

Non-Classical Elements in Epicurean Logic

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In this paper¹, we explore some non-classical elements in Epicurean logic. It seems that Epicurean logic contains elements of pragmatic, relevance and fuzzy logic. We reveal the fuzzy-logical elements in particular and show how they could be used to strengthen some of the Epicurean arguments.

Introduction

In general, the Epicureans did not have logic among their primary philosophical interests. There were some exceptions such as Philodemus whose work ‘On Signs’ is essentially a work on logic. Epicurean logic developed primarily in response to Stoic criticism of Epicurean philosophical positions and argumentation. Among other things, as Philodemus says, Dionysius accuses the Epicureans of ambiguous terminology in using “the word ‘analogy’ first of the common qualities of the apparent and the unperceived”. (De Lacy 1941). In addition, Dionysius claims that Epicureans employ incorrect reasoning because, as Philodemus states, “in some cases we infer about objects by our own method of analogy, and sometimes by contraposition, so that we use both principles”. (De Lacy 1941). Philodemus, in turn, criticizes Dionysius on whether contraposition is the only cogent method to prove a proposition. As he says:

But sometimes the proposition is not proved in this way, but rather, when it is not possible to conceive that the first is or is of a certain character, and the second is not or is not of such character. For example, “Plato is a man, and Socrates is a man.” If this is true, it is true also that, “If Socrates is not a man, neither is Plato a man,” not because by the denial of Socrates, Plato is denied along with him, but because it is inconceivable for Socrates not to be a man and Plato to be a man. But this inference is derived from the method of analogy. (Lacy 1941).

Philodemus responses to Stoic criticism in “On Signs” (De Lacy 1941) and the further elaboration of the Epicurean positions, methods of reasoning, etc, is mainly what one thinks as Epicurean logic. In the section ‘Fuzzy Logic’, we will see how this “ambiguity” Philodemus was accused for contains the seeds of a non-classical logic that the Stoics did not possess.

What constituted Epicurean logic? We summarize its main points in the next session. At first glance though, one sees a clear contrast between Stoic and Epicurean logic. Stoic logic² is conceptual and formal whereas Epicurean logic is based more on immediate experience and practical reasoning. Consider one contrasting point: conditional statements. A conditional statement (implication) “if p , then q ”³ is a-priori and rational for the Stoics whereas it is a-posteriori and empirical for the Epicureans. For example, in the claim “If there is motion, then there is void” the

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Stoics would argue by contraposition that if there no void then there would be no motion, which is false. Therefore, the initial implication is true. But the Epicureans would protest that the conclusion cannot be established this way in an a-priori way. It will be established only when one finds evidence from experience that indeed there is no motion if there is no void. Thus, empiricism and induction set the foundation for the distinctive Epicurean logic whose particularity, range, etc, had a direct effect on all fields of Epicurean philosophy.⁴

Furthermore, unlike Stoic logic, Epicurean logic contains some non-classical aspects worth exploring. Especially elements from multi-valued logic and fuzzy logic. Not much has been written on this topic. It is well known that the Epicureans rejected the principle of bivalence and the law of excluded middle (Inwood & Gerson 1988) but, to my knowledge, no serious connection of Epicurean logic to fuzzy logic has been established. In a short article titled “ ‘Fuzzy’ Logic” (Patzoglou 2011) the author makes an intuitive attempt to connect Epicurean logic to fuzzy logic but the article contains several mistakes and suffers from lack of scientific and philosophical rigor.⁵ In two other articles that came to my attention, (Altas 2015) and (Kardneris 2015),⁶ the authors also attempted to link the Epicurean multiple explanations method to multi-valued and fuzzy logic but with no success. These two articles also contain several mistakes and suffer from lack of scientific and philosophical rigor.⁷ Several other connections between Epicureanism and fuzziness, in various contexts, could be found in some articles⁸ but nothing to explicitly relate Epicurean logic and non-classical logic directly from Epicurean sources. In the following sections, we will try to reveal those non-classical aspects of Epicurean logic giving emphasis on fuzzy logic.

Epicurean Logic

Logic was not part of the Epicurean Canon, namely the epistemological rule-set and conditions on how truth is evaluated. Yet, the Epicureans neither rejected logic altogether nor were ignorant of it as some of their critics (such as Seneca and Cicero) claimed. (Inwood & Gerson 1988). They simply accepted aspects of logic under certain conditions and adapted it to meet their epistemological beliefs. As we already mentioned in the Introduction, Philodemus’ “On Signs” is a work on logic. (Sedley 1982, De Lacy 1941, Crespo 2014).

Epicurean logic could be roughly summarized as follows:

(A) Accepted: (i) *sound arguments*, (ii) *contrapositive (by contradiction)*, (iii) *induction*, (iv) *abduction*. More specifically:

- (i) *sound arguments*: they accepted the conditional $p \rightarrow q$, when p is real fact (i.e. empirical).⁹ (Inwood & Gerson 1988, Sedley 1982).
- (ii) *contrapositive*: they accepted $\neg q \rightarrow \neg p$ to go from ‘ $\pi\rho\acute{o}\delta\eta\lambda\omicron\nu$ ’ (obvious/observed fact) to ‘ $\acute{\alpha}\delta\eta\lambda\omicron\nu$ ’ (not obvious/unobserved fact). (Inwood & Gerson 1988, Sedley 1982). Also, they accepted the ‘by contradiction’ method when something contradicted with the facts (Inwood & Gerson 1988).¹⁰

- (iii) *induction*: Epicurus, and even more so Zeno and Philodemus, developed a theory of inductive inference which bases the inference on empirical observation. (Marquand 1883). An inference is justified if it is verified directly by the facts (“επιμαρτύρησης”). It is also justified by similarity to other facts (“καθ’ ομοιότητα τρόπος”) (Sedley 1982) or by analogy to other facts (“κατ’ αναλογία τρόπος”) (De Lacy 1941).¹¹ An inference is not justified if it contradicts the facts (“αντιμαρτύρησης”).¹²
- (iv) *abduction*: they accepted the schema if it facilitates explaining real facts (Inwood & Gerson 1988).

(B) Rejected: (i) *Principle of Bivalence (PB)*, (ii) *Law of Excluded Middle (LEM)*, (iii) *abstractions*, (iv) *dialectic*. More specifically:

- (i) *PB*: they rejected this principle and adopted the “multiple method” (“πλεοναχός τρόπος”) which suggests multiple explanations or reasons for something (Inwood & Gerson 1988).
- (ii) *LEM*: rejected this principle for future propositions and in defense of free will. (Inwood & Gerson 1988).
- (iii) *abstractions, dialectic*: detested abstractions and technicalities, deduced things from real facts (Crespo 2014).
- (iv) *dialectic*: definitions, sophisms and dialectical resolution of problems they rejected altogether. (Stocks 1925, Sedley 2018).

Overall, as it is clearly shown from the examples above, Epicurean logical inference relies heavily on facts from experience. As Marquand says:

Both (Epicurus and Zeno) are occupied with the sign-inference, and look upon inference as proceeding from the known to the unknown. Epicurus, however, sought only by means of hypothesis to explain special phenomena of nature. Zeno investigated generalizations from experience, with a view to discovering the validity of extending them beyond our experience. This resulted in a theory of induction, which, so far as we know, Epicurus did not possess. In the system of Aristotle, induction was viewed through the forms of syllogism, and its empirical foundation was not held in view. The Epicureans, therefore, were as much opposed to the Aristotelian induction, as they were to the Aristotelian syllogism. It was Zeno who made the first attempt to justify the validity of induction. (Marquand 1883)

Hence, in a sense, the Epicureans adopted and promoted a more “realistic/pragmatic” logic which contained the seeds for later modern logics such as Mill’s empirical logic (Marquand 1883), relevance logic¹³ (Sedley 1982) and Pierce’s semiotic logic (Bellucci 2016).

Interestingly, one can observe a fuzzy logical framework in Epicurean inference. In (Aristidou 2023) the author took a closer look on some matters from (A), especially (A) (iii) (i.e., induction) which was a main feature of Epicurean logic. In the next section, we look into matters from (B), in particularly (B) (i, ii), and see how fuzzy logical elements are embedded in the Epicurean reasoning, thus enhancing further the Epicurean rejection of the PB and the LEM.

Fuzzy Logic

There are certainly elements of fuzzy logic in the Epicurean logic.¹⁴ In the ‘Letter to Herodotus’, Epicurus stated:

Moreover, there is an unlimited number of cosmoi, and some are *similar* to this one and some are dissimilar. For the atoms, which are unlimited (as was shown just now), are also carried away to *very remote* distances. (Inwood & Gerson 1988).

Also:

In addition to these points, one must not believe that there can be an unlimited number of masses – no matter how *small* – in any finite body. (Inwood & Gerson 1988)

In Bailey (1926), the above quote is translated as:

Besides this we must not suppose that in a limited body there can be infinite parts or parts of every *degree of smallness*. (Bailey 1926)

Furthermore, in ‘On Signs’, Philodemus also says:

It is neither necessary to take into consideration every phenomenon in our experience, nor a few cases at random; but taking *many* and various phenomena of the *same general kind*, and having obtained, both from our observation and that of others, the properties that are common to each individual, from these cases may we pass to the rest. (Marquand 1883)

The terms in italics in the quotes above are mine and they are to show that these terms are typical fuzzy terms. The Epicurean “*similarity method*” (“καθ’ ομοιότητα τρόπος”) (Sedley 1982) is also typical fuzzy reasoning. Let us look at some examples of typical Epicurean arguments, which we improve using fuzzy logic:

Example 1 Since Epicurus is a man

 Metrodorus is a man
 (Sedley 1982)

The argument has the form:

x is in M

 y is in M

because x and y have high degree of similarity, i.e. the similarity of being men. More specifically, let “man” be a fuzzy set denoted by M and $M(x)$ the degree of membership of x in M . The degree of similarity of x and y is understood by the metric $SM(x,y) = 1 - |M(x) - M(y)|$.

So, more rigorously the argument could be formulated as:

Since x is in M	(by experience)
$SM(x,y)$ high	(by experience)

y is in M	

where all the above depends on the weight of evidence from x 's and y 's biology, behavior, etc.

Example 2

Men in our experience are mortal

Men outside our experience are mortal

(Sedley 1982)

The argument has the form:

All x in our experience are MO

All y outside our experience are MO

because men x in our experience and men y out of our experience have high degree of similarity of being men (fuzzy set M). Hence, they would be all belong to the same (non-fuzzy set) MO (mortal). Using the same sets and metric as in Example 1, we have:

All x in our experience are MO	(by experience)
x, y are in M	(hypothesis)
$SM(x,y)$ high	(hypothesis)

Men outside our experience are MO	

where all the above depends on the weight of evidence from x 's biology, evidence from historical records, etc, and the fact that men outside our experience are *still* men.

Example 3 In the Letter to Herodotus, Epicurus said:

Moreover, one should not believe that atoms have every [possible] magnitude, so that one may avoid being testified against by the appearances. But one should believe that there are some differences in magnitude. [...] But [to suppose] that every magnitude exists is not useful for [accounting for] the differences of qualities, and at the same time it would be necessary that some atoms reach the point of being visible to us-which is not seen to occur nor can one conceive how an atom could become visible. In addition to these points, one must not believe that there can be an unlimited number of masses-no matter how small-in any finite body. [...] For obviously these unlimited masses must be of some size or other; and no matter how small they might be, the magnitude [of the whole object] would for all that be unlimited. And since the limited has an extreme which can be distinguished even if it cannot be observed on its own, it is impossible not to conceive that the thing next to it is of the same character and that by

moving forward from one point to the next in this fashion it turns out that one will in this fashion reach the unlimited conceptually. (Inwood & Gerson 1988)

So, the argument could be:

All physical objects have a variety of magnitudes
 The smaller the object, the smaller its magnitude
 Atoms are very small (or are the smallest) objects

 Atoms have very small (or have the smallest) magnitudes

The argument has the form:

All x 's have a variety of m 's
 The smaller the x , the smaller its m
 a 's are very small (or are the smallest) x 's

 a 's have very small (or have the smallest) m 's

where x 's are physical objects, a 's are atoms and m 's are magnitudes of x 's. Let $S(x)$ and $M(x)$ be the degrees of membership of x in the fuzzy sets "small" and "magnitude".¹⁵ Then, as $M(x)$ is low when $S(x)$ is high, the argument becomes:

All x 's have a variety of m 's (by experience)
 If $S(x)$ low, then $m(x)$ low (by experience)
 $S(a)$ is low (or the lowest) (a 's compose everything else)

 $M(a)$ is low (or the lowest)

The term "small" in the argument is subjective by experience.

Example 4 Sedley in (2020) discusses the question of how objects change color. He draws from the quite ambiguous excerpt in the *Letter to Herodotus* by Epicurus which says:

Moreover, the atoms themselves must be considered to exhibit no quality of things evident, beyond shape, weight, size, and the things necessarily included in the nature of shape. For all quality changes. But the atoms do not change at all, since something solid and indissoluble must survive the dissolution of the compounds to ensure that the changes are not into, or out of, the nonexistent, but result from rearrangements within many things, and in some cases from additions and subtractions of certain things. Hence those things that do not admit of (internal) rearrangement must be indestructible, and must lack the nature of that which changes. (Sedley 2020)

The main question in Sedley (2020) was whether atoms have color at all.¹⁶ Leaving the controversial case of atoms aside, what about other things that have *significant* change in color? Or things that *slightly* change color? Say the leaves of a cypress tree. Epicurus could have argued as follows:

Most things change color
 If a thing has a change in color, then it is due to rearrangements
 of the atoms involved
 Not many rearrangements of the atoms involved occur in cypress' leaves
 in the winter

Cypress' leaves do not significantly change color in the winter

The argument has the form:

Most T 's change C
 If a T has a change in C , then it is due to rearrangements
 of the atoms involved
 Not many rearrangements of the atoms involved occur in CL
 in the winter

CL do not significantly change C in the winter

where T is "things", C is "color" and CL is "cypress leaves". And the argument could be formalized using fuzzy functions as follows:

$C_T(x)$ (by experience)
 If $C_T(x)$, then it is due to $RA_T(x)$ (hypothesis)
 $RA_{CL}(x)$ low in the winter (by experience)

 $C_{CL}(x)$ low in the winter

where $C_T(x)$ is "color change of T ", $RA_T(x)$ is "rearrangements of the atoms of T ", $C_{CL}(x)$ is "color change of cypress leaves" and x is "time" (or the relevant factors that cause the changes).

It is important to emphasize once more that the Epicureans did not present or suggest any of the arguments in the analytic form we showed above. After all they did not have the fuzzy-logical tools that we possess today. But, perhaps they could have used similar, but less formal, quantified line of reasoning to reinforce their arguments.

Conclusions

The Epicureans did not have logic in their primary philosophical interests and they did not produce a formal system of logic. Yet, the Epicureans neither rejected logic altogether nor were ignorant of it. They adopted an empirical canon on how truth should be evaluated, based on sensory experience and induction. Almost two centuries after Epicurus Epicurean logic started becoming more precise and have a clearer "shape". Philodemus responses to Stoic criticism in "On Signs" (De Lacy 1941) and the further elaboration of the Epicurean positions, methods of reasoning, etc, is mainly what one understands as Epicurean logic. Empiricism and induction set the foundation for the distinctive Epicurean logic.

Some obvious problems with Epicurean logic are the following:

1. Even though induction is important in mathematics, it is not enough for doing mathematics (Polya 1973; Aristidou 2023). Yet mathematics is important in science which was a field of utmost interest to Epicureans. Furthermore, Epicurean logic has to dismiss mathematics for empirical reasons, yet mathematics can take one where his senses cannot. For example, one needs mathematics to build x-ray machines, microscopes, telescopes, GPS systems, etc. Epicureans most likely would not oppose to mathematics done and used for practical and useful purposes, but mathematics many times is very much abstract and “useless” before it finds applications.
2. The Epicurean rejection of PB and LEM and the informal and empirical nature of Epicurean logic drew criticisms from other schools. The Stoics accused them of using ambiguous terms and non-rigorous reasoning. As De Lacy (1941) says:

The discussion of the relation of deductive and formal logic to inductive logic and empirical method is, of course, very inadequate in Philodemus. (De Lacy 1941).

Nevertheless, if one looks a bit deeper one can uncover valuable elements from Epicurean logic, something that the Epicureans themselves did not advocate for. Some are the following:

1. Epicurean logic contains elements of later modern logics such as Mill’s empirical logic, Pierce’s semiotic logic, relevance logic and fuzzy logic. Fuzzy logic turned out to be very important in technology and in making people’s lives easier and more comfortable, something which the Epicureans would have certainly condoned.
2. The Epicurean critique of formal logic and their detest of abstraction is felt even today, as ‘informal logic’ is what characterizes mathematics in practice and in education (Aristidou 2020b) Informal logic is also what characterizes better human natural reasoning.

In this article, we explored some fuzzy-logical elements in Epicurean logic and indicated how some Epicurean arguments could be strengthened and made more rigorous by implementing those fuzzy-logical tools. We gave a few examples how this can be done. Linking Epicurean logic to fuzzy logic seems like a natural logical and historical progression as the Epicurean sources show a clear relation of Epicurean logic and method of inference to non-classical logics. Certainly, the Epicureans did not implement those fuzzy-logical tools as they were unavailable to them in those times, but we are quite confident that had they known them they could perhaps made a good use of them. Modern Epicureans could certainly enhance some of the old or new Epicurean arguments, on a variety of other topics of philosophical importance, drawing from the fuzzy-logic toolbox. This is important, as some of the ancient discussions and controversial issues are actually quite contemporary. We live in an era where AI (Artificial Intelligence) started being integrated into our

everyday lives more often. Fuzzy logic is central to AI because is a method of reasoning that allows a system to reason under uncertainty and imprecision using degrees of truth instead of crisp truth values. Fuzzy systems do well mimicking human reasoning, making AI technology more dynamic and powerful, opening up a whole new set of scientific and ethical dilemmas.

Notes

1. This paper is based on a talk entitled “The Epicureans on Mathematics: Some Lessons on Axioms, Logic, Experiment and Proof”, given by the author at the ‘7th Panhellenic Conference on Philosophy of Science’ (University of Athens, December 1-3, 2022).
2. Stoic logic, developed primarily by Chrysippus, is a propositional logic that analyzes the relationships between propositions and their truth values. It also focuses on the structure and validity of arguments rather than the specific content of those arguments. Stoic logic is considered a precursor of modern 20th ce propositional logic whose main chapters is what students study in logic classes in schools or colleges today. The Stoics analyzed connectives, such as negation, conjunction, implication, etc, and had ways of evaluating arguments for their validity that share similarities with today’s propositional logic. Yet, they did not have today’s truth-tables and some of their treatment of some connectives (e.g. implication) was not strictly truth-functional.
3. Formally expressed $p \rightarrow q$, where p is the antecedent (hypothesis) and q is the consequent (conclusion). The implication was referred to as “sign-relation” by Stoics (“σημείωσις”). (Sedley 1982). The antecedent and the consequent were referred to as “sign” and “signified” respectively. The statement $p \rightarrow q$ is formally logically equivalent to $\neg q \rightarrow \neg p$ (contrapositive) which is what the Stoics called contraposition (“ανασκευή”). Similar to Chrysippus’ later notion of “συνάρτησις”. (Sedley 1982).
4. It had also an effect on their stance on mathematics. Zeno’s methodological critique of the foundations and logical structure of Euclidean geometry was a consequence of Epicurean logic. For more on this see (Aristidou 2017, 2020a).
5. One of the mistakes is that the author confuses fuzzy logic, i.e. the mathematical theory (or logical system) with its various applications in technology with the “fuzzy” (vague, imprecise, non-formal, etc) way humans reason. One attempt to model human reasoning using fuzzy logic was done in (Aristidou 2004). Another problem with the article is that the Epicurean (empirical) logic that the author alludes to is not clear how it relates to fuzzy logic. There are also several misconceptions of what classical logic is (the author calls it “absolute”), misrepresentation of Aristotle, the nature and scope of mathematics, over-generalities on various social issues, etc.
6. Their titles are “Multiple Explanations Method” (Altas 2015) and “Multi-Valued Logic” (Kardnerinis 2015). Both articles, as well as the one by Patzoglou (2011), are in Greek and their titles are my translations. All three articles appeared in the proceedings of two different symposia organized by the same organization, namely the Friends of Epicurean Philosophy in Greece.

7. One mistake in both articles is that the authors confuse fuzzy logic with multi-valued logic. It seems that both authors think that multi-valued logic is fuzzy logic. This is incorrect and the distinction is clear in Greek texts too. For example, in the high school logic textbook (Anapolitanos et al. 1999). In the articles in question the authors translate multi-valued logic as fuzzy logic and they mistakenly claim that multi-valued logic is based on fuzzy sets (Altas 2015) and that multi-valued logic “has many more values and states (between 0 and 1) that possibly change over time” (Karderinis 2015), etc. There are also historical inaccuracies in both articles. Altas (2015) thinks that the roots of fuzzy logic are found in the Ionian philosophers and in particular in Heracleitus. Karderinis (2015) thinks that Epicurus is the founder of fuzzy logic. Even though there are fuzzy logic elements in Epicurus, as we show in this article, he is certainly not the founder. In Aristotle’s ‘Organon’ and ‘Metaphysics’, as we explain in Note 14 below, there are also fuzzy logic elements which preceded the ones by Epicurus.

8. For example, in Greig (2021) the author ascribes fuzziness to the Epicurean notion of death:

While there is a distinct difference between life and death, I contend that the boundary the Epicureans upheld is fuzzy; the process of dying - the move from life to death - could be an uncertain and potentially painful process that ought to be feared.

9. More on the “paradoxes” of the conditional see Lewis (1917); Farrell (1979); Mansur (2005). Also, for students’ difficulties with the conditional see Hoyles and Küchemann (2002); Romano and Strachota (2016). On whether we need formal logic altogether in mathematics classes see Aristidou (2020b).

10. It is not very difficult to see that the Epicurean logic is not the most appropriate logic for mathematics. Truth in mathematics has stronger conditions and it is tied to proof. So, insisting on contrapositive/contradiction does not guarantee proof. Consider the following two theorems:

- (i) “If a and b are rationals, then $a + b$ is rational”. The proof here is direct. If one tries to prove it by contrapositive/contradiction, i.e., by supposing that $a + b$ not rational, then it does not lead anywhere.
- (ii) “If n is even, then n^2 is even”. Just like the example above, the proof here is also direct. If one tries to prove it by contrapositive/contradiction, i.e., by supposing that n^2 is not even (i.e., it is odd), then it does not lead anywhere.

11. De Lacy seems to be translating “similarity” as “analogy” (De Lacy 1941), something which Sedley pointed out in (Sedley 1982). The two concepts are not identical. Similarity involves inferring something from an observable known sample to a larger observable or unobservable population of the same or similar *kind*. For example, inferring that since humans in our experience are mortal then all humans are mortal, even humans outside our experience. Analogy involves inferring something from certain observable things to other observable or unobservable things that might be different in kind. For example, inferring that since physical bodies have shape then atoms have shape. I believe Philodemus knew the difference too. (see De Lacy 1941).

12. Just as the contrapositive/contradiction (see Note 10), induction is useful but it has a limited scope. For example, the sequence 12, 121, 1211, 12111, 121111, ..., etc., consists of composite numbers. But, surprisingly, the number 1211...1, i.e. the

one that has 136 1's after the 12, is prime. (Jones 2011). This is a number larger than 10^{136} . By induction alone one had to conclude that the statement "the sequence of numbers above consists of composite numbers" is true, which is not. That is because one would have checked sufficiently large number of positive cases to believe that confirm it. How large? Would humanity's lifetime suffice? Life on Earth is estimated to end in about 5 billion years (i.e., $5 \cdot 10^9$ years). The number 1211...1,

though, is way bigger than 10^{100} years. So, induction might not always carry one to the right facts.

13. Read (2012) says that:

The idea that validity requires a relevant connection between premises and conclusion has a long history. It certainly featured in Greek discussion on the nature of conditionals, since Sextus Empiricus, in his history of Pyrrhonism, speaks of 'those who introduce connection or coherence assert that it is a valid hypothetical whenever the opposite of its consequent contradicts its antecedent' (Pyrrhoneiae Hypotyposes, ii 111). He does not say who held this view.

Neither Sextus nor Read mention the Epicureans.

14. In Aristotle too. In the 'Organon' Aristotle spoke of degrees of membership and in his 'Metaphysics' he spoke of degrees of truth. (Belohlavek et al. 2017). Fuzzy logic is based on fuzzy sets which were first rigorously introduced by Zadeh (1965). We keep the technicalities on fuzzy sets and logic to a minimum here. (for an easy introduction see (Klir 1997). Briefly: A classical set A is a collection of objects, for which an element x is either a member of A or not. A *fuzzy set* A^* is a set for which an element x is a member of A^* in some degree. The degree is measured by a membership function $A^*(x)$ which takes values in $[0,1]$. For example, T^* be the fuzzy set of "tall" NBA players, then among others T^* includes Michael Jordan with $T^*(\text{Michael Jordan}) = 0.8$, Yao Ming with $T^*(\text{Yao Ming}) = 0.95$ and Spud Webb with $T^*(\text{Spud Webb}) = 0.3$. The membership-degree function is subjective and can be modelled by a variety of linear functions, piece-wise functions or other curves. *Fuzzy logic* is the logic that uses fuzzy sets to build inferences, arguments, etc. Similarly, one can employ fuzzy sets such as "distance" and "speed" in Robotics and implement fuzzy logic to build "smarter" robots, say for example cars that can park themselves, etc.

15. Magnitude is a fuzzy quantity because is a fuzzy number. A *fuzzy number* x is defined as the fuzzy set of numbers "near" x . Near is captured by a function d (degree of membership) with values from $[0,1]$ which quantifies how near to x a number is. For example, fuzzy 5 contains numbers such as 4.8, 4.2, 5, 5.3, etc, with say $d(4.8) = 0.9$, $d(4.2) = 0.7$ and $d(5) = 1$.

16. As Sedley says, that was a "question put to a later generation of Epicureans, around 100 BCE, by the Stoic Dionysius". (Sedley 2020). Sedley argues as follows:

All colors change
 If a body changes color, this is due to rearrangements
 of its constituent parts
 The parts of an atom cannot be rearranged

 Individual atoms are colorless
 (Sedley 2020)

Suspensions could be raised on the validity of the argument as due to the second premise one could conclude that *atoms do not change color*, not that they are colorless. Which of course would contradict the first premise. So, perhaps one more premise would be required here saying something like “things that do not change colors are colorless”.

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