson et al. 2007), distance between social horizons of understanding a practice is heard as being 'contested diagnosis between CFS (chronic fatigue syndrome) and depression' (856).

Confronted with his doctor's diagnosis of depressive behaviour (such as not sleeping) their research participant recalls, 'I said "It's not depression, I know my own body and I know how I'm feeling and I know this is not depression."' Source and subject of reflecting know the same self, advancing *phronesis*. Embodied self tells a story, exercised with practical understanding, phenomenologically, enactive, motor or 'operant' meaning making, represented in affective articulation of his narrative.

Coping with chronic pain pursues corporeal *phronesis*. Manifested in recurring concern, its demanding focus can displace a fuller engaging with living to far distant horizons of understanding. Research participants, Osborn and Smith (1998) record, 'remained preoccupied with a sense of confusion, loss and threat' (Osborn and Smith 1998, p. 76). Hermeneutically regarded, *phronesis* here implicitly assumes a (dis)enabling environment of equipment rather than entities (*simpliciter*). In reflective interviewing, pain may be explicitly interpreted as being signal of dysfunctional 'equipment'. 'I always thought you had pain to tell you when there was something wrong' (female) (Osborn and Smith 1998, p. 69), a bodily malfunction.

For Ricoeur, hermeneutically, spatio-temporal separation from a practice can support critical 'distanciated' (Ricoeur 1981b) interpreting. Discussing a research participant's affective distanced horizon in critically understanding experience of embodied pain, Smith and Osborn (2007) consider 'pain as an assault upon the self'. An interviewee reflectively dissects her account, so separating the source and sufferer of pain (albeit 'me'), positioning herself on a horizon of affective interpretation from where she critically distances herself from source of both pain and a 'mean me', 'all sour and horrible', distinct from herself as a person suffering, a 'nice person'. 'It's the pain, it's me, but it is me, me doing it but not me do you understand what I'm saying?' A different contribution installs a similar spatio-temporal distance between 'selves' in its narrative, enabling a 'refiguring' (Ricoeur 1988) of identity: 'it's like living with this guy who follows you around all the time.' In narratives of ethical positioning, *phronesis* is a 'space of moral possibilities' (Yanchar and Slife 2017, p. 146).

Societal horizon of understanding embedded in daily practices can be physically as well as meaningfully established, denoted as bearing the multiple status in research participant 'discursive consciousness' (Giddens 1979, p. 5). Moving to his desert island, the chronic pain sufferer in Smith and Osborn's (2008) research would 'still be a miserable old git but it wouldn't matter' (73), since nobody would 'come around'. He would be alone in the island enabling 'refigured' (Ricoeur 1988) existence, goal-oriented in 'practical coping' (Heidegger 1962), 'all day'. He reflects affectively on achieving this 'easier' material as well as metaphorical horizon of living: 'just be yourself it doesn't matter what you do'. 'Self/identity and relationships define pain experience' (Smith and Osborne 2008, p. 74). This interviewee presents *phronesis* in caring for self (*Selbstsorge*) (Heidegger 1962).

In the article discussing 'the personal experience of chronic benign lower back pain: an interpretative phenomenological analysis', Osborn and Smith (1998) reflect on their research participant Linda's concerned understanding of her

diminished practices, her *Selbstsorge* as a grounding aspect of her being-in-the-world, her reduced embodied knowing-how, practical understanding ‘cut down’:

Linda’s hermeneutic *phronesis* is ‘goal-directed’, ‘caring’. ‘I can’t do half of what I used to do’. Her bodily ‘equipment’ (or *Zeug*, Heidegger 1962) is not ‘ready-to-hand’ (ibid), has been ‘cut down’. Her caring is informed by perspectival re-presentational ‘cultural horizon’ (Gadamer 1975):

‘they say life begins at 40’, people ‘flying their kite’, ‘I used to work like a horse’.

Linda dissects her *phronesis* as generic ‘text’ establishing ‘sense-content’ (Ricoeur 1981a):

‘I’m only 50 and I should be doing this and that and the other’. Her practices ‘configured’ (Ricoeur 1988) her desired relational identity, ‘I just think I’m the fittest’. She affectively distances herself or ‘distanciates’ (Ricoeur 1981b) from her ‘refigured’ (Ricoeur 1988) embodied self :- ‘I can’t do half of what I used to do’. Her corporeal being-in-the-world *matters*.

Understanding-in-practice projects and maintains meaning, constituting ‘object’ as enabling equipment, functioning items seen from an affective horizon of understanding embodied in generic, goal-oriented behaviour. Investigating compulsive hoarding, Garza and Landrum (2015) also show how psychological horizon of understanding, ‘meaning horizons’ (138), can be spatially evident or ‘constitutive of a “world” of hoarding’ (144). Thus for a person who hoards, ‘horizons of meaning’ are (in)forming a ‘relational space’, materialising the ‘basis of an experienced gulf between herself and the others in her life’ (146). For the interviewer, horizons of *phronesis* form focus for insight.

Smith and Osborn (2008) discuss a ‘patient’s experience of renal dialysis’, generic narrative instantiating their embodied and equipped experience. Exploring *phronesis*, they seek their patient’s affective temporal horizon of understanding around ‘coping’: ‘What does the term “illness” mean to you? How do you define it?’; ‘Do you think about the future much?’ Considering the experience as identity-constructing, as a text ‘refigured’ by the patient in constructing ‘self’ they ask: ‘Has having kidney disease and starting dialysis made a difference to how you see yourself?’ Embodied horizon of understanding can be non-verbally evident as when one participant in Willig’s (2007) research on ‘extreme sport’ repeatedly releases ‘audible intakes of breath in order to reinforce her descriptions of the tense and exciting nature of her experiences’ (220), re-presenting a powerful immersion.

As we have noted articulated in our analysis throughout this article hermeneutic philosophy provides spatiotemporal concepts (e.g., ‘horizons of (practical) understanding’, ‘fusion of horizons’, participant ‘projection’ of possibility and ‘refiguring’ narrative), shown as situating the analysis of social dimensions of meaning and materiality in participant *phronesis*. Participant contributions to Shaw et al. (2016) writing on ‘extra care’ housing for older adults equally respond to hermeneutic concept. Horizons can be materially or ontologically as well as epistemically evident in practices.

Care home participant *phronesis* incorporating ‘refigured’ (Ricoeur 1988) identities shaped by ‘coming to terms with having what feels a little bit invasive’,

maps out their agentic embodied horizons of understanding affordances as being enclosing ‘equipment’ (Heidegger): ‘four walls to see, four walls closing in on me’ or ‘sit(ing) on the chairs outside, my hat on, and people com(ing) past’. They form materially (dis)enabling affective conditions of living in community housing.

This research narrative is evidently a ‘prefigured’ (Ricoeur 1988) or anticipatory account of a formed ‘fore-understanding’ (Heidegger 1962). ‘And this is like your cabin and you come out you see and you’ve got everything there for you’. ‘I do like to do a little walk about each day ... around the perimeter’, marking out material horizons of a socially extended understanding in *phronesis*.

The author’s Malaysian hermeneutic research contains the comparable invoking of material/metaphorical positioning societal horizon of practical understanding, on this occasion the *phronesis* of studying in a university campus. Watching a brief video blogging narrative, a student comments:

The scenery is so beautiful, no matter it’s morning, noon, evening or night. I feel (it would be) very relaxing if I could study in this university. From the video, I could feel the air is so fresh, it could reduce my stress on study (male, Chinese).

This discourse establishes a metaphorically spatiotemporal social horizon of understanding place as ‘beautiful’, ‘fresh’, a site of potent narrative where ‘I could study’, activity ‘refigured’ as nourishing identity (‘very relaxing’ to ‘reduce my stress on study’). Anticipating potent *phronesis*, recurring routines, projecting meaning he celebrates an ‘embodied experience’ (Willig 2007, p. 209). Denoting Malaysian Borneo ‘relaxing’ material horizons, ‘scenery’ surrounding public university, the ‘embeddedness’ of reflective responding in a ‘local world’ (Todorova 2011, p. 36) is evident, as a ‘frame of reference’ (Martin and Sugarman 2001b, p. 196) from where he contemplates such study. Dwelling encompassed by such a ‘beautiful’ life-world, therein enjoying a ‘relaxing’ familiarity of embodied goal-oriented *phronesis*, would embody/emplace affective tacit horizon of understanding ‘study in this university’ as being enjoyably ‘settled/still within the physical environment’ (Shaw et al. 2018, p. 29). Elsewhere, ‘stress on study’ lacks a ‘horizon that can give respite’ (Shaw et al. 2018, p. 32).

Listening to this interviewee account from a ‘horizon of expectation’ (Jauss 1982) that it would reference a university campus, the researcher accessed student ‘configured’ (Ricoeur 1988) narrative. ‘Blanks’ or ‘indeterminacies’ (Iser 1978) were absent with such an affective anticipating of study. Embodied horizons of understanding encircle the life-world (in)forming therein participant narrative, ‘beautiful’ circumstances lived pre-reflectively, evident in his anticipatory participating.

The hermeneutic discussion locates (‘foregrounds’) the interviewee position on a horizon of affective understanding embedded within practices as ‘framing assumption’ (Yanchar 2015, p. 107), a reference point, stance from which participant experience is being tacitly expected and evaluated. Such horizons of embodied understanding can be followed, discerned as encircling a life-world, to be informing *phronesis*, as the ‘centrality of temporal-narrative themes’ (Yanchar

2015, p. 107): ‘the air is so fresh, it could reduce my stress on study’ and ‘scenery is so beautiful (... ) I feel (it would be) very relaxing’.

This celebratory instantiating, albeit as counterfactual, of a generic goal directed narrative presents personal identity, a less stressed ‘self’. We hear ‘implicit and embodied understanding of the good life that (could be) manifested in social practice’ (Christopher and Campbell 2008, p. 677), there displayed affectively, viewed from ‘horizon of narrative inquiry’ (Rosiek 2007). Embodied and encircling horizons of understanding, perspectives in *phronesis*, (in)form life-worlds.

Dwyer et al. (2019) in their article endorsed by Nizza et al. (2021) as being a ‘very good IPA paper’ (2), discuss ‘young adults’ experiences of residing in nursing homes following acquired brain injury’ (ABI). This article identifies some ‘young adult’ narrative themes exemplified by quotation. Furthering a hermeneutic analysis of Dwyer et al.’s (2019) theme, ‘existential prison of the nursing home: stagnated lives’, one can see that ABI adult ‘horizons of understanding’ care homes are structurally material, their being both physically emplaced, surrounded by ‘imprisoning’, as well as by sharing affective perspectives re-presenting a nursing home as experiential ‘stagnation’. Adults are existing on the mortal edge, horizon of death: “‘Like, so many people have died there, I’m in their company and helping people here and there and next thing you know they are after dying.” (Liam, 178–189)’ Materially and intersubjectively, ‘we (can) speak of narrowness of horizon’ (Gadamer 1975, p. 301) within which a caring practical wisdom, a limited *phronesis* is being exercised.

Horizons of understanding articulating this identity are built or established through personal vocal affective narrative. “‘You see that empty chair where they used to sit and slowly, one by one, they are disappearing ... you say, ‘is it coming around to me?’” (Sarah, 1450–1460)’ Here, Sarah’s dis-equilibrating narrative ‘fore-conception’ ‘refigures’ (Ricoeur 1988) her identity as ‘threatened’. There then continues Sean’s uneasy narrative re-equilibrium: “‘I felt like I’d come out of the crypt. That I was Lazarus, awoken from the dead. ... It was like I was brought back to life.” (2904–2910)’, a phronetic insight realising self-caring.

‘Horizons of understanding’ are established, marked out or traced linguistically, conveying close or differentiated affective perspectives: ‘David’s repeated use of “I had to” illustrates his lack of control over a fundamental aspect of his daily life (meal times)’ (Dwyer et al. 2019), constructing an alternative point-of-view from Sean who ‘stated that he found the meal structure consoling’ (Dwyer et al. 2019).

In another ‘very good IPA paper’ (Nizza et al. 2021), Conroy and de Visser (2015) discuss the ‘importance of authenticity for student non-drinkers’. Here, ‘horizons of understanding’ emerge hierarchically, signifying ethically, personally, or socially elevated greater vision with wider insight, as empowering authenticity. Michelle’s horizontal ascension augments her awareness: ‘The less and less I drank the more I realised that I didn’t like who I was when I was drunk, so the less I drank still.’ Likewise, Katie attains an ethically self-elevated horizon of understanding alcohol consuming: ‘you’re tainting yourself’. Both experience here ‘an expansion of horizon, of the opening up of new horizons’ (Gadamer 1975, p.

301). Fear articulates, emplaces a phronetic ‘horizon of expectation’ in Paul: ‘The reason I don’t drink might be because I am afraid of what I might say or do, if I drink.’

In short, I suggest ‘horizons of understanding’ are implicit in research participant narrative. The care home example also illustrates well how a material horizon of understanding is established by economic decision-making, thereby relating IPA to addressing political analysis. In these papers, engaging with the spatio-temporal metaphor ‘horizons of understanding’ ‘illustrates a close meshing of theoretical and empirical instantiation of the search for experiential meaning’ (Smith 2018, p. 15). Martin and Sugarman (2001a) write: horizons of “self” understanding connect particular being to the life-world in ways that respond to the cares and concerns of embodied agents’ (110), their self-caring realising of *phronesis*, horizontal practical wisdom in securing personal salvation:

Implicit ‘horizons are the conditions that provide the meaning for (experience), conditions which need to be made conscious for a proper understanding of the (experience). As providing the key to understanding our perceptions they draw us forward, inspiring investigation’ (Vessey 2009, p. 536).

### **Hermeneutic *Phronesis* Perspectives: Analysis of Research Participant Practical Wisdom**

Doing IPA inevitably involves being hermeneutic in the general sense; doing it well involves more particular hermeneutics (Smith 2011a, p. 58).

Interpretative Phenomenological Analysis, firmly asserts its initiating author, is ‘strongly connected to the interpretive or hermeneutic tradition’ (Smith 2004, p. 40). Can insight exercised in hermeneutic *phronesis* analysis, in perceiving embodied understanding as ‘practical consciousness’, function as schema enabling the application of core concepts analysing research participants’ narratives? Can participants’ emerging interpretative ‘horizons’ of (in)sight be regarded as their reflecting on a care-directed *phronesis*? Presenting the earlier case studies, the author has sought affirmative response.

Analysed within a philosophical (in)forming, participant meaning making *phronesis* can be considered as placed within perspective, horizon of understanding, diversely implicit in embodied action. What is their anticipated ‘horizon of possibilities’ (Papadimitriou et al. 2018, p. 13) heard in research participants’ self-regarding understanding? How is this behaviour being seen? From what conceptual horizons of understanding participant behaviour is this research ‘co-constituted’ (Gupta 2020, p. 12)? Thinking philosophically, it becomes clear that the Gadamerian ‘fusion of horizons’ with participant perspective mediating researcher reflecting (in)forms the ‘interactional “business” of the interviews’ (Coyle 2010, p. 79). During the process, horizons of understanding can be augmented, or as if in a tunnel of constrained comprehension, narrow and shrink. What is being heard? *Phronesis* is ‘about a dialogue or conversation about those

perspectives that offer the best theoretical answers to practical questions' (Duvenage 2015, p. 85).

Constituting meaning may be viewed 'horizontally', from close analysis to far away, from defining human behaviour to the hermeneutic background of making sense. Experientially, 'we are only able to make sense of and interpret the world within these horizons' (Burton et al. 2017, p. 376), the ubiquitous 'horizons of experience' (Burton et al. 2017, p. 378). Within research participants' 'meaning-making resources' (Coyle 2010, p. 81), the tacit 'horizon of meaning' becomes the sense making 'background that is usually taken for granted' (Githaiga 2014, p. 403) by interviewees, but their understanding can be discursively 'fore-grounded' in discussing. A participant's retreat to a back room from the public space can be closely considered by the researcher as taking 'refuge', reducing bodily pain.

Constituting meaning can be considered temporally, as being in process. People understand via different routes, agreeing or disagreeing. Ideational location as a social horizon of understanding in a group, may be shared or individually occupied, so seen in shifting ground, a 'chronological and cumulative process of sense-making' (Tomkins and Eatough 2010, p. 255) in discussions.

Smaller groups enable potentially lengthy 'visits' to perhaps darkened horizons of concerned understanding, enabling each participant to 'discuss their own experiences in more detail' (Phillips et al. 2016, p. 299). Self-caring (*Selbstsorge*) hermeneutic embodied *phronesis*, enabling coping with debilitating chronic pain can emerge during reflecting, there viewed as emplacing moral horizon of understanding, establishing interviewee 'refigured' (Ricoeur 1988) identity, individually or group.

Davidson (2013) asserts the importance of sighting and reflecting on a broader or wider ideational participant 'horizon of meanings' (Davidson 2013, p. 324), with the distant (perhaps the 'refuge' as in reducing pain) being equally significant as the immediately evident meaning of the experience ('pain'). The account may involve a 'hermeneutic circle of understanding' by integrating horizontal context with *phronesis*. Establishing distant meaning, discussion can proceed from a horizon of uncertain vision.

Hermeneutically, *phronesis* is generic, recurring (akin to texts (Ricoeur 1981a)) as types of behaviour. Considering participant narratives as reflecting on earlier generic goal directed practices enables the latter's acknowledging as behavioural type embodying roles, actions more or less successfully accomplished - again, for instance, in pain reducing behaviour. Narratives may be characterised by disturbances or disequilibria before returning to equilibria or calm.

Smith (2007) writes: 'I think there is considerable scope for developing, and extending hermeneutic theory to help its application to the activities of researchers in the human sciences' (Smith 2007, p. 4). Such development could be initiated by noting that narrative derived research participant content may be interpreted as 'configuring' (Ricoeur 1988), establishing experientially their position on 'horizon of understanding' (Gadamer 1975), with the latter thereby appropriately explored during interview. How, to revert to the earlier example, do adult brain injured persons perceive resident care homes? Interviewers can suitably and sympathetically search within 'horizons of implicit meaning' (Finlay 2014, p. 138)

- for there are equally 'embedded meanings to consider' (Finlay 2012, p. 185). How does someone adversely affected by arthritis re-present their diminished experiences more widely? How is their disabling generic experience diffracted across a diversity of identity establishing behaviour? Exploring embodied horizons of understanding around recurring *phronesis* shows 'concepts, habits, routines, expectations, and norms may be disrupted or even destroyed by illness' (Carel 2016, p. 17).

Interpretative Phenomenological Analysis (IPA) is informed by hermeneutic philosophy. So, for instance, the researcher's interpreting participant narrative of self-understanding is characterised as being a 'double hermeneutic' (Smith 2011b, p. 10) augmenting the former's perspective through 'fusion of horizons' (Gadamer 1975), therein aligning a question and response. Content established from listening to participant *phronesis* should be 'grounded' in 'enough particularity' yet equally contain 'enough abstraction' to allow a 'conceptual' organising of interview narratives (Smith et al. 2009, p. 92). This article has sought to suggest 'concepts'.

Using hermeneutic spatio-temporal metaphor, this research protocol may be re-interpreted as recommending locating, positioning particular participant accounts within conceptually appropriate horizons of understanding, points of viewing 'grounded' *phronesis* heard during an interview. IPA's presenting research through the increasingly abstract relationship between 'superordinate concept' and 'themes' can be re-presented as being an epistemological hierarchy of 'horizons'. Here, seeing gathered qualitative data 'top-down' from far off to focussed views of participants' accounts shapes 'fine-grained analyses of individual lived experiences' (Eatough and Smith 2008, p. 186) from such a reflective wider perspective. 'Themes', in short, can be more or less precise, construed as denoting perceptive framing 'horizons of understanding' *phronesis*, a portal on research participants' - actual, present or preferred alternative - life-worlds. Hermeneutic philosophy is thereby further enacted.

Insights as a themed seeing could be termed 'horizontalisation' (Trondalen 2005 in Lee and McFerran 2015, p. 368), 'horizoning', or the 'horizontal' perception of research interviewee *phronesis*. Doing so signals IPA's hermeneutic underwriting as reflective process in its attending to 'fusion' of interviewer/interviewee 'horizons' (Gadamer, 1975) as interpreting individual participant or focus group 'multiperspective' (Palmer et al. 2010, p. 117) constructing of shared practices. Shaw (2010) points to Gadamer's epistemic of our culturally and historically situating ubiquitous horizontal perspectives as hermeneutics being appropriate 'support for adopting a reflexive attitude in experiential qualitative research' (Shaw 2010, p. 235), or researcher reflecting on preconceptions. Realising participant perspectives, their 'fore-understanding' (Heidegger 1962), establishing interviewee instantiated horizons, should find a critical focus in reflection. 'Objectivity begins by being aware of one's own subjectivity' (De Castro 2005, p. 159) - a distancing oneself from presuming participant assumptions enables a 'critical hermeneutic gaze' (Langdrige 2006, p. 644).

Citing Schleiermacher's hermeneutics, who 'suddenly sounds contemporary', Smith (2007) reminds us that participants as 'author' of their *phronesis* may be considered sources establishing a meaning for their experience (5). An interviewee

account can be analysed as attempting or essaying integration of their episodic experience in a 'hermeneutic circle of understanding' (Gadamer 1975): he or she strives to assemble the fragmented experience into holistic narrative of an issue. Listening, the researcher gains insight avoiding distorting horizontal 'fore-conception' (Heidegger 1962).

Interpretative Phenomenological Analysis is (in)formed philosophically by the hermeneutic narrative of knowing as use (cf. Wittgenstein 1991), as primarily practice. Here, 'understanding' is considered 'practical engagement with the world', involving 'self-reflection and sociality, affective concern, and a temporal, existential location' (Smith et al. 2009, p. 17). Participating in research, the interviewee's affective shared 'self-reflection', their 'discursive consciousness' (Giddens 1979, p. 5) of concerns is heard enunciating their pre-reflective recurring 'practical consciousness' (Giddens 1979, p. 5). IPA shares in listening to contributors the hermeneutic *phronesis* perspective goal, bringing to the fore tacit knowledge-in-use. The latter - and likewise, I suggest, the former - is concerned with socially located recurrent practices. Both exemplify research reflecting upon agential 'embodied concerned involvement in practices' (Yanchar and Slife 2017, p. 147), as in reverting to a room/sanctuary from noise.

Being reflective entails relating finding back to 'practical engagement' as the understanding of experience prior epistemologically - as a form of knowing - to the capacity to issue propositional accounts for researchers. In the latter, as we noted at the outset, evaluative horizon of understanding can be glimpsed or become more fully visible. A sustained trial and error in surmounting issues may be shown to be returning interviewee to habituated 'ready-to-hand' (Heidegger 1962) familiarity.

Analytical results map the 'constitutive structure' (Willig 2007, p. 221) of understanding. Here discursive recollecting of *phronesis* consciousness (as in caring) references embodied constraint and enablement. Shareable generic rules for enacting affectively interpreted purpose, marking a cultural standard of engagement may exist, from participating in 'extra care housing' (as exemplified above) to pain alleviation. Reflecting on health practices, participant enabled social genres emerge.

Hermeneutic IPA contributes to sustaining such qualitative research across 'embodied active situated cognition' (Larkin et al. 2011, p. 319) - finding multiple modes of understanding-in-practice. The preceding section applied diverse 'moments' or aspects of a hermeneutic *phronesis* perspective (HPP) on behaviour to re-viewing, thinking through empirical exemplification of IPA research. Being universal characteristics of 'practical consciousness', they are seen emerging as such in ubiquitous report as well as incorporated, immanent or heard in particular participants' discourse, 'presented as *both a priori* and essential and concrete and variable' (Tomkins and Eatough 2013, p. 4, emphasis in original). Focus group or interviewing display these epistemological moments through responses so furthering the 'consideration of emergent core constructs in IPA' (Brocki and Wearden 2006, p. 100). From hermeneutic horizons, understanding is 'primordially embodied, enabled *phronesis*, tacitly emplacing social perspectives, universally discernible 'moments' in research participant reflection.

Research participant epistemological ‘horizons’ can be glimpsed, then gained more widely. ‘Analysis results in an explicitation of horizons, which enables an access to the life-world’ (Sages and Szybek 2000, p. 155). *Phronesis* is discerned in discussion as discourse shaped philosophically.

To make the case that hermeneutic Interpretative Phenomenological Analysis is enhanced drawing more widely upon spatio-temporal metaphor such as ‘horizon of understanding’, is acknowledging that ‘when we are trying to understand another person or a text, we need to have some idea of the horizon in which the subject matter is intelligible to the author or speaker’ (Vessey 2009, p. 539).

In this way, speaking and writing from implicit, social horizon of understanding, positioned within a perspective, can be seen as conceptually informing the interviewee’s grasp on - or point of view of - a subject matter. If considered temporally, the subject’s ‘entering’ *phronesis* from horizon of understanding, our thus seeing content as being generically constituted, generates an expectation, potentially fulfilled by subsequent sequence. A ‘relaxing’ campus facilitates reduced ‘stress’.

Horizons of understanding are not only the embodied affective perspectives from which we make sense of the world, conceptualise its occurrence, but the material and metaphorical perimeters or limits of vision. ‘Refigured’ (Ricoeur 1988) narrative securing identity for research participant - an experiencing ‘self’ - can substantiate horizon of understanding a room as refuge from pain. So engaging in such a practice, one can maintain embodied ‘home understanding’ (Taylor 1995a, p. 150), refiguring identity ‘equipped’ by enabling familiar habitual supportive concrete circumstance. Thus ‘belonging’ facilitates ‘becoming’ in this narrative of subjectivity in recurring *phronesis*.

### **Conclusion: Interviewing from an Interpretative Hermeneutic *Phronesis* Perspective (HPP): Using Analytical Metaphors Situating Understanding**

Philosophy is tacitly embedded shaping research practice, present within hypothetico-deductive philosophy of science or in the hermeneutic structuring of scientific paradigms. Reflecting upon philosophy-in-practice enhances empirical analysis. Hermeneutic philosophy has informed and shaped social science research activity, not least as practices theory. The consciously culturally located investigator seeks pre-reflective participant self-understanding in *phronesis*: while latent in recurring behaviour, an identity establishing narrative becomes manifest in discussion.

An embodied ‘horizon (of understanding) is not a rigid boundary but something that moves with one and invites one to advance further’ (Gadamer 1975, p. 238).

Presenting research participants as being always already engaged in their familiar habituated life-worlds, achieved as a ‘home’ or as alterable focus of would-be distancing, is philosophically far from empiricism’s narrative of knowing an external world as the passive ‘imprinting’ (Taylor 1971, p. 32) of ‘sense-data’, received impressions constituting the base substratum for assembling

'ideas' or knowledge. Positivism's data are derived and delivered free from any theoretical encumbrance:

the basic building block of knowledge on this view is the impression, or sense-datum, a unit of information which is not the deliverance of a judgment, which has by definition no element in it of reading or interpretation, which is a brute datum (Taylor 1971, p. 7).

Instead, the world in which one is placed is interpretatively regarded, seen from materially located cultural position, an individually nuanced location. While a socially designated horizon of its understanding may be limiting, thus rendering a person 'incapable of framing certain questions, and entertaining certain possibilities' (Taylor 2002, p. 130), points-of-view can discursively broaden.

'Unformulated' (Taylor 1995b, p. 179) during recurring practice, embodied understanding-in-use is habitual resourcefulness available to be 'voiced out' during research interviews. Horizons are ubiquitous: 'doing without frameworks is utterly impossible for us' (Taylor 1989, p. 27).

Interpretative Phenomenological Analysis is reflectively grounded in a qualitative study of participant 'particularity' (Smith et al. 2009, p. 92), rather than physical science (alleged) 'raw data', accounts contextualised by conceptualising theme, rather than contained in law-like nomological generalisation. Interviewees, this paper suggests, reflect on absorbing in hermeneutic *phronesis* of understanding experiences - often as issue-laden encounters - tacit repeated understanding-in-use wherein entities are interpreted as disabling/enabling. IPA's phenomenology is then distant from positivism's subject-object dualism in denying understanding disconnects from its embodiment. 'Being angry is an experience which is lived through the body' (Eatough and Smith 2008, p. 494). 'Once drunk he seemed like God's gift to women!' (participant in Roleston and Shaw 2017, p. 7).

Viewing Interpretative Phenomenological Analysis (IPA) as exemplifying the philosophical hermeneutic perspective in undertaking empirical research, these pages have sought to extend IPA's 'distinctive epistemological framework' (Shinebourne 2011, p. 17) towards achieving 'uncovering of meanings' (Shinebourne 2011, p. 19) implicit in research participant *phronesis*. Reflective philosophy's research can reveal 'involvement' (Heidegger 1962), an interviewee's embodied (Larkin et al. 2011) and implicit articulation of entities as 'equipment' in her or his understanding-in-use. A tacit affective 'horizon of (practical) understanding' (Gadamer 1975) can be traced during the analysis as being a more or less 'shared commonality' (Shinebourne 2011, p. 23) of the interpretative experience.

Reflection situates 'seeing' enabled/ elided by culture (in)forming understanding, this 'shared background of life into which we are initiated' (Martin and Sugarman 2001b, p. 197). Yet participant *phronesis* can limit as well as allow understanding (Langdridge 2006) and may be challenged by a difficulty where 'people reflect and deliberate on how to proceed' (Polkinghorne 2000, p. 457) thereby revising their recurrent activity and identity in self-interpretation (Brinkmann 2008).

The paper has reflected on the foundational conceptual framing of hermeneutically informed Interpretative Phenomenological Analysis. It considered the proposition that revisiting hermeneutic philosophy's multiple spatio-temporal metaphors for the process of understanding would generate a resource further enhancing analysis of research participant *phronesis*. Thus reference to a historical underpinning could shape a current insight. Such a concluding comparison may be made with other communication disciplines. Refiguring this qualitative psychology through the philosophical lens of understanding as an interpretative hermeneutic *phronesis* perspective (HPP), its research participant reflecting on their previously tacit recurring activity can be viewed as narrating an (i) emergent; (ii) embodied; (iii) equipped *phronesis*; (iv) emplacing implicit 'horizons of understanding' (Gadamer) in their (v) generic, care-directed activity (vi) 'refiguring' (Ricoeur) their identity as (vii) a subject of celebratory or 'distanced' (Ricoeur) viewing, (viii) contextually an institutionally consensual or contested 'boundary object' (Star 2010). So 'practical consciousness' (Giddens 1979) emerges in reflection as being tacit, teleological awareness of identity in lifeworld socially positioned subjectivity.

Considering conceptual foundations shaping a hermeneutic Interpretative Phenomenological Analysis I have reviewed this approach to qualitative interview psychology engaging with research participant *phronesis*. Drawing on an epistemological framework advanced by Aristotle, Heidegger, Gadamer and Ricoeur would enable analytical narratives enhancing psychology's empirical insight as a contribution to a 'narrative psychology' (Sools et al. 2015). Hermeneutic philosophy advances practice further articulating theory and psychology, whether research is conducted by individual or a 'team' of investigators (Montague et al. 2020, p. 26). Research participant articulated 'horizons of understanding' as representational, render explicit a tacit pre-reflective awareness of powerful life-world embedded in a participant's meaning-making 'practical consciousness' (Giddens 1979). *Phronesis* as pursuing personally propitious practices 'refigures' (Ricoeur 1988) participant identity.

Interpretative Phenomenological Analysis reflects in partnership with participants upon their tacit understanding in/of recurring behaviour, the hermeneutic dimension of *phronesis*. 'The person becomes the universe of exploration at the outset' (Smith 2021). Acknowledging IPA concern with practices, as an interpretative hermeneutic *phronesis* perspective (HPP), allows participant accounts to be seen as instantiating *a priori* perspectives, a structurally ubiquitous behavioural (in)forming.

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## Sets, Properties and Truth Values: A Category-Theoretic Approach to Zermelo's Axiom of Separation

By Ivonne Pallares Vega\*

*In 1908 the German mathematician Ernst Zermelo gave an axiomatization of the concept of set. His axioms remain at the core of what became to be known as Zermelo-Fraenkel set theory. There were two axioms that received diverse criticisms at the time: the axiom of choice and the axiom of separation. This paper centers around one question this latter axiom raised. The main purpose is to show how this question might be solved with the aid of another, more recent mathematical theory of sets which, like Zermelo's, has numerous philosophical underpinnings.*

**Keywords:** *properties of sets, foundations of mathematics, axiom of separation, subobject classifier, truth values*

### Introduction

Set theory is a field of mathematics that has the peculiar property of allowing the “codification” of many mathematical concepts such as those of number, cartesian product, function, group, ring and many others. Another branch of mathematics that also has this property is category theory, although its “encoding” is quite different from that of set theory. It is mostly this feature shared by both theories that makes them quite amenable to philosophical investigations. Indeed, the word most commonly used is not “codification” or “encoding” but *foundation*, although this latter term tends to have different meanings for different philosophers of mathematics. In 1964 the American mathematician F. William Lawvere published the first axiomatization of the concept of *set* within the framework of category theory<sup>1</sup>. In this paper I shall refer to the more fully developed version he published jointly with Robert Rosebrugh in 2003<sup>2</sup>. For Zermelo's axiomatization I will base my discussion on a paper entitled “Investigations in the foundations of set theory I”, which he published in 1908<sup>3</sup>. The discussion is centered around one axiom, one that deals with relation between certain properties and the existence of subsets of a given set. Zermelo calls it the *Axiom of separation* and *LR* call it *Membership representation via truth values*<sup>4</sup>.

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<sup>1</sup>Reprinted in Lawvere (2005).

<sup>2</sup>Lawvere and Rosebrugh (2003). I shall refer to this axiomatization throughout the text as *LR*.

<sup>3</sup>Zermelo E (1908a). Throughout the main text I will refer to this paper as *Zermelo 1908a*, and to Zermelo (1908) as *Zermelo 1908*.

<sup>4</sup>Within the more general context of topos theory (an important branch of category theory), this is called the Subobject Classifier axiom.

When philosophers of mathematics discuss foundational issues concerning set theory, they usually have in mind what has become to be known as “Zermelo-Fraenkel set theory” or “Zermelo-Fraenkel set theory with the axiom of choice” (abbreviated as ZF and ZFC, respectively). Ever since category theory entered the scene in discussions of foundational issues, ZF (and *per force* Zermelo’s own axiomatization) has been discredited to a large extent<sup>5</sup>. I believe that the categorical approach to set theory still owes something of value to *Zermelo 1908a*, even if only for reacting against it on various fronts, some of which I will present in the section concerning the concept of ‘function’<sup>6</sup>. So by basing my exposition on *Zermelo 1908a*, rather than on current versions of set theory like ZF, I intend this paper to be partly a tribute to Zermelo. Indeed, both *Zermelo 1908* and *Zermelo 1908a* include the axiom of choice, and so does the categorical approach. One of my purposes here is to show the sense in which the categorical approach includes part of Zermelo’s axiom of separation. But my main purpose, and now as a tribute to the categorical approach, is to show how the categorical version of this axiom solves a problem raised by its original formulation. Thus rather than discrediting ZF in favor of the categorical approach, I propose here to see the latter as an instance of conceptual progress within mathematics.

Both theories are rich in consequences, not just mathematical but also philosophical. However, I shall only present those axioms, concepts, theorems and definitions from each theory that are necessary for the exposition or useful for illustrating purposes, thus keeping technical details to a minimum.

### Zermelo’s Set Theory and Properties of Sets

According to *Zermelo 1908a*, set theory is about a certain domain of *individuals* or *objects*, among which are the *sets*. One of the functions of the axioms is precisely to characterize the latter ones. In the introductory part he says that Cantor’s 1895 original definition of a set as “any collection into a whole  $M$  of definite and separate objects  $m$  of our intuition or our thought” (Cantor 1955, p. 85) needs to be restricted, for, as it was well known, it leads to certain contradictions such as the Russell paradox concerning the set of all sets not belonging to themselves. Thus the axioms are meant to serve the function of restricting the notion of ‘set’ so as to prevent the theory to give rise to paradoxes such as Russell’s<sup>7</sup>.

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<sup>5</sup>Ernst (2017, p. 69), e.g., talks of categorical foundations replacing set theory.

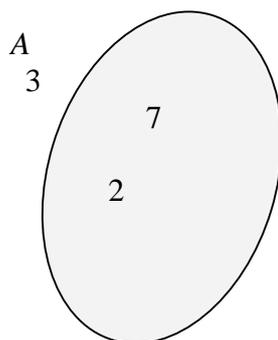
<sup>6</sup>Lawvere (2005) describes its own contents as “a way out of the [Zermelo-Fraenkel] impasse” (p. 6) where the latter’s membership-theoretic definitions and results are “for a beginner bizarre” (ibid.) Almost forty years later the claim is that “[...] from the ongoing investigation of the ideas of sets and mappings, one can extract a few statements called *axioms* [...] The use of this axiomatic method makes naive set theory rigorous and helps students to master the ideas without superstition.” (*LR*, pp. ix–x, emphasis in the original). Be this as it may, I am nonetheless convinced that there is much more of value in category theory, especially for all those interested in philosophy of mathematics.

<sup>7</sup>See, e.g., Fraenkel et al. (1973, p. 18). However and in contrast, Gregory H. Moore has argued extensively in favor of the claim that Zermelo’s main motivation for axiomatizing set theory was instead that of defending his proof of the well-ordering theorem, which he first gave in 1904 and

Set theory is based on the notion of *membership*, which is taken as basic, not definable in terms of any other more basic concepts. From the seven axioms in *Zermelo 1908a*, five of them are about the formation of new sets from other sets previously given or assumed to exist. The exceptions to these are the axiom of the empty set (which postulates the existence of a set with no elements) and the axiom of extensionality. Except for this axiom, all the remaining axioms that postulate the existence of certain sets start from *sets* already given, not from general “objects” of what Zermelo calls the “domain” of set theory, that is, not from objects that may not be sets in the sense of the other axioms. Following most introductory textbooks on set theory, I will ignore this point and consider the universe of the theory as consisting of only sets whose elements are themselves sets, whose elements are also sets, and so forth.

The symbol for the relation of membership between sets is  $\in$ . There are several ways for denoting a set. Suppose that  $A$  is a set whose elements or members are the natural numbers from 1 to 4. We may then write  $A = \{1, 2, 3, 4\}$ . Or, if  $N$  denotes the set of *all* natural numbers, we may also write  $A = \{x \in N \mid 1 \leq x \leq 4\}$ , where the symbol ‘ $\mid$ ’ is read as “such that” or “with the property that”. This notation is useful, for example, when one cannot list all the elements of a set, either because they are too many for it to be feasible in practice or because the set is infinite. The set of even numbers, for instance, is infinite, so we can denote it simply by writing  $\{x \in N \mid x \text{ is even}\}$ . Now, in order to express that, for example, a certain number is *not* an element of  $A$ , we use the symbol ‘ $\notin$ ’. So for instance, we have that  $5 \notin A$ .

Sometimes it is useful to picture sets by means of what are called Venn diagrams. For example, if  $A = \{2, 3, 7\}$  we may draw the following diagram



Zermelo’s first axiom gives us a criterion for determining whether or not any two given sets are equal.

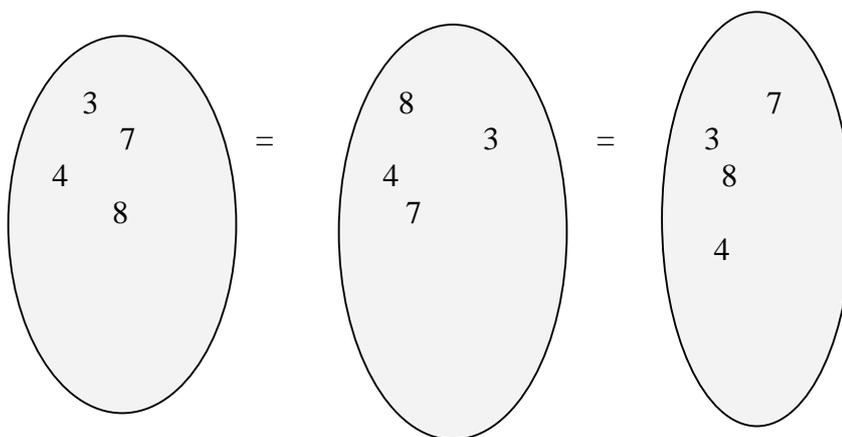
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then later modified in Zermelo (1908). See, for instance, Moore (2013), especially Chapter 3. I tend to favor Moore’s view, if only because it makes Zermelo’s efforts to prove the well-ordering theorem—which of course include his axiomatization of set theory—more interesting from a mathematical, philosophical and historical perspective.

*Axiom I (Axiom of Extensionality)*

Given any two sets  $M$  and  $N$ , if every element of  $M$  is also an element of  $N$  and vice versa, then  $M = N$ .

One may see this axiom as saying that a set is completely determined by its elements or, equivalently, as saying that, for any two given sets, in order for them to be different, there *must be* at least one element belonging to one of them but not to the other set. One consequence of this axiom is that, for example, the sets  $\{3, 7, 4, 8\}$ ,  $\{8, 3, 4, 7\}$  and  $\{7, 3, 8, 4\}$  are all one and the same set. Venn diagrams may illustrate this more clearly<sup>8</sup>:



So the order in which the elements of a set are listed does not affect its identity. One might say that what gives a set its “substance” are its elements, regardless of the order in which one thinks of them. The categorical approach to sets contrasts with this way of thinking about sets; for within category theory, the focus is not so much on what the elements of a set are but on how it relates to other sets.

In the next section we will see how one can capture the idea of *order* in terms of the membership relation. The remaining three axioms I will be considering are of a different nature than the axiom of extensionality since they all postulate the existence of certain sets.

*Axiom II (Axiom of Elementary Sets)*

There exists a set that has no elements at all. If  $a$  is a set, there exists a set  $\{a\}$  containing  $a$  as its only element. If  $a$  and  $b$  are sets, then there always exists a set  $\{a, b\}$  whose only elements are the sets  $a$  and  $b$ .

One question immediately arises: How many sets with no elements are there? If we suppose there are at least two different empty sets, let us call them  $A$  and  $B$ , then, according to Axiom I, one of them must have at least one element that does

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<sup>8</sup>In principle, Venn diagrams can be of any shape and size, although they are most commonly drawn as above or as circles. See e.g., *LR*, p. 1.

not belong to the other (empty) set. But clearly this is not possible, since by hypothesis neither  $A$  nor  $B$  have any elements. In this way, we have arrived at our first theorem:

**Theorem** (Uniqueness of the empty set)

There is only one empty set.

□.

Since there is just one empty set, we are justified in using a special symbol for it. As it is customary, I will use the symbol  $\emptyset$ , although it is sometimes also denoted as  $\{\}$ .

Before presenting the axiom of separation, which is Zermelo's Axiom III, I shall present one more axiom of existence which, together with Axiom II, is useful for illustrating the nature of the set universe as well as the special encoding set theory allows for defining mathematical objects such as the natural numbers. However, I shall retain Zermelo's numbering for his axioms.

*Axiom VII (Axiom of Infinity)*

There is a set that has the empty set as one of its elements and it is such that whenever a set  $a$  belongs to it, so does the set  $\{a\}$ .

With Axiom II at hand, we can prove, for example, the existence of the sets  $\{\emptyset\}$ ,  $\{\emptyset, \{\emptyset\}\}$  and  $\{\{\emptyset\}\}$ . Clearly, none of these three sets is equal to the empty set: they all have either one or two elements. Axiom VII then postulates the existence of the following set

$$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, \dots\}$$

and this is a set that can be used to encode the natural numbers. For we simply let the number 0 be the empty set, 1 the set  $\{\emptyset\}$ , 2 the set  $\{\{\emptyset\}\}$ , and so on. The usual operations of addition and multiplication can be defined for these sets and in such a way that the elementary properties of arithmetic (such as commutativity and associativity) hold. Moreover, equations such as  $1+1 = 2$  are theorems of the theory, that is to say, they can be proved within the theory (after all, they only involve the relation of equality between sets which in turn is given in terms of the membership relation).

There are, however, other ways for encoding the set of natural numbers. Consider, for example, the following

$$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\{\emptyset, \{\emptyset\}\}\}, \dots\}$$

In this case, we let  $0 = \emptyset$ ,  $1 = \{\emptyset\}$ ,  $2 = \{\emptyset, \{\emptyset\}\}$ ,  $3 = \{\emptyset, \{\emptyset\}, \{\{\emptyset, \{\emptyset\}\}\}$ , etc. Here too, one can define the operations of addition and multiplication and prove the usual arithmetical properties of them.

It is tempting to see either definition of the natural numbers as providing an answer to the question of what (natural) numbers are<sup>9</sup>. There are, however, strange well-known consequences one can draw from either one. For example, in both cases it is true that  $0 \in 1 \in 2 \in 3 \in \dots$ . And assertions such as these simply do not make sense outside the context of the set-theoretic encoding of numbers, neither are they taught as the basic properties of numbers<sup>10</sup>. This is indeed one of the criticisms that has been raised against set-theoretic foundations of mathematics: they are at odds with mathematical practice<sup>11</sup>. In fact, the categorical approach to sets aims at bringing set theory closer to, and useful for, the actual practice and development of mathematics<sup>12</sup>. But mathematics evolves and so do its practice, problems and tools, as I hope will illustrate the comparison I will draw between Zermelo's axiom of separation and its categorical version.

For stating the last axiom that I shall be considering, we need the following definition.

### Definition

Let  $M$  and  $N$  be given sets. If every element of  $N$  is also an element of  $M$ , then we say that  $N$  is a *subset* or *part* of  $M$ , and we denote this by  $N \subseteq M$ . If  $N \subseteq M$  but  $N$  is different from  $M$  (that is, if there is at least one element of  $M$  that does not belong to  $N$ ), we say that  $N$  is a *proper subset* of  $M$ , and write  $N \subset M$ <sup>13</sup>.

For example, let  $M = \{1,2,4,7,9\}$  and  $N = \{2,7\}$ . By using Venn diagrams, we can picture that  $N \subset M$  as follows

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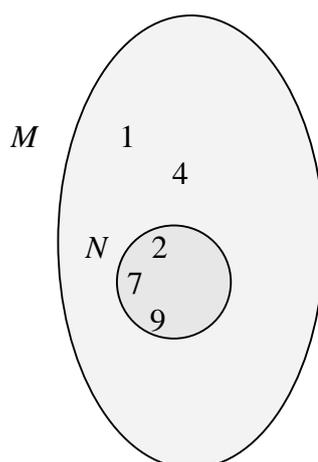
<sup>9</sup>Benacerraf (1965) develops a now classical argument concerning the issue of whether numbers can be sets or not.

<sup>10</sup>However, within set-theoretic foundations, one can make sense of this assertion: for example, for the set  $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, \dots\}$  one can simply *define* or reduce the relation " $n$  is less than  $m$ " to the assertion " $n \in m$ ". In this way one introduces an order among the elements of "the" set of natural numbers.

<sup>11</sup>Leinster (2014), besides giving a concise introduction to the category of sets, also presents in a straightforward manner some of these criticisms.

<sup>12</sup>See, e.g., McLarty (2017), especially pp. 11ff.

<sup>13</sup>The concept of *proper subset* allows us to define a relation of order among the elements of the set  $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}, \{\{\emptyset, \{\emptyset\}\}\}, \dots\}$ : we say that  $m < n$ , if and only if  $m \subset n$ .



Suppose now that  $M$  is a non-empty set. Then clearly, for each  $x \in M$ ,  $x$  is an element of  $M$ . Hence,  $M \subseteq M$ . Now, in order for the empty set to *not* be a subset of a given set  $M$  (whether  $M$  is empty or not), the empty set must, according to the above definition, have at least one element that does not belong to  $M$ . But this is clearly not possible: by definition, the empty set does not have any elements. So, we arrive at the following

### Theorem

For any given set  $M$ ,  $\emptyset \subseteq M$ .

□

As a particular case, we have the following

### Corollary

$\emptyset \subseteq \emptyset$ .

□

Thus, *any* given set  $M$  distinct from the empty set has at least *two different* parts or subsets:  $M$  itself and the empty set  $\emptyset$ . And, if  $M$  is the empty set, then it has just one part:  $M$  itself. We now come to our last axiom, the axiom of separation or the subset axiom. In *Zermelo 1908* this is phrased as follows:

All elements of a set  $M$  that have a property  $F$  well-defined for every single element are themselves the elements of another set,  $M_F$ , a “subset” of  $M$  (Zermelo 1908, p. 183).

So, this is an axiom linking the process of forming new sets with (well-defined) *properties*<sup>14</sup>. For his second formulation of the axiom, *Zermelo 1908a* introduces the following definition:

<sup>14</sup>In Fraenkel et al. (1973), the authors assert that the axiom of separation “turned out to be inconsistent and therefore cannot be used as an axiom of set theory. However, since this axiom is so close to our intuitive concept of set we shall try to retain a considerable number of [its] instances”

A question or assertion  $F$  is said to be *definite* if the fundamental relations of the domain, by means of axioms and the universally valid laws of logic, determine without arbitrariness whether it holds or not. Likewise a “propositional function”  $F(x)$ , in which the variable  $x$  ranges over all individuals of a class  $R$ , is said to be definite if it is definite for *each single* individual  $x$  of  $R$ . Thus, the question whether  $a \in b$  or not is always definite, as is the question whether  $M \subset N$  or not.<sup>15</sup>

The second formulation of the axiom in *Zermelo 1908a* is then as follows:

*Axiom III (Axiom of Separation or Subset Axiom)*

Whenever the propositional function  $F(x)$  is definite for all elements of a set  $M$ ,  $M$  possesses a subset  $M_F$  containing as elements precisely those elements  $x$  of  $M$  for which  $F(x)$  is true (Zermelo 1908a, p. 201).

Zermelo then observes

[...] In the first place, sets may never be *independently defined* by means of this axiom but must always be *separated* as subsets from sets already given; thus contradictory notions such as “the set of all sets” or “the set of all ordinal numbers” [...] are excluded. In the second place, moreover, the defining criterion must always be definite in the sense of our definition [...] (that is, for each single element  $x$  of  $M$  the fundamental relations of the domain must determine whether it holds or not), with the result that, from our point of view, all criteria such as ‘definable by a finite number of words’ hence the ‘Richard antinomy’ [...] vanish.<sup>16</sup>

With this axiom, Zermelo had then two aims: on the one hand, to restrict separation to separation within an already given set in order to exclude totalities such as the set of all sets that are not members of themselves (a totality that leads to the well-known Russell’s paradox); and, on the other hand, to exclude “illegitimate” properties such as “ $x$  is a number definable in a finite number of words in natural language” (a property that leads to the Richard’s paradox)<sup>17</sup>. Indeed, as a direct application of this axiom, Zermelo proves the following

**Theorem**

Every set  $M$  possesses at least one subset  $M_0$  that is not an element of  $M$ .

*Proof.* For every element  $x$  of  $M$  it is always definite whether  $x \in x$  or not (at least the axioms do not exclude the possibility that there may be a set  $x$  such that  $x \in x$ ). The negation  $x \notin x$  is therefore also a definite propositional function. By the axiom of separation, the collection  $M_0 = \{x \in M \mid x \notin x\}$  is therefore a set. We have then

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(p. 32)—where the instance the authors used for deriving a contradiction was the (non-definite!) property of not being a member of itself:  $x \notin x$ . My point is, however, that even those dismissing Zermelo’s notion of definiteness, they consider it essential to the concept of set itself that it is tightly connected with *properties* of its elements.

<sup>15</sup>Zermelo (1908a, p. 201), emphasis in the original.

<sup>16</sup>*Loc.cit.*, emphases in the original.

<sup>17</sup>See Ebbinghaus’ introductory note to Zermelo (1929) in Zermelo (2010, pp. 352–357). Concerning Richard’s paradox, see Taylor (1993, pp. 547–549).

that either that  $M_0 \in M_0$  or  $M_0 \notin M_0$ . If  $M_0 \in M_0$ , then  $M_0$  must satisfy the propositional function  $x \notin x$  that characterizes all the elements of  $M_0$  and this means that the assertion  $M_0 \notin M_0$  is true. But this contradicts our hypothesis that  $M_0 \in M_0$ . We conclude then that  $M_0$  is *not* an element of  $M_0$ . If, finally,  $M_0 \in M$  then, then  $M_0$  would satisfy the condition (given by the propositional function  $x \notin x$ ) characterizing all the elements of  $M_0$  and would thus belong to  $M_0$ , a possibility which we have already excluded. Therefore,  $M_0$  is a subset of  $M$  such that  $M_0 \notin M$ .  $\square$ .

Zermelo then argues that this theorem implies that *not* all objects of the domain of individuals or objects that set theory is about, can be elements of one and the same set. In other words, the theorem implies that this domain is *not* itself a set, so that there is no such thing as the set of all sets. With this result at hand, Zermelo manages to exclude the existence (as a set *of the theory*) of the collection of all sets that are not members of themselves, which is the starting point of the Russell paradox. As Zermelo points out, and except for the case of certain axioms of existence (including the three ones previously given here), “sets [...] must always be *separated* as subsets from sets already given”<sup>18</sup>, and this separation is to be carried out by means of certain properties.

Besides assertions of the form  $x \in y$ ,  $x = y$ ,  $x \notin y$  and  $x \neq y$ , we may safely conclude from other applications Zermelo gives of the subset axiom, that at least the following are also *definite* propositional functions that we may use in order to form new sets out of previously given sets: if  $F(x)$  and  $G(x)$  are definite propositions, so are their corresponding negations  $\neg F(x)$  and  $\neg G(x)$ , their conjunction  $F(x) \wedge G(x)$  and their disjunction  $F(x) \vee G(x)$ . So the property of being *definite* is closely connected to logic at a syntactical level. But it is also connected to logic (to *classical* logic, as it turns out) at a semantical level. For according to Zermelo an assertion is definite if “the fundamental relations of the domain, by means of the axioms and the universally valid laws of logic, determine without arbitrariness whether *it holds or not*”<sup>19</sup>. Thus, if  $F(x)$  is a definite propositional function and  $M$  is a given set then, for each element  $x$  of  $M$ , the assertion “ $F(x) \vee \neg F(x)$ ” is always true. Or, in other words, if  $F(x)$  is a well-defined property of  $x$ , then either  $x$  has that property  $F$  or it does not.

To say of these definite or well-defined properties that the laws of logic determine whether they hold or not, is to say at least that such properties are *bivalent*: if  $F$  expresses a property meaningful for all elements  $x$  of a given set  $M$  then, either the assertion  $F(x)$  is true or it is false and never both. In logic, if an assertion  $F(x)$  is true (or false, as the case may be) we say of  $F(x)$  that its *truth value* is the value *true* (or, accordingly, the truth value *false*).

In Fraenkel AA, Bar-Hillel Y, Levy A (1973) the authors give a succinct account of what they see is the problem with Zermelo’s notion of definite

<sup>18</sup>Zermelo (1908a, p. 202), emphasis in the original.

<sup>19</sup>*Loc.cit.*, my emphases.

property<sup>20</sup>. They first give the following reformulation of Zermelo's axiom of separation

For any set  $a$  and any condition  $B(x)$  on  $x$  there exists the set that contains just those members of  $a$  which fulfil the condition  $B(x)$  (Fraenkel et al. 1973, p. 36).

The authors then explain why, in spite of appearances to the contrary, this formulation is different from Zermelo's. In the first place, they say, the notion of a 'condition  $B(x)$  on  $x$ ' is a well-defined notion, for they have previously defined what is called in logic the 'object language' in which these conditions on  $x$  are to be expressed. This language is, of course, a first-order language with equality and the basic notion of membership. It is a language with precise rules for forming "conditions on  $x$ " out of the basic membership relation with the aid of the logical connectives and the quantifiers and in such a way that given any finite sequence of symbols of the language, one can always determine whether it is well-formed or not, that is to say, whether it is in fact a condition on  $x$ <sup>21</sup>. Zermelo did not make a distinction between the object language and what is called the metalanguage (which is the natural language one uses for expressing or explaining axioms, theorems, and so forth). This is one of the reasons why he rejected solutions such as the one proposed by these authors, for the well-formed expressions in the object language are always *finite* sequences of symbols, and Zermelo thought that one of the purposes of set theory was precisely to give an account of the concept of (finite) number (see Zermelo 1929, pp. 359, 363). Thus, the authors continue, since "Zermelo did not have any particular object language in mind [...] his notion of a statement  $B(x)$  was quite vague" (Fraenkel et al. 1973, p. 37). Once one is clear about the object language, according to them one can then interpret Zermelo as follows:

What Zermelo meant by saying that the truth or falsity of [a statement]  $B$  is determined by the primitive relations of the system is [...] that once the primitive relation of the system (namely, the membership relation) is "given" then the very meaning of  $B$  makes it either true or false. Using modern terminology we can say that  $B$  is definite if it belongs to a formal system with an interpretation which makes  $B$  true or false; likewise,  $B(x)$  is definite for a class  $R$  of objects of the system if  $B(x)$  belongs to an interpreted formal system which makes  $B(x)$  true or false for every member of the class  $R$ . (*Loc.cit.*)

If Zermelo did indeed not make a clear distinction between the object language and the metalanguage, it seems difficult to interpret him as in the above passage. In the final section, I will present Zermelo's own account of the concept of definite property, an account which he gave in 1929 and in response to the criticisms his 1908a version received. Those criticisms amount basically to claiming that his notion of definiteness is imprecise, that it lacks in mathematical rigor. Let it suffice it for the moment to say that the three mentioned authors

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<sup>20</sup>For a thorough historical account of Zermelo's notion of *definiteness*, see Ebbinghaus (2003).

<sup>21</sup>For an historical account of the relationship between logic and axiomatic set theory, see Moore (1980).

thought their version was similar to, although not identical with, Zermelo's (1929) new formulation of the concept of definiteness (*Loc.cit.*, fn. 2).

The current situation concerning the presentation of the axiom of separation in introductory textbooks may be divided into two broad groups: those who simply leave the concept of (well-defined) property unanalyzed, and those along the lines proposed by the Norwegian mathematician Thoralf Skolem in 1922 (Skolem 1922). After introducing the symbolism for three logical connectives or operations of negation, conjunction and disjunction together with the two quantifiers (existential  $\exists$  and universal  $\forall$ ), Skolem makes the following definition:

By a definite proposition we now mean a finite *expression* constructed from elementary propositions of the form  $a \in b$  or  $a = b$  by means of the five operations mentioned (*Op.cit.*, pp. 292–293, italics mine).

Notice here the word 'expression' he uses. The focus is no longer on properties but on the (logical) language used to refer to them. What is usually done in modern textbooks of set theory, is to first introduce a first-order language and then define what are its (well-formed) *formulas*, which are always finite sequences of *symbols* of the language. In the case of set theory, this language includes the primitive or basic (binary) relations of equality and membership. A variable within a well-formed formula may be free or bounded by a quantifier. For example, in the *formula*  $\forall A \forall x \in A (y \in x)$  the variables  $A$  and  $x$  are bounded whereas the variable  $y$  is not, that is, the variable  $y$  is free. In this way, Zermelo's axiom of separation became an axiom *schema* with infinitely many instances, one for each formula of the first-order language with equality and membership. It is customary to indicate, for a given formula, what its free variables are by writing them between parentheses. So, for the above example, we might denote the formula by  $\varphi(y)$ . In this way, the modern version of Zermelo's axiom of separation is usually stated as follows: For any given set  $M$  and any given first-order formula  $\varphi(x)$ , the collection  $\{x \in M \mid \varphi(x)\}$  is a set. Although this version has many mathematical advantages<sup>22</sup>, it is clearly not in the semantical spirit of Zermelo's original formulation, for he was concerned with truth values, and not (so much) with the syntax of the underlying logical language or the so-called object language. Indeed, in his 1929 paper he explicitly says that,

What I intended to do was to derive the main theorems of set theory from the smallest possible number of assumptions and by means of the most restricted expedients. I recognized that in order to do so, the unrestricted use of "propositional functions" would be [...], on account of certain "antinomies" [...] dangerous. At the time, a universally acknowledged "mathematical logic" on which I could have relied did not exist—nor does it exist today when every foundational researcher has his own logistic. With my primary tasks being different, however, it hardly would have been appropriate for me to develop in extenso such a logistical foundation, particularly at a time when most mathematicians still harbored suspicions about any kind of logistic. But I believed that my explanation of the concept in question, and in particular its

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<sup>22</sup>For one thing, it no longer makes any references to other axioms.

applications, had at least made sufficiently clear how it was meant (Zermelo 1929, p. 359).

Certainly, from the applications *Zermelo 1908a* gives of the axiom of separation, it is clear at least that “definiteness” is closed under negation, conjunction and disjunction. If  $F(x)$  and  $G(x)$  are any two *definite* propositional functions, then to say that the following three are also definite propositional functions

$$\begin{aligned} &F(x) \wedge G(x) \\ &\neg F(x) \\ &F(x) \vee G(x) \end{aligned}$$

is to say that, just like  $F(x)$  and  $G(x)$ , each one has one and only one of the two truth values for each value of  $x$ : each one is either true or false and never both (for each value of  $x$ ). And to say the latter is to say that these three connectives are just being classically defined, as it is usually done by means of the so-called truth tables.

What I propose here is to show that what the categorical version of the axiom retains from Zermelo’s *semantic* formulation of the axiom in terms of definite properties, is the idea that parts of a given set  $M$  may be separated from it by means of properties, where these properties are such that the elements of  $M$  *always* either have them or don’t have them (and never both!). And that as a consequence, any one of these special properties divides  $M$  into two disjoint parts which together exhaust all of  $M$ <sup>23</sup>.

### Interlude: Functions

Just as Zermelo set theory has primitive notions, so does the category-theoretic approach to the concept of set. And just as sets are almost everywhere in mathematics, *functions* too are ubiquitous. The intuitive idea behind the notion of function involves three ingredients: two collections of “things” and a correlation between the things in one collection and those in the other one. For example, let us suppose we have a group of five students and we want to write down their final grades. The group of students is called the *domain* of the function which we might call “final grade”; we usually take as the *codomain* of this kind of functions the set of numbers from 0 to 10. The codomain is thus where the function takes its values for each element of the domain. The function “final grade” then associates to each student one *and only one* number between 0 and 10. The important point here is that *each* student in the domain gets assigned *one but only one* final grade. Notice that it may happen in one specific scenario that no student gets, for example, the number 7 as his or her final grade; in such cases we still have a function from the set of students to the set  $\{0, 1, 2, \dots, 10\}$ . If instead of the set of numbers between 0 and 10, we take a larger set of numbers, say the set of *all* natural numbers, we

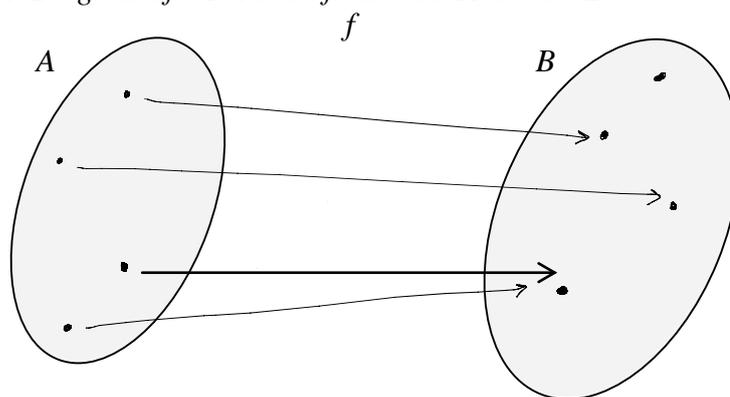
<sup>23</sup>See Figure 3 in the section on the categorical approach to sets.

can also define a function assigning final grades to students but this function would have a different codomain than the first one, and so it will be a *different* function, even though the “rule” for the assignment and the domain remain the same as in the first case. An analogous thing happens if we change the domain of the function; in such a case, one gets two different functions even if the codomain and the rule of assignment remain unchanged. So, one requirement for two given functions to be equal, is that they at least have the same domain and the same codomain. We will shortly see why this is so, but I will first give one example of a correlation that is *not* a function.

Let us consider for the domain  $D$  of our counterexample a family consisting of five members: the parents, two boys and one girl. For the codomain  $C$ , let us take the same family together with the immediate families of each one of the parents. If we assign to each person in  $D$  his or her brother, the girl in the domain will be assigned two people in the codomain  $C$ . Thus the rule “the brother of  $x$ ” is not, for this particular domain, a function: at least one element of the domain is assigned to two elements in the codomain. It could also happen that for this particular domain at least one of the parents does not have a brother, and this would be another reason for the assignment “the brother of  $x$ ” not to be a function for these particular groups of people  $C$  and  $D$ . In sharp contrast, the assignment “the father of  $x$ ” is indeed a function from  $C$  to  $D$ .

Using Venn diagrams, we may picture a function  $f$  from a set  $A$  to a set  $B$  as in Figure 1.

**Figure 1.** Venn Diagram of a Function from a Set  $A$  to a Set  $B$



And an assignment or correlation that is *not* a function from  $A$  to  $B$  could look like this

**Figure 2.** Venn Diagram of a Correlation from a Set  $A$  to a Set  $B$  that is not a Function

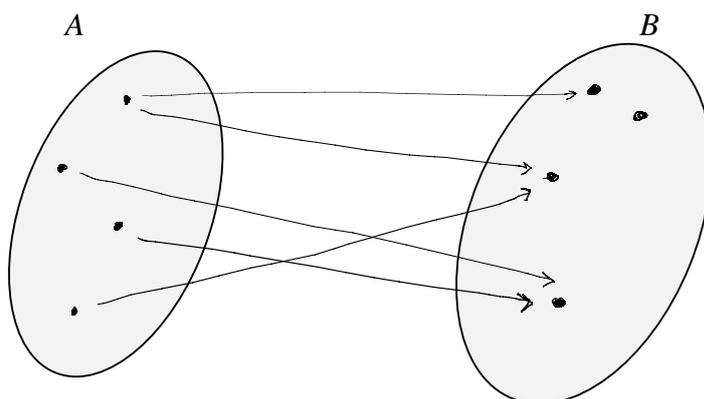


Figure 2 does not depict a function because there is (at least) one element in  $A$  that was assigned to more than one element in  $B$ , but *not* because there is an element in  $B$  that is not correlated with some element in  $A$ . The concept of function does not require that all elements of the codomain are correlated with some element of the domain.

Another way for denoting functions together with their corresponding domains and codomains is the arrow notation. So if  $f$  denotes a function with domain  $A$  and codomain  $B$ , we may write

$$f: A \rightarrow B \quad \text{or} \quad A \xrightarrow{f} B$$

If  $x$  is any given element of  $A$ , the member of  $B$  that the function  $f$  assigns to  $x$  is denoted by  $f(x)$ . Usually, when the sets involved are sets of numbers, the functions mathematicians are interested in are given in terms of arithmetical operations. So, for example, if the set  $A$  is the set of natural numbers including the number 0, and  $B$  is the set of natural numbers without the number 0, we can define the function with domain  $A$  and codomain  $B$  by simply writing

$$f(x) = x + 1.$$

This function has the property that for *each element*  $y$  in the codomain  $B = \{1, 2, 3, \dots\}$ , there is one element  $x$  in the domain  $A = \{0, 1, 2, 3, \dots\}$  such that  $f(x) = y$ :

$$1 = f(0), 2 = f(1), 3 = f(2), \dots, y = f(y - 1), \dots$$

If we now change the codomain, and take instead of  $B$  the set  $A$  but keep the same domain and the same rule of assignment, the above property  $f$  no longer holds, namely, that each element of the codomain  $\{1, 2, 3, \dots\}$  of  $f$  “comes from” (via  $f$  itself) one element of the domain  $A = \{0, 1, 2, 3, \dots\}$ . For in this new case the number 0 is an element of the codomain and there is no element  $x$  in  $A = \{0, 1,$

$2, 3, \dots\}$  such that  $x + 1 = 0$ . The property that  $f$  has is called *surjectivity* and we say of  $f$  that it is *surjective*. Thus when we changed the codomain but retained the same domain and rule of assignment, we obtained a different function, one that, unlike  $f$ , is not surjective. Figure 1 also illustrates a function that is not surjective. This distinction gets lost in the set theoretic definition of a function as we will now see.

Functions in set theory are encoded or defined in terms of *ordered pairs*. With the notion of an ordered pair, denoted by  $(a, b)$ , one can capture the idea that the element  $a$  is assigned only to the element  $b$ . If  $a$  and  $b$  are sets, then by the axiom of elementary sets,  $\{\{a\}, \{a, b\}\}$  is also set. The ordered pair  $(a, b)$  is defined precisely as the set  $\{\{a\}, \{a, b\}\}$ . It is important to notice that this latter set is different from the set  $\{a, b\}$ . The idea that “ $a$  comes first” or that the order matters, is expressed in the following theorem which is easy (but somewhat tedious) to prove with the axiom of extensionality

### Theorem

For any given sets  $a, b, c$  and  $d$ ,  $(a, b) = (c, d)$  if and only if  $a = c$  and  $b = d$ .

□.

We can now capture the idea that each element  $x$  of the domain of a function gets assigned to one *and only one* element of the codomain in the following terms: if  $(x, b) = (x, d)$ , then  $b = d$ . By means of an axiom called the Union Axiom<sup>24</sup> and the Power Set Axiom<sup>25</sup>, it can be proved that, if  $A$  and  $B$  are sets, then the collection  $\{(x, y) \mid x \in A \text{ and } y \in B\}$  is a set. Thus a function, say  $F$ , is defined in set theory as a certain set of ordered pairs, for instance  $F \subseteq \{(x, y) \mid x \in A \text{ and } y \in B\}$ , such that for any  $x \in A$ , if both  $(x, b)$  and  $(x, d)$  are elements of  $F$ , then  $b = d$ .

We saw earlier that the function  $f: \{0, 1, 2, \dots\} \rightarrow \{1, 2, 3, \dots\}$  given by  $f(x) = x + 1$ , is surjective, whereas the function  $g: \{0, 1, 2, \dots\} \rightarrow \{0, 1, 2, \dots\}$  given by the same rule, that is, by  $g(x) = x + 1$ , is not. However, according to the set-theoretic definition of function, both functions  $f$  and  $g$  are encoded in one and the same set, namely, the set

$$\{(0,1), (1,2), (2,3), \dots, (x, x+1), \dots\}$$

So the set-theoretic account of the concept of function misses an important ingredient of it. As we will see, this is a point that the categorical approach emphasizes: every function comes equipped with both a specific domain and a specific codomain. We saw earlier that the empty set is a subset of *any* set. So, if  $A$  and  $B$  are given sets,  $\emptyset \subseteq \{(x, y) \mid x \in A \text{ and } y \in B\}$ , and the empty set vacuously satisfies the condition for being a function from  $A$  to  $B$  for *any* sets  $A$  and  $B$ . This is an extreme example that shows how the definition of the concept of function as a certain subset of ordered pairs neglects the importance of taking into account what

<sup>24</sup>This axiom states that for any given set  $M$  the collection of all the elements of the elements of  $M$  is also a set.

<sup>25</sup>This postulate guarantees the existence, for any set  $M$ , of the set of all of the subsets of  $M$ .

the domain and the codomain are: the empty set is a function from *any* set to *any* set.

There is another significant property a function may have. Consider the following two sets:

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

and let  $f: A \rightarrow B$  and  $g: A \rightarrow B$  be defined as follows:  $f(1) = f(2) = 3$ ,  $g(1) = 3$  and  $g(2) = 4$ . One difference between these two functions is that  $f(1) = f(2)$  even though  $1 \neq 2$ , whereas  $g(1) \neq g(2)$ . In other words, for all elements  $x$  and  $y$  in the domain  $A$ , if  $x \neq y$  then  $g(x) \neq g(y)$ . This property is called *injectivity*. So the function  $f$  is *not* injective. A function that is both injective and surjective is called *bijective*. Thus the function  $g$  is bijective. Using the same sets  $A$  and  $B$  from this example, we can define another bijective function: let  $h: A \rightarrow B$  be given by  $h(1) = 4$  and  $h(2) = 3$ . Figure 1 is an example of a function that is not injective.

Bijective functions are quite useful in many ways. For instance, they allow us to determine, without counting, whether any two given sets have the same number of elements. A striking example is given by the set  $N$  of natural numbers  $\{0, 1, 2, \dots\}$  and the set  $E$  of even numbers  $\{0, 2, 4, \dots\}$ . Let  $f: E \rightarrow N$  be given by  $f(x) = x \div 2$ . This is indeed an injective function (recall that all basic arithmetical operations are functions). Let, then,  $y \in N$ . In order to show that  $f$  is surjective we need to find an element  $x$  of  $E$  such that  $f(x) = y$ . Well, since  $y \in N$ , then  $2y$  is an even number, and so it is an element of  $E$ . Moreover,  $f(2y) = 2y \div 2 = y$ . And this shows that  $f$  is also surjective. So, despite appearances to the contrary, there are as many natural numbers as there are even numbers.

But bijective functions are also important from an epistemological point of view, a fact that we will use extensively when presenting the categorical version of the axiom of separation. It often happens, like in the above example, that the bijective function is given or defined in such a way that it allows to *construct* an object or element of the codomain *from* an element of the domain, *and vice versa*, even when these elements are not numbers but more complicated mathematical objects. And this in turn allows to rethink or reconceptualize a mathematical object, sometimes in simpler ways that are more amenable to mathematical calculations. In this way, one can move back and forth from two different ways of conceiving of some mathematical object, according to one's needs.

Let us consider now an arbitrary set  $A$ . We can always define a function with domain and codomain the set  $A$  itself, namely, the function  $f: A \rightarrow A$  given by  $f(x) = x$  for all  $x \in A$ <sup>26</sup>. This function is called *identity on A* and it is usually denoted by " $id_A$ ". And given any two functions where the codomain of one is the same as the domain of the other function

$$f: A \rightarrow B \text{ and } g: B \rightarrow C$$

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<sup>26</sup>If the set  $A$  is empty, then this is vacuously true.

we can always define another function, called their *composition*, with domain  $A$  and codomain  $C$ . For suppose that  $x \in A$ . Therefore  $f(x) \in B$  and hence we can apply  $g$  to this element of  $B$ ,  $g(f(x))$ , obtaining in this way an element of  $C$ :

$$\begin{aligned} gf: A &\rightarrow C \\ fg(x) &= g(f(x)) \in C \end{aligned}$$

For example, let us consider again the set of natural numbers  $N = \{0, 1, 2, 3, \dots\}$  and the set  $E$  of even numbers  $\{0, 2, 4, 6, \dots\}$ . Let  $f: N \rightarrow E$  and  $g: E \rightarrow N$  be defined as follows for each  $x \in N$  and each  $y \in E$

$$f(x) = 2x \quad \text{and} \quad g(y) = y + 3$$

Let us now take, say, the element  $7 \in N$ . Then

$$f(7) = 14 \quad \text{and hence} \quad g(f(7)) = g(14) = 14 + 3 = 17.$$

More generally, for any  $x \in N$ ,  $fg(x) = 2x + 3$ . And this is certainly a function from  $N$  to  $N$  since there is no natural number  $x$  for which  $2x + 3$  would give us two *different* results.

Let us again consider two arbitrary functions  $f: A \rightarrow B$  and  $g: B \rightarrow C$  and suppose we also have a third one, say  $h: C \rightarrow D$ , where  $D$  is just another arbitrary set. Since the codomain of the composition  $gf: A \rightarrow C$  is the same as the domain of  $h$ , we can again compose these two functions

$$h(gf): A \rightarrow D.$$

But we can also consider the composition of just  $g: B \rightarrow C$  with  $h: C \rightarrow D$ :

$$hg: B \rightarrow D.$$

And we can now compose  $f: A \rightarrow B$  with  $hg: B \rightarrow D$ , thus obtaining another function from  $A$  to  $D$ :

$$(hg)f: A \rightarrow D.$$

When the sets involved are sets of numbers and the functions are defined by the usual basic arithmetical operations,  $h(gf): A \rightarrow D$  and  $(hg)f: A \rightarrow D$  are the same function. However, when one takes the concept of *function* as primitive (as it is done in the categorical approach), the equality  $h(gf) = (hg)f$  must be postulated as an axiom. The equality  $h(gf) = (hg)f$  is called *associativity of composition*.

Finally, let  $A$ ,  $B$  and  $C$  be given sets, and let  $id_B: B \rightarrow B$ ,  $f: A \rightarrow B$  and  $g: B \rightarrow C$ . We then have the compositions

$$(id_B)f: A \rightarrow B \text{ and } g(id_B): B \rightarrow C.$$

Consider now an arbitrary  $x \in A$  and an arbitrary  $y \in B$ . Then, by the definition of identity functions, we have the following equalities

$$(id_B)f(x) = f(x) \text{ and } g(id_B(y)) = g(y).$$

That the equalities  $(id_B)f = f$  and  $g(id_B) = g$  actually hold, is also postulated as an axiom in the categorical approach. Existence of composition (as long as the codomain of one function coincides with the domain of the other one), existence of identities for every set, associativity of composition, and these two last equalities constitute the basic axioms when the concept of function is taken as primitive. As we shall see in the next section, there are other axioms that characterize the category of sets (and thus distinguish it from other categories). But before presenting the categorical approach, we need to look at a few more special kinds of functions.

Let us consider an arbitrary set  $A$ , a set with just one element, for example, the set  $\{\emptyset\}$ , and a set with exactly two elements, for instance, the set  $\{\emptyset, \{\emptyset\}\}$ . Let us call ' $t$ ' any one of the elements of the set  $\{\emptyset, \{\emptyset\}\}$ . For example, we can let  $t = \emptyset$ . We can now define a function  $f$  with domain  $A$  and codomain the one-element set  $\{t\} = \{\emptyset\}$  by simply letting  $f(x) = t$  for all  $x \in A$ . Moreover, this is the *only* function from  $A$  to the set  $\{t\}$  since there are no other elements in  $\{t\}$  that we can assign to the elements of  $A$ . This result is in fact more general: for *any* one-element set and any given set  $A$ , there is always one *and only one* function from  $A$  to the one-element set.

Let us now consider functions from the one-element set  $\{t\}$  to  $A$ . We have that, for each  $x \in A$ , we can define a function  $f_x: \{t\} \rightarrow A$  by letting  $f_x(t) = x$ . But conversely too. Each function  $g: \{t\} \rightarrow A$  determines a unique element of  $A$ , namely,  $g(t) \in A$ . This *bijective* correspondence between functions from a one-element set to  $A$  and elements of  $A$ , is the point of departure for reconceptualizing, in categorical terms, the notion of element, as we will see in the following section.

Suppose now that we are given a function  $f$  from  $A$  to the two-element set  $\{t, \{\emptyset\}\}$ . By Axiom III, the collection of all  $x \in A$  such that  $f(x) = t$  is a subset of  $A$ , even when there is no  $x \in A$  such that  $f(x) = t$ , for in this case the subset of  $A$  in question would be the empty set. At the other extreme, if  $f(x) = t$  for all  $x \in A$ , then the subset in question would be  $A$  itself. Intuitively, all the cases in between these two extremes, should give us *all* of the subsets of  $A$ . In other words, there is a close connection between *all* functions from  $A$  to the two-element set  $\{t, \{\emptyset\}\}$  and subsets of  $A$ . Indeed, given a subset  $S$  of  $A$ , we can always define a function  $f_S$  from  $A$  to the set  $\{t, \{\emptyset\}\}$ : for any given  $x \in A$ , we simply let  $f_S(x) = t$  if and only if  $x \in S$ . This (bijective!) correspondence between subsets of  $A$  and functions from  $A$  to a two-element set is at the heart of the categorical version of the axiom of separation, as we will shortly see.

There is one more relationship we need to look at between the concept of function and the notion of subset. Consider any two sets  $A$  and  $B$ . If  $A$  is a subset

of  $B$ , we can always define a function  $i: A \rightarrow B$  by simply letting  $i(x) = x$  for all  $x \in A$ . And this function is clearly injective. But moreover, if we are given an injective function, say  $j: C \rightarrow B$ , then it determines (by Axiom III) a subset of  $B$ , namely the set

$$\{y \in B \mid \text{there is } x \in C \text{ such that } y = j(x)\} = \{j(x) \in B \mid x \in C\}.$$

Thus we have a correspondence between, on the one hand, subsets of a given set and, on the other hand, injective functions with codomain the set in question<sup>27</sup>. And this correspondence is bijective. It will be useful to keep in mind this observation when we come, in the next section, to the definition of *subset* in the categorical approach.

### The Category-theoretic Approach to Sets

Category theory is a branch of mathematics that is nowadays at the forefront of developments in the foundations of mathematics<sup>28</sup>. It is a highly abstract theory so I hope this section can also serve the purpose of giving a first approximation to it.

As mentioned earlier, the categorical approach starts from axiomatizing the concept of *function* or, as I will call it from now on, *map*. Maps always come “attached” to two objects<sup>29</sup>, one called its *domain* and the other one its *codomain*. So the concepts of domain and codomain are also primitive. Certainly, any given object can be the domain of one map but the codomain of another map. The point is that the properties of these domains and codomains will be given, not in terms of their elements, but in terms of maps from and to them<sup>30</sup>. As it is usual in category theory, either notation:

$$f: A \rightarrow B \quad \text{or} \quad A \xrightarrow{f} B$$

stands for “ $f$  is a map with domain  $A$  and codomain  $B$ ”.

The categorical approach to set theory starts then from what in *LR* are called *abstract sets* (or simply *sets*) and certain maps between them. The first three axioms capture the basic properties of functions presented in the previous section.

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<sup>27</sup>Notice that the set  $\{j(x) \in B \mid x \in C\}$  is a subset of  $B$ , not of  $C$ , where  $B$  is the codomain of the injective function  $j: C \rightarrow B$ .

<sup>28</sup>The title of Landry (2017) is a good testimony to this. It also contains a wealth of references concerning relatively recent debates on categorical foundations for mathematics.

<sup>29</sup>The word ‘object’ in this context is a technical term referring exclusively to the domains and codomains of maps or, as they are called in general, ‘morphisms’ of a category.

<sup>30</sup>In fact, not all categories are categories *of sets*. For example, given *any* category, one can always form the category whose objects are the maps or, more generally, morphisms of the given category.

*Axiom (Existence of Composition and Identity Maps)*

Every map has associated a set called its domain and a set (not necessarily distinct from the domain) called its codomain.

For any given maps  $f: A \rightarrow B$  and  $g: B \rightarrow C$ , there is a map  $gf: A \rightarrow C$  called their *composition* (there may of course be other maps from  $A$  to  $C$  besides the composition map  $gf$ ).

For any given set  $A$ , there is a map  $id_A: A \rightarrow A$ , called the *identity map on  $A$*  (besides this identity map, there may also be other maps from  $A$  to  $A$ ).

*Axiom (Associativity of Composition)*

For any given maps  $f: A \rightarrow B$ ,  $g: B \rightarrow C$  and  $h: C \rightarrow D$ ,  $(hg)f = h(gf)$ .

*Axiom (Identities are Neutral with Respect to Composition)*

For any given map  $f: A \rightarrow B$ ,  $f(id_A) = f$  and  $(id_B)f = f$ .

*Axiom (Terminal Set)*

There is a set, denoted by  $1$ , with the property that for any given set  $A$  there is one *and only one* map  $A \rightarrow 1$  from  $A$  to  $1$ . This special map is denoted by the symbol  $!_A$ .

The set-theoretic counterparts of this set  $1$  are, as we saw in the previous section, the one-element sets. In that section we also saw that there is a bijective correspondence between functions from one-element sets to an arbitrary set  $A$  and elements of  $A$ . So we have the following

*Definition (Elements of Sets)*

Let  $A$  be an arbitrary set. Then an *element* of  $A$  is a map  $1 \rightarrow A$ <sup>31</sup>.

By the definition of the set  $1$ , there is only one map from  $1$  to itself, the special map denoted by  $!_1; 1 \rightarrow 1$ . But by an axiom, there is also the identity map  $id_1: 1 \rightarrow 1$  from  $1$  to itself. So, the special map  $!_1$  and the identity map  $id_1$  must be the same. In this way we arrive at our first theorem:

**Theorem**

The set  $1$  has exactly one element.

□.

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<sup>31</sup>One interesting consequence of this definition, which I hope can serve as a motivation for the concept defined below of ‘membership in a part of a set’, is that if  $A$  and  $B$  are different, then they cannot have any elements in common: for if  $a: 1 \rightarrow A$  is an element of  $A$ , it cannot be equal to any element  $1 \rightarrow B$  because the codomains of these maps are, by hypothesis, different from each other—recall that a necessary condition for two functions to be equal is that they at least have the same domain and the same codomain.

For the counterpart of *subset* in the category-theoretic approach, we need the following definitions:

*Definition (Monomappings)*

Let  $A$  and  $S$  be given sets and let  $m: S \rightarrow A$  be a given map. Then  $m$  is a *monomapping* if the following holds: for any given set  $T$  and any given maps  $f_1: T \rightarrow S$  and  $f_2: T \rightarrow S$

$$\text{if } mf_1 = mf_2 \text{ then } f_1 = f_2.$$

If in the above definition we take  $T$  as the set  $1$ , we arrive at the categorical version of *injective function*. The generality of the definition is due to the fact that concepts in category such as that of *monomapping* are defined for arbitrary categories, so in particular for categories that may not have a terminal set or, even more generally, that are not about sets at all.

In the previous section we saw that for any given set  $B$  there is a bijection between its subsets and injective functions with codomain  $B$ . We now make use of this bijection to reconceptualize in terms of maps the notion of subset.

*Definition (Parts of Sets)*

Let  $A$  be a given set. Then a *part* of  $A$  is a monomapping with codomain  $A$ .

Since identity maps are monomappings, every set is a part of itself.

For any given set  $A$ , maps in general with codomain  $A$  are called *generalized elements* of  $A$  and they contrast with what are called *global elements* of  $A$ , that is, maps with domain a terminal object and codomain  $A$ . The definition of *membership in a part* (like the definition of monomapping), in *LR* is given for generalized elements, partly because it is meant to apply in arbitrary categories. However, and for the purposes of keeping close the analogy with Zermelo's axioms, I will confine the following definition to the case of maps with domain the terminal object  $1$ .

*Definition (Global Membership in a Part)*

Let  $A$  and  $U$  be given sets,  $a: 1 \rightarrow A$  an element of  $A$  and  $m: U \rightarrow A$  a part of  $A$ . Then we say that  $a$  is a *member* of the part  $m$  of  $A$ , if there is an element of  $U$

$$m_a: 1 \rightarrow U$$

such that the composition  $mm_a: 1 \rightarrow A$  is equal to the map  $a: 1 \rightarrow A$ .

When an element  $a: 1 \rightarrow A$  is a member of a part  $m: U \rightarrow A$  of  $A$ , we write  $a \in_A m$ . We could think of the above definition as saying that if the element  $a: 1 \rightarrow A$  of  $A$  is a member of the part  $m: U \rightarrow A$  of  $A$ , then there is an element  $m_e: 1 \rightarrow U$

of  $U$  that  $m$  interprets as the element  $a: 1 \rightarrow A$  of  $A$  in the sense expressed by the equality  $mm_a = a$ . The map  $m_a: 1 \rightarrow U$  may also be thought of as a “witness” of the fact that  $a: 1 \rightarrow A$  is indeed a member of  $m: U \rightarrow A$ .

In order to be able to incorporate Zermelo’s semantic notion of “definite property” *within* the theory, that is to say, without appealing to syntactic notions such as “expressions in a first-order language”, we need the following

*Axiom (Truth-value Set)*

There is a set, denoted by  $2$ , with exactly two elements,  $t: 1 \rightarrow 2$  and  $b: 1 \rightarrow 2$ <sup>32</sup>.

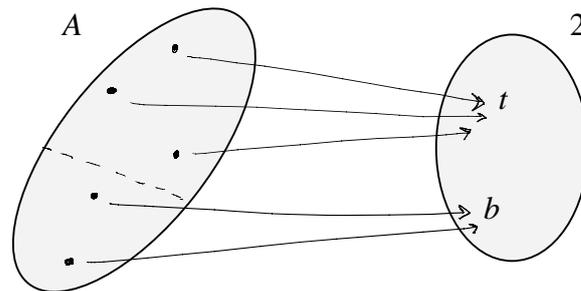
The key idea is to now think of the elements of  $2$  as truth-values; that is why one of them (it really does not matter which one) is usually called  $t$  for “true”. Now, in order to capture the bivalence of Zermelo’s definite properties, we make the following

*Definition (Properties of Elements of a Set)*

Let  $A$  be a given set. Then a *property* of elements of  $A$  is a map from  $A$  to  $2$ .

Notice that what this definition implies is what Zermelo’s definite properties *do*: they divide a set into two mutually disjoint parts which together exhaust the whole set; one part consists of those elements of the set that are assigned the map or value  $t$ , that is, those that do have the property in question, and those elements that are assigned the other element or value  $b: 1 \rightarrow 2$  of  $2$ , that is, those elements that do not have the property in question, as the following diagram illustrates

**Figure 3.** Venn Diagram of a Property of Elements of a Set  $A$



Given a set  $A$ , an element  $x: 1 \rightarrow A$  and a property  $\varphi: A \rightarrow 2$  of elements of  $A$ , to say now that  $x$  has property  $\varphi$  is to say that the composition  $\varphi x: 1 \rightarrow 2$  is equal to the element “true”  $t: 1 \rightarrow 2$  of  $2$ . By simply being maps with codomain this special object with exactly two elements, it is always definite *tout court* whether any given element of the domain has or does not have the property in question. More precisely, the composition

<sup>32</sup>The formulation in *LR* of this axiom is different. I changed it since the original one requires more concepts than are necessary for the exposition. See *LR*, pp. 19, 27–28.

$$\varphi x: 1 \rightarrow 2$$

has the same domain and the same codomain as the only two elements of the set 2. So the map  $\varphi a: 1 \rightarrow 2$  must be equal to one and only one of these two elements of the set 2. In other words, it must be equal either to the truth value *true* or to the truth value *false*. In the first case, we will say that the element  $a: 1 \rightarrow A$  of  $A$  has the property  $\varphi: A \rightarrow 2$ , and in the other case we will say that it does not.

As we saw in the previous section, there is a bijection between subsets of any given set  $M$  and functions from  $M$  to any two-element set, functions which now in this context correspond to properties of elements of  $M$ . We will now characterize this bijection in more precise terms. Consider then a part  $i: U \rightarrow A$  of  $A$  and a property  $\varphi: A \rightarrow 2$  of elements of  $A$ . What we want to capture *solely* in terms of maps is the idea that  $a: 1 \rightarrow A$  is a member of the part  $i: U \rightarrow A$  if and only if  $a: 1 \rightarrow A$  has the property  $\varphi: A \rightarrow 2$ . And to say in terms of maps that  $a: 1 \rightarrow A$  has the property  $\varphi: A \rightarrow 2$  is to say that the composition  $\varphi a: 1 \rightarrow 2$  is equal to the truth value *true*  $t: 1 \rightarrow 2$ . In other words, we simply say that the equality  $\varphi a = t$  holds, which in turn we can rephrase by saying that the statement “ $a$  has property  $\varphi$ ” is true. The following definition is given in *LR* in terms of generalized elements but, again only for the purposes of this discussion, I give it only terms of global elements. The definition is the category-theoretic way of expressing that “the element  $a$  of  $A$  has a certain property  $\varphi$  if and only if  $a$  is a member of the part  $i: U \rightarrow A$ ”.

### *Definition*

Let  $A$  be a given set,  $i: U \rightarrow A$  a part of  $A$  and  $\varphi$  a map from  $A$  to the set 2. We say that  $\varphi: A \rightarrow 2$  is a *characteristic function* of the part  $i: U \rightarrow A$  if for any element  $a: 1 \rightarrow A$  of  $A$ , the following holds

$$a \in_A i \text{ if and only if } \varphi a = t$$

In other words, a map or property  $\varphi: A \rightarrow 2$  is a characteristic function of a part  $i: U \rightarrow A$  of  $A$  if, for any element  $a: 1 \rightarrow A$  of  $A$ ,  $a \in_A i$  if and only if  $a$  has the property  $\varphi$ . We can now state the category-theoretic version of Zermelo’s axiom of separation.

### *Axiom (Properties and Parts of Sets)*

Let  $A$  be a given set. Then any property  $\varphi: A \rightarrow 2$  of elements of  $A$  is the characteristic map of a part of  $A$ . And any part of  $A$   $i: U \rightarrow A$  has a unique characteristic map  $\varphi_i: A \rightarrow 2$ <sup>33</sup>.

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<sup>33</sup>Given a property  $\varphi: A \rightarrow 2$ , the part of  $A$  corresponding to it is sometimes denoted as  $\{x \mid \varphi\}$  (see, e.g., Goldblatt 1984, p. 107).

This axiom expresses *exclusively* in terms of the semantic notion of map, the bijection that, as we saw, exists (via the axiom of separation) within Zermelo's set theory between subsets of a given set and functions from it to a two-element set. It thus establishes a clear and close connection between properties of elements of a set with its subsets or parts, the connection I believe Zermelo intended to express with his notion of *definite property*.

The axiom that characterizes the particular set 2 is a special case of a more general concept called *subobject classifier* which some categories have and some do not. I am not claiming that this concept arose with the purpose of clarifying Zermelo's notion of definite property, it did not (see e.g., McLarty 1990). What I am claiming is that by taking seriously Zermelo's emphasis on the semantic aspect of the axiom of separation, one can see the particular subobject classifier:

$$1 \xrightarrow{t} 2 \xleftarrow{b} 1$$

as removing the imprecision in Zermelo's notion of definite property while at the same time keeping its semantic spirit, thus showing an instance of mathematics' own process of evolution. For category theory arose almost 40 years later than Zermelo's axiomatization and it was not until 1964 when the first categorical approach to set theory was published. Moreover, the last axiom does not, unlike Zermelo's axiom of separation, make any references to a background logic nor to other axioms of the theory (except, of course, an implicit but necessary reference to the axioms characterizing the sets 1 and 2).

### Concluding Remarks

As mentioned previously, it was not until 1929 that Zermelo responded to the proposed solutions concerning the vagueness in his concept of definiteness. Regarding a solution like Skolem's, Zermelo writes:

[...] It is not concerned with the proposition *itself* but its *formation* or *generation* [...]. It *runs counter* to the nature of the axiomatic method and therefore it is really, in my opinion, just as out of place in any axiomatics [...] (Zermelo 1929, p. 363, emphasis in the original).

What Zermelo wanted was an axiomatic characterization of the concept of definiteness, where 'axiomatic' contrasts with what he called 'genetic' or 'constructive'. Genetic accounts of the notion of definiteness are precisely those that are concerned with the formation of *expressions* of an underlying object language. So what he did was to give axioms characterizing the concept of definiteness as a system closed under the logical connectives and the quantifiers. Using modern notation, this system is axiomatized as follows<sup>34</sup>:

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<sup>34</sup>I am omitting the clause concerning quantification over propositional functions since it seems unnecessary for the purposes of this discussion.

Let  $B$  be a domain and  $R$  a system of *fundamental relations* written as  $r(x, y, z, \dots)$ , where the variables  $x, y, z, \dots$  range over the domain  $B$ . Then we say of a proposition  $p$  that it is “*definite with respect to  $R$* ”, and write  $Dp$ , if the following is the case:

- I.  $Dr(x, y, z, \dots)$  for every relation  $r$  of  $R$  and any variables  $x, y, z, \dots$  ranging over the domain  $B$ . In other words, all fundamental relations are definite.
- II. If  $Dp$  holds, then also  $D(\neg p)$  holds. Definiteness is closed under negation.
- III. If both  $Dp$  and  $Dq$  hold, then so do  $D(p \wedge q)$  and  $D(p \vee q)$ . Definiteness is closed under conjunction and disjunction.
- IV. Let  $f(x, y, z, \dots)$  be a propositional function with free variables  $x, y, z, \dots$  ranging over the domain  $B$ . If  $Df(x, y, z, \dots)$  holds for all  $x, y, z, \dots$ , then so does  $D(\forall x, y, z, \dots f(x, y, z, \dots))$ . Likewise, if  $Df(x, y, z, \dots)$  holds for all  $x, y, z, \dots$ , then so does  $D(\exists x, y, z, \dots f(x, y, z, \dots))$ . Definiteness is closed for propositional functions under universal and existential quantification.
- V. If  $P$  is the system of all “*definite*” propositions, then it has no proper subsystem  $P_1$  that, on the one hand, contains all fundamental relations from  $R$ , while already including, on the other hand, all negations, conjunctions, disjunctions and quantifications of its own propositions and propositional functions. Thus  $P$  is the largest system containing all and only definite propositions. Any proposition not obtained by this inductive procedure would not count as definite<sup>35</sup>.

It is clear that this axiomatization of a system of definite proposition is the semantical counterpart of the inductive characterization of a first-order formula or expression, for we should bear in mind that Zermelo’s “pre axiomatic” notion of definite proposition is still semantic, that is, even 21 years after the publication of his “Investigations in the foundations of set theory I”, a definite proposition is for him one that is either true or false and never both (and of course, what determines what its truth value is, are the axioms and the universally valid laws of logic). His thorough reluctance to place the focus on the syntax, together with his insistence on *the* truth value a definite proposition *always* has, makes Zermelo’s axiom of separation close in spirit to its categorical formulation. It is unfortunately beyond the scope of this paper to explain how, given two properties  $\varphi, \psi: A \rightarrow 2$  of elements of  $A$ , their corresponding negations, their conjunction and their disjunction are also properties of elements of  $A$  and hence correspond to certain parts of  $A$ <sup>36</sup>.

<sup>35</sup>Before giving this axiomatization, Zermelo briefly discusses models of axiomatic systems and says that definiteness is “what is decided in every single model” (Zermelo 1929, p. 361). Although models play no role in his axiomatic characterization of the notion of definiteness, this passage supports my claim that this notion remained for him thoroughly semantic.

<sup>36</sup>Needless to say, the case of the quantifiers is even more complicated, but I believe it can be specialized to the category of sets and presented along the lines proposed here.

I suggest then that by looking at other of Zermelo's axioms from 1908 with modern mathematical tools such as category theory, one may glimpse a continuum of ideas rather than a straightforward repudiation of some of them in favor of new ones. Moreover, it seems to me that this way of looking at old mathematics may also serve the purpose of teaching and learning new mathematical concepts and theories. The case of category theory is of particular importance within philosophy for it helps us to take a general and novel look at various parts of mathematics, including set theory.

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## William Blake: Mental Slavery and his Visions of Mental Freedom

By Alice Reininger\*

*William Blake belongs to one of those visionary artists who, during his lifetime, did not receive much recognition from society, was not understood and therefore even marked as “crazy”, his art “odd”. Nevertheless, a small circle of sensitive connoisseurs favored and supported him. But fortunately, his work was not completely forgotten. It seems that today William Blake is being highly valued. An extensive exhibition of his fine, masterful artwork was shown at Tate Britain in London till February 2020. His pictorial, sensitive works are of inestimable value to us today as they address areas that are so important for the further development of mankind nowadays. For William Blake, imprisonment of a human being does not mean only the physical, as a slave, but above all the spiritual, the mental. Unfortunately, mental slavery is not recognized as such by mankind, as Blake let us know, this is influence of sly forces. From early childhood he had visions of God and angels. For him it was clear and undoubted that man should be innocent as a small child and therefore is under God’s protection. However, according circumstances emanating from negative forces, man is enthralled and drowned in it.*

**Keywords:** religion, mental slavery, mental freedom, love for mankind, salvation

### Introduction

William Blake was born in London, 28 November 1757, and was baptized in the church St. James, Piccadilly. He was the son of a craftsman, a hosier and haberdasher, the third child born into this family. From early childhood he had visions that accompanied him throughout his life. His first visions he had when he was four years old. He saw God. God put his head to the window and made the little boy screaming. But the little boy quickly understood what his task would be, which way he had to go. When, as a boy wandering around in the countryside of London, he looked up and saw a tree filled with angels. These imaginations became the inspirations of his marvelous sensitive works. Blake saw angels, he saw God, he saw the glory in his visions, but he also saw negativity, as it took over the whole of humanity. Later his wife Catherine told people when asked about her husband, that her husband is always in heaven, meaning his attention was on the one ultimate power of universe.

In 1863, Blake’s biography was published by Alexander Gilchrist, his book became the source of this outstanding artist. By that time when the book was published still some people were alive who could talk about Blake, who knew him well. Gilchrist (1942, p. 200) wrote about Blake’s early visions: “Trembling I [Blake] sit day and night, my friends are astonish’d at me, / Yet they forgive my

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wanderings. I rest not from my great task! / To open the Eternal Worlds, to open the immortal Eyes / Of Man inwards into the Worlds of Thoughts, into Eternity ...”

As soon he could hold a pencil in his small child’s hands, he began to draw, copied prints he could get. His parents supported him and later he became an engraver’s apprentice. After years of learning Blake became a painter, a poet, an engraver, a printer, last but not least: he was a prophet. In printing which he brought to perfection and excellency, he saw a medium in which he could spread his visions to humanity. He knew about the power of print media and how quickly news spread. This medium was able to bring his visionary art work among the crowd, to inspire them, that they recognize, and to warn them, and thus breakthrough for a new path, and to be freed from slavery, mentally and physically.

### **His Mission and Visions: Harsh Critics on Religion, Ancient Philosophers, Society and Arts**

William Blake is not easy to understand. He is radical. There is great beauty in his lyrics, but also simplicity, some might call them naive, but they are not. One has to deal with his work, understand it with the heart and not with the mind. Simple: to go beyond, *ultima sapientia*, to go beyond the limits of human mental limitation. Only then one can, like Blake himself, see the immeasurable glory of heaven. Human mind, the ratio, only a hinderance: “There is no natural religion ... Reason, or the ratio of all we have already known, is not the same that it should be when we know more” (Keynes 1984, p. 97).

Rationalism was an obstacle to William Blake, because there should only be one thing for man, namely: the spiritual. To what extent Jacob Boehme, Emanuel Swedenborg or other religious or occult writings influenced Blake, he left undecided. He analyzed them, criticized the texts and brought them into connection with the only truth, with the world that he saw in his visions. He knew his mission, through pictures, but also through his writings, to show humanity the right way. He recognized the power of the visual, he knew how to use it in a targeted manner. He denounced grievances: slavery in all its facets, cruelty, materialism, corruption, atheism, excesses of art, literature, etc. Blake was the prophet; he saw a religious devotion in his artistic work. He was convinced of himself, stable in himself, and therefore endured severe criticism from his contemporaries, including not being accepted by his fellow artists and being portrayed as crazy. Mind promotes ego, materialism, suffering of mankind in a neverending variety. This way of thinking is evident in the poem “London” in the volume “Songs of Experience”: “... In every cry of every Man. / In every Infants cry of fear. / In every voice; in every ban. / The mind-forg’d manacles I hear.” (Keynes 1984, p. 216).

The captivity of a human being does not mean only the physical but above all the spiritual. For him, who already had missions from God in his earliest childhood, it was clear and undoubted that man should be innocent as a small child. By circumstances, however, starting from negative forces, man is drawn into and becomes involved. “I went to the Garden of Love, / And saw what I never had seen; / A Chapel was built in the midst, / Where I used to play on the green.”

To his horror Blake had to realize that his Garden of Love had completely changed. Instead of flowers gravestones, the ground now consisting of graves and over it, in black gowns scary acting monks walking over: "... And binding with briars my joys and desires" (Keynes 1984, p. 215). Garden of Eden, paradise, was lost, Blake let us know in his poem "The Garden of Love" in the volume "Songs of Experience". He knew how much humanity longed for this circumstance of simplicity, of innocence, which he regarded as the most original state.

Religions, of whatever kind, are signs of falsehood, prison, which contributed that the innocence of humanity lost thereby. It is the priests who extend their tentacles and hinder others in their spiritual development, who contributed to the fact that this most original state was lost. However, the Christian idea, in Blake's eyes, is different from the Christian religion because religion is man-made and barely has anything, or very little, to do with the divine words from the mouth of the Most Highest of the Universe. Even the representatives of religion are also prisoners in this system who act blindly in it.

Blake is the mystic who let us see the divine, who leads us humbly and sensitive to the Most High. Blake is deeply religious; the Bible is his constant companion. For Blake the whole Bible was filled with imaginations and visions. Religion is not necessary for him. He believed in one universal religion, official church as an institution is dominant and cruel. "All Religions are One," he wrote in a series of aphorisms and etchings in 1788 (Keynes 1984, p. 98). Inner spirit is connected with the outer body. Therefore, he opposed this institution as all religions have only one source, natural religion does not exist as well. Reflecting his baptism as a newborn child he was the opinion that the priest cursed his head, because God himself laid "... his Hand on my [Blake's] Head & gives blessings to all my works ..." as he wrote from Felpham in a letter to Mr. Butts on April 25, 1803, (Gilchrist 1942, p. 162).

As classical culture meant war and materialism, these conditions were in an absolute contrast to Christian idea, the Christian attitude that Jesus Christ personally taught to man. In Jesus he saw all virtues gathered in one, existing only out of pure light. Unfortunately, these values have been lost through reason and selfishness. Blake also criticizes the attitude of people who identify themselves as "Christians", who exclude other people with regard to a dark complexion, or different culture. For these so-called Christians, these dark-skinned people also possess a dark spirit and must be brought to the Christian religion by all means and, if necessary, with fire and sword.

Blake was aware of images of slavery which were produced and spread during his lifetime as well. They showed cruelty in a most brutal way and affected him deeply. He referred to these publications in etchings and engravings<sup>1</sup> and a poem "The Little Black Boy" in Songs of Innocence: "My mother bore me in the southern wild, / And I am black, but O! my soul is white; / White as an angel is the English child, / But I am black, as if bereav'd of light." For William Blake existed

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<sup>1</sup>In 1796 J. G. Stedman published "Narrative of a Five Years Expedition Against the Revolted Negroes of Surinam". This book included 80 illustrations by Stedman himself. About thirteen Blake engraved, etched and signed from Stedman's watercolour and drawings. And: Bindman, D. (2007) *Mind-forg'd manaces: William Blake and slavery*. London: The Hayward Gallery Publishing.

no differences between mankind, a dark skin did not represent darkness of the soul, a spiritual darkness which excluded automatically from divine light. In this poem he showed in his sensitive, inimitable kind that both, the "Little Black Boy" and the "white" English child are equal, both are loved by God without any difference: "... we shall hear his voice, / Saying: 'Come out from the grove, my love & care, / 'Around my golden tent like lambs rejoice.' ..." (Keynes 1984, p. 125) (Poem: "The Little Black Boy" in Songs of Innocence).

He denounced the fatal situation in urban life as well. He saw people in their misery, he knew only too well what it meant to be poor. He himself suffered most time of his life as he did not make out any profit of his art. But: he was happy in his poverty, as he saw the spiritual sun. The misery of life shows mental slavery and chains, consequence of materialism, industrialism, social grievances, injustices. Enslavement is carried out through the whole world. People are frustrated, suffering, a constant struggle, are imprisoned in a net of their own devising, of jealousy. For mankind world in this situation is a prison, only liberation is death from the body. The soul leaving the body, rises up to the golden Door of Death. The soul returns to real life. He also painted "The River of Life"<sup>2</sup>, showed how all souls are travelling down the river into the bright, holy sun, a passage out of the Book of Revelation. The thread of life is cut by Fate. The poem "The Chimney Sweeper" talks about child's labour in those days but in the end, they are liberated by an angel who came by "...who had a bright key, / And he open'd the coffins & set them all free; ..." (Keynes 1984, p. 117 (Poem: "The Chimney Sweeper" in Songs of Innocence).

Highly acclaimed philosophers of antiquity, such as Plato, Homer, and others, whom he opposed, did not bring mankind closer to the Divine, but promoted only ego and blinded the true spirit. However, in later years, he turned to shocking, hard-spoken words about ancient philosophers because he did not want to hide the ultimate truth anymore, he had seen and his task was to spread. Their writings were perverted for Blake: "The Stolen and Perverted Writings of Homer & Ovid, of Plato & Cicero, which all Men ought to contemn, are set up by artifice against the Sublime of the Bible; but when the New Age is at leisure to Pronounce, all will be set right ...". These harsh words he wrote to his work "Milton". Further: "... Shakspeare [sic!] & Milton were both curb'd by the general malady & infection from the silly Greek & Latin slaves of the Sword" (Keynes 1984, p. 480). He blamed Plato for twisting the four cardinal virtues -- justice (iustitia), moderation (temperantia), bravery (fortitudo, magnitudo animi bzw. virtus) wisdom (sapientia/prudentia) -- into "the four pillars of tyranny" as he knew nothing. Ancient philosophy was just a delusion and "blinds the Eye of Imagination." He saw no meaning in it as empty idioms, fictional fights of the mind, production of power and egoism, forgetting the very roots. "The Greek & Roman Classics is the Antichrist. I say Is & not Are as most expressive & correct too" (Keynes 1984, p. 786). These roots, however, Blake showed in his pictures. Sensitive and unmistakable. Even Newton still sits on the bottom of the sea, not able to emerge;

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<sup>2</sup>The River of Life, (C. 1805), ink and watercolour on paper; Tate Gallery London.

as he is, all the others are, still engaged with themselves, egoistic. These philosophers do not speak about truth, truth about God, the *ultima sapientia*.

One exception: Socrates. Socrates used “pretty much the same language”, Blake told when asked about his visions on him, and added: “... I was Socrates ...” and after a pause “... or a sort of brother.”<sup>3</sup> To criticize these Greek thinkers was and still is a sacrilege that philosophers dismiss with horror and react with incomprehension. For Hegel Plato was one of the world-historical individuals, his philosophy one of the world-historical existences, which have had the most important influence for the formation and development of the spirit from its origin in all subsequent times. Blake did not care about.

Man’s mental captivity, according William Blake, is a result of that classical culture. For him, classical culture meant war and materialism, a total contrast to traditional thinking. Therefore, Blake knew no mercy when it comes to the only truth existing in the world. He was of the opinion that ancient thinkers and others were unjustly worshiped because their scriptures in no way testify to the truth that comes from God. For him, these thinkers seek definitions only, build a powerful ego, and cannot give answers. They cannot answer the question of what man is. Their ideas: purest tautology according to Blake! For Blake man is the image of God, which he sets out graphically in the person of Albion who finally is liberated.

This opinion about ancient philosophy were written down by Blake in several treatise in later years, when he clearly as radically spoke about the divine truth. At a young age, Blake already read many books of ancient philosophers, felt that these thinkers had all the capabilities to be taken seriously, but they failed. To our astonishment: Blake did not change his opinion when he grew older, he stuck to his earlier statements written in his notebooks.

Even against rationalism represented by John Locke, Isaac Newton (as mentioned before), Burke and others, Blake turned. For him, they brought no closer to humanity that state man should have: only one single task to praise God and follow his words. The same way as ancient philosophers did. If man is in this state, he develops the love he should feel, for that he obtains his salvation:

Burke’s *Treatise on the Sublime & Beautiful* is founded on the Opinions of Newton & Locke; on this Treatise Reynolds has grounded many of his assertions in all his Discourses. I read Burke’s Treatise when very Young; at the same time I read Locke on Human Understanding & Bacon’s *Advancement of Learning*; on Every one of these Books I worte my Opinions, & on looking them over find that my Notes on Reynolds in this Book are exactly Similar. I felt the Same Contempt & Abhorrence then that I do now. They mock about Inspiration & Vision. Inspiration & Vision was then, & now is, & I hope will always Remain, my Element, my Eternal Dwelling place; how can I hear it Contemned without returning Scorn for Scorn? (Keynes 1984, pp. 476-77).

If this is not the case, human egoism and materialism will take over. This becomes clear in the drawing a writings “Urizen”. Urizen spreads the book of

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<sup>3</sup>Blakes refers to Lord Byron in Byron’s writings on the ethics of Christ. Blake’s comment: *If morality was Christianity, Socrates was the saviour*, Cf: Keynes (1984, p. 786), Gilchrist (1942, p. 97), Raine (2002, p. 75).

materialism to lure mankind. And: Blake made portraits of evil forces. For some time, he lived in John Varley's<sup>4</sup> house who sat next to him, when Blake had his visions. In these nocturnal sittings Blake made pencil drawings like portraits, spoke with these entities. Sometimes great historical persons appeared before him. When these spirits visited Blake, he described them to the small auditorium, while he was making the drawings. The next day he reaffirmed his drawings: "... Oh, it's all right! ... I saw it so." (Gilchrist 1942, pp. 264-266).<sup>5</sup>

Blake also criticized artists who were famous in his time and well appreciated, f. e. Reynolds, he criticized poets such as Wordsworth and others. Art should be done out of inspiration as art is inspiration. Michelangelo, Raffael, Duerer and some others more were, according Blake, of such a caliber, such greats, as genius is above ages and time. Blake had never been abroad to study great Italian artists, he simply knew theirs works. Criticized artists found Blake's words as hostility towards them. But these words were an honest contempt matched with high spiritual aims. He did not condemn them because they did not understand the sublime. In "Discourse VIII" he wrote:

I see in Wordsworth the Natural Man rising against the Spiritual Man Continually, & then he is No Poet but a Heathen Philosopher at Enmity against all true Poetry or Inspiration. .... Natural Objects always did & do now weaken, deaden & obliterate Imagination in Me. Wordsworth must know that what he Writes Valuable is Not to be found in Nature. Read Michel Angelo's Sonnet, vol 2 p. 179: [Heaven-born, the Soul a heaven-ward course must hold; / Beyond the visible world She soars to seek, / For what delights the sense is false and weak / Ideal Form, the universal mould.] (Keynes 1984, p. 783).

### **Last But not Least, There is Hope: The Golden Age brings Liberation and Salvation**

Blake wouldn't be Blake if he shows us the glorious future, which he clearly depicted in the art work "Allegory of the Spiritual Condition of Man" (Figure 1). We should be centered, our attention should be on the Spirit, the Allpervading Power of the Universe, otherwise we are lost and will not reach the ultimate stadium being in God's glory. However, there is the hope in a new age, the Golden Age, that all this negativity will be eradicated, that a new generation of young, born out of innocence, who have regained their innocence through divine grace, will prepare the way of the future human beings and through the activity of these lead the right way to God. Blake's spiritual world is clarity, purity, and light. To gain this state

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<sup>4</sup>John Varley (1778-1842) was an artist himself, a landscape designer and a well known professional astrologer In the 19th century, worked with Blake on visionary heads, where hundreds of paintings were produced.

<sup>5</sup>Ghost of a flea: "... "During the time occupied in completing the drawing, the flea told him that all fleas were inhabited by the souls of such men as were by nature blood-thirsty to excess, and were therefore providentially confined to the size and form of insects ..." According Gilchrist: "... The devil himself would politely sit in a chair to William Blake, and innocently disappear .."

men must throw of the chains and, as it is written in the Bible, must be born again through the spirit.

**Figure 1.** *Allegory of the Spiritual Condition of Man*



Source: Blake 1811.

Blake does not see himself as a judge, he does not want anything, as he got everything. Blake urges us to trust Jesus Christ, the eternal, everlasting innocence, for it is he who gives us the wisdom that leads us, through the narrow door of the kingdom of heaven, for here too, to the words of the Bible “A camel goes through the eye of a needle rather than teaching through the gate to the kingdom of heaven.” We are judged finally by the ultimate, neverending power of the universe (Figure 2). In Blake’s eyes God had created nature perfect, but man perverted this order and the elements now are filled with the “Prince of Evil” (Keynes 1984, p. 388).

**Figure 2.** *The Vision of the Last Judgement*

Source: Blake 1808.

“The Last Judgement is one of these Stupendous Visions. I have represented it as I saw it; to different People it appears differently as everything else does; for tho’ on Earth things seem Permanent, they are less permanent than a Shadow, as we all know too well. ... This World of Imagination is the World of Eternity; it is the Divine bosom into which we shall go after the death of the Vegetated body. This World of Imagination is Infinite & Eternal ...” (A Vision of the Last Judgement from the Note-Book. For the Year 1810. Additions to Blake’s Catalogue of Pictures.)

## Conclusion

Kathleen Raine wrote about William Blake: “But we are able to recognize only what we are qualified to discover ...” (Raine 2002, p. 2. The quotation below the image “The vision of the last judgement”: Keynes 1984, p. 605). It is divine permission, what

humanity should know and depends on the level one can bear and understand. Blake, it looks like it, did not reveal everything to us, the mystic is still there, which we have to discover for ourselves in its depth and complexity.

Blake is unique. He touches our hearts when reading his poems, inhale the spirit of his artworks. His criticism on arrogance on humanity, on self-satisfaction and weakness, should be taken seriously, as he was a man of great force and – great love for all mankind. His soul, his knowledge derived from ancient times which have nothing to do with widespread esoterics and different New Age movements.

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## Crime as Language II – Hyperviolence and Georges Bataille's Concept of the Sovereign

By Claudia Simone Dorchain \*

*In political philosophy, trust, legality and violence are interdependent, with different weights, connecting and excluding. Trust structures suffer most from an anticipation of violence or violence itself. Violence systematically takes place in three stages, according to the German sociologist Jan-Philipp Reemtsma: expulsive, abusive, and homicidal violence, all of which have their distinctive and recurring verbal and nonverbal equivalents. The hyperviolence phenomenon goes beyond this, however, and even mutilates the dead body, whether actually physically, or through massive propaganda that declares the enemy a killable non-human. The goal of hyperviolence is a demonstration of absolute power over victim and bystander by means of a traumatizing violation of trust, reason, and individuality. At the same time, hyperviolence is an atavism in modernity (a historical regression); therefore the practitioner of hyperviolence is the anti-modern type par excellence. The French philosopher Georges Bataille introduces here the concept of "sovereignty" and the "sovereign" as the perpetrator also of hyperviolence. The sovereign's sovereignty is the immediate experience of the moment, free of any responsibility, with a rhetoric of mock justification of each of the three types of violence including hyperviolence. With the ahistorical view of escalating violence and the denial of any rational legitimation by the sovereign as the actor of hyperviolence, the structure of trust and the contract system of democracy itself is also endangered. So the question is, how does one unmask the "sovereign" on his way to undermine legal relations already in his linguistic patterns, in his non-linguistic codes, and how can escalation be prevented by early detection?*

**Keywords:** *violence, hyperviolence, criminal linguistics, Georges Bataille, sovereign*

A philosophical discourse on the "language of violence" has gained a frightening topicality in 2022 due to the unmediated Russian invasion of Ukraine with its war crimes in February of that year. Thousands of Ukrainian civilians have since been shot dead in Kiev, Mariupol, Butcha and elsewhere, some victims brutalised, raped and tortured before their deaths, individual corpses mutilated, cut up or disemboweled after their deaths. These crimes are "excessively excessive violence", which is the phenomenological definition of *hyperviolence*. This form of violence is at the same time a cultural-historical atavism as something that no longer seemed to have a place in current Western societies, as it originates from a much earlier era and transports norms remote from the present. Moreover, it is also a non-verbal but differentiated "language" which, according to Wilhelm von Humboldt (1836), conveys a "world view" and, like any language, is both "ergon" (work) and "energeia" (will). Work and will in the sense of excessive violence and

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the corresponding attitude are personified here in a type of criminal sociopath that has a name in Georges Bataille: the *sovereign*. To what extent does the sovereign use this language - how can it be recognised at an early stage in order to prevent its escalation?

### **Sociological Violence Research: Three Stages of Escalation, Transgression and Hyperviolence**

Not all violence is the same: recent sociological research assumes several forms of violence that are either mutually dependent - as *reciprocal violence*, when victim and perpetrator stand in a fighting relationship to each other and measure their forces in counter-defence and repetition of attacks - or, on the other hand, increase as *escalatory violence* - when the victim is no longer capable of counter-defence and the perpetrator can act out without restraint. Even this rough distinction between reciprocal and escalatory violence shows that research on violence, in terms of its classification of phenomena, is at the same time also an implicit measurement of the resources of victim and perpetrator. The corresponding findings on dyadic relationships or "asymmetrical wars" (see Münkler 1992), as the historian Herfried Münkler calls them, are fruitful sociologically, psychologically and above all philosophically, because what is at stake here in the broadest sense is the human condition, in its unenlightened and dark side. The Hamburg sociologist Jan-Philipp Reemtsma deals with such fundamentally asymmetrical situations between perpetrator and victim, which according to their dynamics are no longer reciprocal but have an escalating effect, and in his philosophical-sociological research on violence he assumes three types of violence: *place-changing violence*, *abusive violence* and *murderous violence* (Reemtsma 2006, p. 132ff).

**Place-shifting violence** changes the location of the victim - in the case of individual crimes, it is the isolation of the victim through confinement, encirclement, confinement, imprisonment; in the case of war crimes, it is the annexation or occupation of land. **Abusive or "raptive" violence** - the term goes back to Walter Benjamin and his extensive philosophical critique of violence from 1921 - is violence that damages, abuses, mutilates or rapes the body. In today's academic criminology, in addition to naming this physically active type of violence, which Reemtsma classifies as abusive, there are also different degrees of damage with regard to the duration of the damage to the victim, but these are not relevant here.<sup>1</sup> Finally, **autotelic violence is homicidal violence**, applied to the individual as execution, applied to a collective as genocide. The three types of violence according to Reemtsma occur in practice, in individual crimes as well as in acts of war, mostly building on each other as a gradual escalation of violence, always when there is a fundamental asymmetry of resources between the perpetrator and the victim and when resistance remains fruitless and only incites the aggressor to greater violence.

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<sup>1</sup>Damage degrees were described in detail by Cohen and Felson (1979, p. 588f).

**Hyperviolence** is, according to the French encyclopaedia, "violence that goes excessively beyond habitual violence" (Encyclopédie française 2022), i.e., exceeds everyday and customary violence, which in turn allows for a fine differentiation and proves its excessive character. In the cases considered here, which are to be classified as typical, it **even goes beyond autotelic violence**. In this respect, it is no longer included in Reemtsma's three-stage scheme and represents a fourth escalation stage that occurs when the autotelic violence has already been carried out and the victim is dead. Hyperviolence is the violence that not only kills the victim, but also mutilates it or renders it unrecognisable, making the victim, as it were, posthumously a non-human, deprived of all dignity and individuality. For reasons of scientific precision, we distinguish here and in the following between the systematic and historical levels of hyperviolence: systematically, hyperviolence is the continuation of Reemtsma's three types of violence and aims at posthumous aggression that escalates the violence of execution. Historically, hyperviolence can be understood as a very old, archaic and barbaric form of violence that was already known in the earliest cultures and was also depicted in written and pictorial evidence.

The problem of excessive, escalating violence, which is not even satisfied with the death of the victim, i.e. which goes beyond the autotelic level of violence, was depicted in European literature at an extremely early point in time. It can already be found in Homer's "Iliad", which according to Diodorus was written at the time of the Trojan War, i.e., in the 7th century BC,<sup>2</sup> when the barbarian Achilles drags the body of the slain Hector across a field in a triumphal procession and cruelly disfigures it by making the corpse take on the colour of the soil and rendering it unrecognisable.<sup>3</sup> Hyperviolence, however, does not remain in antiquity, and already there as the non-verbal "language" of the barbarian, but continues through further stages of historical development with ever new protagonists as perpetrators, and it is precisely these perpetrators who often keep the world and posterity in suspense with their inhuman deeds. In 1961, the French philosopher Georges Bataille reported on the true criminal case of how Marshal Gilles de Rais (1405-1440), known as Retz, a Breton knight and companion in arms of Joan of Arc, kidnapped more than 200 children - according to other estimates even more than 600 -, locked them up in his castle, sadistically sexually abused them, hanged them and ritually executed them, devoutly contemplating their torn-out entrails (Bataille 1961, p. 107). On a systemic level, characteristic of hyperviolence as a type of escalatory violence is the total asymmetry of perpetrator and victim, which cannot be reciprocal anywhere, but is represented by a total imbalance of power: on the one hand, the experienced, heavily armed marshal Retz, on the other, defenceless children. Writer and painter Pierre Klossowski examines similar "asymmetrical wars" and takes a philosophical look at the controversial works of the Marquis de Sade with their numerous references to torture and hyperviolence, which for de Sade himself - who spent a large part of his adult life in a mental institution or in prison for violent crimes - represented

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<sup>2</sup>Diodorus 7,1.

<sup>3</sup>Homer's *Iliad* referred to by Schadewaldt (1975).

both philosophical-literary theory and empirical practice (see Klossowski 1984). Klossowski comes to the conclusion that de Sade's works, in addition to autobiographical references to an undoubtedly disturbed and highly criminal personality, also contain references to the political sadism of feudal rulers (Klossowski 1984). This insight - from the perpetrator to the system - will be examined later.

It is rare to find such perpetrators in empirical research who, like de Sade, take the active and descriptive perspective: literary monstrosities. Historians and war correspondents know not fictional but real hyperviolence: in the concentration camps of the Second World War, where violent dressing up was part of the tactics of psychological warfare (see Dorchain and Wonnenberg 2012), and currently in the occupied territories of Ukraine, where civilian victims in the villages of Butcha and the surrounding area were and still are mocked or mutilated even after their death (Amnesty International 2022). The violence, which extends beyond death, is also intended to disfigure the victim for posterity and thus constitutes *a kind of extended execution*, with a partial or total prevention of memorialisation through disfigurement or destruction of the corpse. In this way, too, the victim is posthumously denied its human dignity and the grieving family is repulsed - hyperviolence tears apart the bond of remembrance and thus the culture of remembrance as a specifically human form of recognition.

### **Hyperviolence and the History of Ideas in Western Philosophy: A Barbaric Transgression Criticized Since Homer's Time**

The philosophical critique of the cultural phenomenon of hyperviolence was already evident in the earliest times of cultural history and can be found in writers of antiquity, who on the one hand describe hyperviolence and on the other hand already find words of rebuke, since they too feel the rupture that this excessive violence has on memory, family, dignity, form - all *systems of meaning*. The philosophical critique of violence increases in the course of history: while antiquity already condemned this form of excessive violence and assigned it to culturally inferior actors, early modernity found contemptuous criticism for it as a generally inhumane procedure, and postmodernity relegated it as an atavism outside its own sphere of validity: anyone who used hyperviolence was not modern, was not a contemporary. This corresponds to Max Weber's laconic conclusion that violence was simply part of everyday life in most cultures and epochs - but that only modernity problematised it, and that therefore the excess of violence in particular did not become a fact of modernity, but a phenomenon with which modernity had a problem (Weber 1999, p. 18). If we follow the systematic development of the historical arguments from literature and philosophy, we actually find the triad "barbaric - unchristian - unmodern" as a progressive pattern of rejection within the critique of violence. First of all, Homer contributes to this by portraying hyperviolence as unmistakably "barbaric", since it marks the barbarian Achilles as such as a non-verbal act and distinguishes him negatively from the more civilised Greeks. Hyperviolence as an act thus already marks a

special dyadic relationship of asymmetry in antiquity and at the same time a cultural difference between perpetrator and victim. If one considers how important Homer's epics were in Greek culture and in the education of children, according to recent research by Thomas Szlezák, one can assume that at least ancient Greece took the cultural condemnation of hyperviolence as a violation of norms seriously (see Szlezák 2010, p. 13). The cultural condemnation of the perpetrator of excessive violence beyond death was preserved in the centuries following antiquity, but found its way into literature only sporadically. In the eighteenth century, but even before Immanuel Kant's questioning of all instances in the state with regard to their legality, also and especially in their exercise of power, a radical ideological change took place: violence and above all hyperviolence became a problem - not only for individual authors like Homer and Hesiod, but in general, as a movement that took hold of large parts of the intellectual community.

In 1746, on the eve of the Enlightenment, which was soon to sweep through Europe like a beacon of light, the Catholic church superior Dom Calmet found astonishingly critical words with regard to such war crimes: Hyperviolence is "unchristian and unethical" (Calmet 1746). The inconvenient cleric supports this judgement with reference to the patristic church fathers Origenes and Tertullian and states that no one could be a Christian who would approve or sham-legitimise war violence or criminal violence of this kind - this was courageous in a time when field preachers were still inciting soldiers in war or even justifying all means in a Macchiavellian way for the figure of the "just war". According to Walter Benjamin, the problem of hyperviolence in early modern thought arose systematically from the problematisation of natural law, which was no longer accepted unquestioningly or even identified with Christian norms (Tiedemann (1992, p. 105). In 1977, Michel Foucault found the latest and at the same time strongest criticism of these excesses of violence, which are found in war among unleashed militaries, in peace among pathological lone perpetrators: Hyperviolence is first and foremost "pre-modern"<sup>4</sup> and, according to its phenomenology, not a modern murder, nor really a medieval practice (although it does occur in the Middle Ages as a form of punishment), but systematically an ancient form of sacrifice rooted in ritual and cult. In this respect, such a perpetrator is a pre-modern person who has fallen out of the present and personifies a reversion to an earlier, sacred cultural level with all its negative, Dionysian attributes.

### **Back to the Future: The Hyperviolent Offender as "Sacrificing Priest" or the Anti-Modern Type par Excellence**

Hyperviolence is found, according to Foucault, as a relic of ancient forms of sacrifice in modernity, or as "blood violence over life" (Marcuse 2009, p. 60) in Walter Benjamin's words, when the perpetrator assumes the role of the sacrificial priest, as it were, and reduces the victim to mere life, the naked body. Here, finally, for the researching criminologist and psychologist or forensic scientist,

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<sup>4</sup>See Michel Foucault's criticism of violence and the judiciary system (Foucault 1977).

there is the disturbing phenomenon of cultic self-aggrandisement or self-deification as the dominant personality trait of the perpetrator in phenomenology, but not as the motive for the crime: this is uninhibited, time-lasting exercise of power. This motive for action in the context of the cult also fascinates lay people.

The sacrificing priest in the film and the human sacrifice is also a dazzling media figure today, which raises the question: Why are film villains, for example in James Bond, often perpetrators of hyperviolence?<sup>5</sup> The appropriate thesis is: because their psycho-moral structure represents an anti-modern type - the antitype to modernity per se, the *other* in culture, atavism in the present or the return of the sacrificial priest as a haunting figure from the past. Hyperviolence remains, viewed in modern terms, a monstrous provocation, be it media-fictional or real-political. Why does this excess of violence shatter the self-understanding of an epoch? Hyperviolence as an individual act (i.e., the perpetrator-victim dyad) as well as a structural act by supra-individual perpetrators (i.e., institutions, parties, corporations) is an atavism, reverses the process of modernity:

- Modernity as a *successive process of secularisation* with thinkers such as Montaigne, Descartes, Spinoza etc. growing out of the philosophical spirit of scepticism is denied, hyperviolence nevertheless again wants the archaic violence of the sacrificing priest as irrational metaphysically transfigured power beyond death.
- Modernity as the *identity of reason and rule* is threatened, since Hyperviolence denies all these achievements and, in an irrational act, establishes total violence beyond the violence of murder.
- Modernity as *individuality and the elevation of the ego* is endangered, since hyperviolence destroys this ego identity to the point of de-individualising the victim.

Hyperviolence as a figuratively staged rejection of secularisation, reason and individuality as the declared paradigms of modernity is thus the pre-modern figure par excellence.

### **Personal Tyranny: Georges Bataille and his Concept of the Sovereign**

Is there a prototype for this pre-modern figure of the excess offender who is not even satisfied with the execution of his victims and who traumatises the modern by violating all their protective self-understandings? Surprisingly, there is such a typology that assigns the act and the perpetrator. In philosophy, there is not only a centuries-old tradition of critique of violence and especially of critique of

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<sup>5</sup>See James Bond: *Goldfinger* from 1965, third part of the film series based on a novel of the same name by Ian Fleming. The name Goldfinger comes from the Phrygian king Midas, who supposedly turned everything he touched into gold. In the film, Bond's villain Auric Goldfinger (from latin aurum = gold) is Bond's adversary. There is a famous scene in the film where Bond's lover Jill Masterson is painted with gold paint and suffocates. This killing, however, is ritual violence: it alludes to the gold-painted human sacrifice of the aztecs.

excessive hyperviolence, but also a typology of the perpetrator, a figure who personifies hyperviolence and is the declared perpetrator: we find it in a prominent place in the french philosopher Georges Bataille's work on norms, norm decay and modernity, it is called **sovereign** (Bataille 1953, p. 264). The "sovereign" as a psychopathological type in Bataille's work (not to be confused with the adjective sovereign state power, which in democracies emanates from the people, or the sovereign in the sense of a psychologically healthy, self-confident normal citizen) is a threat to the public, to order and to peace. He is unrestrainedly violent, even excessively so, he lives completely in the moment, is accountable to no one and creates and dominates the state of exception. Hyperviolence is the violence of the sovereign, because in this figure the phenomenon of the excess of violence is, as it were, accumulated, or in other words: those who exercise hyperviolence stage themselves as sovereigns (Bataille 1953). Only the sovereign exercises hyperviolence - hyperviolence is the violent "language" of the sovereign.

Let us therefore look at the concept of the "sovereign" in Bataille and see what makes him tick: the sovereign is a barbaric concept of ruler, he rules without justification in Max Weber's sense (neither traditional, nor legal, nor charismatic), he does not recognise any limits to action and is completely self-sufficient in his destructive actions. His rule has clearly visible fascist features. The sovereign possesses the power to reduce the Other to the naked body, to "bare life" (Agamben 2002), as Giorgio Agamben defines it. For him, the biopolitical body of his victim is only a projection and image surface, or as Niklaus Largier calls it, a "tableau vivant"<sup>6</sup> for ruling power relations. Here the sovereign is simply an extreme perpetrator of violence that reduces the body, as Reemtsma already defines it: "The reduction to the body carried out by the act of violence is the reason why violence must always be understood as primarily physical" (Reemtsma (2006, p. 124). But hyperviolence does more than reduce the victim to the body; it also manifests the cultural difference between the perpetrator and the victim, which Homer already saw, and eternalises the power relations and, finally, makes it impossible to give back the victim. The sovereign is a merciless judge of the victim beyond death and thereby assumes a role that does not exist in civil law in the 21st century, even in state systems that still know death sentences for delinquent citizens. Whoever engages in hyperviolence, as the alleged master of mere life, undoes the systematic cultural developments in the western understanding of civilisational progress, the sequence of which has already been historically set out in the civilisational drama "Oresteia" by Aeschylus – an itinerary development from the chaotic primordial state to the consensual liberal state contract. *The sovereign lives ahistorically and completely in the moment*, and therefore also denies modernity as a structurally linear process of development and gradual renunciation of violence in the individual, the collective of every kind and the state in particular. Only the connection of the two philosophical structural concepts "hyperviolence" and "sovereign" shown here makes this epoch-breaking dynamic clear. For the ahistorical sovereign represents a culturally earlier stage or atavism in his person, his type and his activity, and his unrestrainedly violent and

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<sup>6</sup>See Niklaus Largier on the aesthetics of violence (Largier 2001).

demoralising action harbours the danger of catapulting the culture he tyrannically afflicts back to a more barbaric state.

### **The Language of the Sovereign: "I am your Lord and God"**

So far, in this phenomenologically underpinned philosophical investigation of language and crime, violence has been symbolically assumed to be non-verbal language, and this striking equation also makes sense with regard to the sovereign as the perpetrator of violent excesses, because he usually "speaks" with weapons. But with the reductionist equation of violence as non-verbal language, the actual dynamic and thus also motivational level of human language development remains untouched and thus not open for insights into its development. For the philosopher and linguist Wilhelm von Humboldt in 1836, language was not merely a totality of non-verbal, sonic or semiotic signs, but essentially always a "weltanschauung" (world view) (Di Cesare 1998), and consisted fundamentally of the two interrelated components

*ergon* (i.e., visible system of signs, word, sound, alphabet)  
*energeia* (i.e., basic mental attitude, attitude, system of norms).

Humboldt's idea was that the "ergon" of a language only changes if the "energeia" of a language has already changed, i.e., the worldview underlying the language has changed significantly. The violent language of the sovereign as "ergon" is thus preceded by a maximally narcissistic, delusional and sociopathic change of mind as "energeia", which seemingly legitimises him, and this may be true for individual offenders as well as for collectives (Klossowski 1984). However, these apparent legitimations are not only thought but often verbally formulated when sovereigns of old and new times manipulate their future victims and suggestively announce their deeds, not infrequently by already implying asymmetrical relations in language. The later asymmetry of warfare through ever more escalating physical violence is linguistically prepared by seemingly metaphysical justifications and exaltations of one's own person - here again Foucault's sacrificial priest becomes recognisable, who gives himself a nimbus - as well as through irrational claims to power that are not amenable to rational argumentation and through generalising formulations that exclude the individuality of the victim, which, as it were, anticipate the unrecognisability of the individual in the language. Here we find linguistic references to *the three self-conceptions of modernity, which the sovereign contradicts: secularisation, unity of reason and rule, and individualism*. Not only the violent actions of the sovereign, but also his speech patterns express his distance from the self-awareness and value consciousness of modernity.

Apparently metaphysical justifications of one's own person and role with narcissistic-grandiose exaggerations remind the attentive listener of the sacrificing priest, irrational justifications devoid of any logic and rationality or even sophisticated exaggerations of reason as such prove that the unity of reason and rule so typical of modernity is vehemently denied. A generalising, equalising language

(e.g., "civilian failures", "average types", "people of the world") denies modernity's strong reference to individuality and represents a rhetorical conditioning of the victim, who is supposed to feel as nothing special, mass-like and indiscriminate. Significantly, this speech pattern of sovereignty was already identified in 1989 by the US-american linguist Noam Chomsky as a rhetorical technique of mass suggestion in modern media, i.e., it is part of everyday life that is seldom questioned and desensitises the contemporary to such presumptions (Chomsky (2002, p. 5). Degrading, equalising addresses of the other as a future victim, irrational claims to power, finally self-deprecation - all these manipulative rhetorical patterns are not about the factual level, but about the relational level, specifically about the degradation and destabilisation of the counterpart, who is thus to be psychologically prepared for the planned "**asymmetrical war**", which may soon escalate in physical form. Accordingly, one has to pay attention to these speech patterns as anticipatory, sham legitimations of violence in order to preventively counteract the escalation of the three types of violence according to Reemtsma and their transgression through hyperviolence. Excessive violence can only be stopped from a criminological point of view if it is recognised in the early stages (Cohen and Felson 1979), which can still be grasped linguistically and therefore linguistically, before it finally becomes, with Hannah Arendt, "violently mute" (Arendt 1981, p. 195) and abusive hands or outright instruments of destruction speak.

One last interesting question on the connection between the hyperviolence of the sovereign and its language should be mentioned here: what actually stands at the beginning of the fatal "energeia" of the sovereign, what triggers it? The answer is: in his perverted understanding of himself and of the world, the sovereign has an initial cause that lies in his deviant understanding of time. His "energeia" is constituted in particular by a strongly disruptive reference to time: he lives completely in the present moment and instant (Bataille 1953) without traditional or legal justification (Weber 1999), he wants to re-enact historical developments or reverse processes of modernity through violent action (Agamben 2002), he creates and controls the "state of emergency"<sup>7</sup> with arbitrary special legislation (Schmitt 1996) and he intends to found a "new time" in which everything is different from now on. As a rule, a dictator has a distorted reference to time, which is exposed linguistically in the run-up to acts of violence, in that the perpetrator of excessive violence announces his action and at the same time mystifies it in an unprecedented yesterday or a utopian tomorrow. Any acute reference to time, which in philosophical ethics would be a rational basis for responsibility, is missing in the understanding of being of the absolute offender, who is accountable to no one. The "energeia" of the sovereign, which is largely based on disruption and provides irrational illusory legitimations for violence, is thus systematically the refusal of those paradigms which, at the latest since modernity, have been regarded as culture-forming in their self-understanding: *renunciation of violence by the individual and the state, dignity, justification, reason, preservation of individuality*. The sovereign is therefore, as it was in Homer's time, only today

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<sup>7</sup>Carl Schmitt quoted by Agamben (2002, p. 27).

much more appalling, an indicator of cultural degradation: wherever hyperviolence occurs and its typical actor, the sovereign, modernity and with it the project of democracy is endangered by this anti-modern, anti-democratic type.

## Conclusion

If citizens, institutions and states want to protect democracy today, they must become more sensitive to recognise the violent criminal type of sovereign at an early stage, because hyperviolence is its characteristic behaviour pattern and will come to light sooner or later, due to its inherently excessive nature. The sovereign as self-appointed "lord of the state of emergency" is a danger not only to peace, but also to the cultural and legal achievements of modernity, its understanding of itself and of norms and its liberal conception of man.

An ironic break in this argumentation, however, lies in the question of whether totalitarian modernity as such can still have a normative claim at all (see Zitelmann 1994, p. 1) - or whether it is not based on normopathy or does not perhaps even promote sovereignty with its excessive use of force. The historical project of civilisation (fragile and by no means linear), and its temporary light line enlightenment (always a work in progress), depends on whether its seemingly epoch-spanning shadow, the sovereign, remains recognisable as an antitype and does not become a new habit or even an ideal.

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