Rameau's Adjusted Mean-Tone Tuning

In his monumental opus entitled 'Traité de l'harmonie'[1] of 1722, Jean Philippe Rameau established the foundation of tertian theory and brought about the most significant change in the way music was understood and taught since Pythagoras first defined the ratios of consonant intervals in the sixth century BCE. In the 'Traité' he proposed that the triad was the basic building block of music and that melody was born of harmony. As a result of his revolutionary thinking, Rameau was known as the Isaac Newton of harmony.[2] This work differed from other treatises on music at the time, in that Rameau not only addressed theoretical aspects of music but also practical aspects, more specifically composition and accompanying. In fact, in the 'Traité', Rameau dealt forthrightly with virtually all aspects of the music of his day, except for one, tuning.

Pythagoras had discovered the incompatibility of perfect 2/1 octaves and perfect 3/2 fifths around 600 BCE, which is known as the Pythagorean comma. When music was primarily monophonic, the issue was not so difficult to manage; however, with the advent of polyphony, beginning around 1000 CE, it became necessary to temper certain pitches in order to avoid certain out-of-tune intervals. The common system of tuning at the time was called mean-tone tuning, which tempered the tuning so that keys with few or no sharps and flats were in relatively good tune, while leaving those with many sharps and flats somewhat out of tune, with some even severely out of tune. Yet, in the same year that Rameau published his 'Traité', Johann Sebastian Bach published his 'Well-Tempered Clavier' with its 24 preludes and fugues, one in each of the major and minor keys.[3] These and other compositions exploiting tonalities in the far reaches of the tonal system were becoming increasingly prevalent by the early eighteenth century, thus demanding more sophisticated adjustments to the current system of tuning.

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[1] Jean-Philippe Rameau, *Traité de l'harmonie réduite à ses principes naturels,* (Paris: J. B. C. Ballard, 1722).

[2] Thomas Christensen, *Rameau and Musical Thought in the Enlightenment*, (Cambridge: Cambridge University Press, 2006), xxx.

[3] Johann Sebastian Bach, *Das Wohltemperierte Klavier*, 1722. (Bach produced a second set of 24 preludes and fugues in each of the 12 major and minor keys some 20 years later, which is usually published as volume II of *The Well-Tempered Clavier*, BWV 846-893.)

Then, in 1737, Rameau published his 'Génération harmonique'[4]*, in which he boldly endorsed equal temperament as his preferred tuning system, in order to resolve this inherent problem in tuning.*

There is, however, a little-known system of tuning proposed by Rameau in his second treatise of 1726, entitled 'Nouveau système de musique théorique'[5]*, that stands somewhere between mean-tone tuning and equal temperament, which he called 'adjusted mean-tone' tuning. In it, he proposed a series of adjustments to 'mean-tone' tuning in order to resolve most of the dissonance associated with this system. In his proposal, however, he did not provide any mathematical explanations for these adjustments, only a list of verbal directions for the practitioner. It is the purpose of this study to examine 1) the inherent problem in tuning and why temperament is necessary; 2) the evolution of tuning systems prior to the time of Rameau; 3) Rameau's 'adjusted mean-tone tuning', including mathematical implications; and 4) the role of Rameau's 'adjusted mean-tone tuning' system in his evolution as a music theorist.*

The Inherent Problem in Tuning

When Pythagoras defined the consonant intervals in music through the process of string division as he laid out the theoretical basis for music theory in the sixth century BCE, he predicated his scale on the principle of stacked 3/2 fifths as shown in Table 1.[6]

Table 1. *Cycle of Pythagorean Fifths*

F	c	g	d'	a'	e''	b''
2/3	1	3/2	$(3/2)^2$	$(3/2)^3$	$(3/2)^4$	$(3/2)^5$

[4] Jean-Philippe Rameau, *Génération harmonique, ou Traité de musique théorique et pratique* (Paris: Prault, 1737), 94-104.

[5] Jean-Philippe Rameau, *Nouveau système de musique théorique* (Paris: J. B. C. Ballard, 1726), 107-12.

[6] Murray Campbell, Clive Greated, *The Musician's Guide to Acoustics*, (Oxford: Oxford University Press, 1994), 172-73.

Assuming the identify of octaves, when reduced to a single octave these stacked fifth result in the following ratios:

Table 2. *Pythagorean Scale*

c	d	e	f	g	a	b	c′
1	9/8	81/64	4/3	3/2	27/16	243/128	2

In this system the well-known circle of fifths is not a circle at all but rather a spiral of fifths, since by taking this progression to the twelfth 3/2 fifth, B-sharp, and comparing it to the seventh 2/1 octave, C, there exists an excess on the part of the B-sharp known as the *Pythagorean comma*, which may be expressed mathematically as: $(3/2)^{12} \times (2/1)^{7} = 531441/524144$.

However, since our harmonic system is a closed tonal system and B-sharp must be compatible with C, the twelfth fifth (e-sharp to b-sharp) must be diminished by almost a quarter of a semitone so that octaves may retain their pure 2/1 quality. Because of this discrepancy, the Pythagorean scale existed more as a theoretical scale than a practical scale.[7] Consequently, over the centuries many systems of tuning have evolved in an effort to meet the needs of increasingly complicated music. In order to fully understand Rameau's proposed system of tuning, a brief survey of the tuning systems in use prior to Rameau is essential.

[7] Christiaan Huygens, "Musique et mathématique," *Oeuvres complétes*, Vol. XX, (The Hague: Nijhoff, 1940), 76-77.

The Evolution of Tuning Systems Prior to Rameau

Aristoxenus (ca. 300 BCE) reportedly figured a system of temperament that divided the octave into twenty-four equal quartertones, and was distraught that it was not generally followed.[8] Claudius Ptolemy (ca. 90-168 CE), argued that tuning should be predicated on simple mathematical ratios and empirical observations. He borrowed the 3/2 fifth from Pythagoras but based his thirds on 5/4 and 6/5 proportions to create the tuning system known as *just intonation*, in which the C major scale is derived from the proportions found in Table 3.[9]

Table 3. *Intervallic Sources for the Just Intonation Diatonic Scale*

C	D	E	F	G	A	B	C_1
	(2 fifths)	(maj third)	(-fifth)	(fifth)	(maj third-fifth)	(maj third+fifth)	(octave)
1/1	9/8	5/4	4/3	3/2	5/3	15/8	2

By adhering to perfect 3/2 fifths and pure 5/4 major thirds the resulting triads are perfect as in nature (4:5:6), at least those found on the first, fourth and fifth scale degrees. Therefore, these chords sound more harmonious than the equivalent chords using the *Pythagorean* system. For this reason *just intonation* became the tonal ideal of the renaissance period and, thus, the norm by the sixteenth century.[10]

Assuming the tuning starts on C, fifths found on pitches other than C, F and G are problematic in *just intonation*. For instance, the ratio of the fifth between D-A is dissonant because it is 1/54 smaller than the pure 3/2 fifth, and the dissonance is even more pronounced in fifths found in the distant reaches of the tonal

[8] Armand Machabey, Sr., "Jean-Philippe Rameau et le tempérament égal" in *La revue musicale*, numéro spécial 260 (1964), 115.

[9] Jon Solomon, *Ptolemy* Harmonics*: Translation and Commentary* (Leiden: Brill, 2000).

[10] Ben Johnson, *"Maximum Clarity" and other writings on music*, edited by Bob Gilmore, (Urbana and Chicago: University of Illinois Press, 2006), xiv.

system. Additionally, this system contains two different sizes of whole tones. The ratio of the whole tone between C and D is 9/8, whereas the whole tone between D and E has a ratio of 10/9. This difference in whole tones created such tuning difficulties that some keyboards had split keys.[11]

The system known as *five-limit tuning* uses pure 5/4 thirds and 3/2 fifths to generate chromatic *just intonation* scales, of which there are numerous possibilities. For instance, two fifths above C is D, with a ratio of 9/8, while two fifths below C is a B-flat, the ratio of which is 16/9. A major third above D is F-sharp, with a ratio of 45/32, while a major third below the B-flat produces a G-flat, with a ratio of 64/45. Since the F-sharp and G-flat are not compatible, one or the other would be chosen but not both. Several such incompatibilities throughout the scale results in many possible combinations for the chromatic *just intonation* scale. Table 4 demonstrates one often-cited version.[12]

Table 4. *An Example of a Five-Limit Just Intonation Chromatic Scale*

C	$D^{\flat}$	D	$E^{\flat}$	E	F	F#	G	$A^{\flat}$	A	$B^{\flat}$	B	C_1
1	16/15	9/8	6/5	5/4	4/3	45/32	3/2	8/5	5/3	9/5	15/8	2

Considering the out-of-tune fifths that occur on pitches other than those found on the first, fourth and fifth scale degrees, it must be concluded that this scale, like the *Pythagorean* scale, exists more for theoretical use than practical use, and is the reason singers traditionally tempered certain tones when singing and instrumentalists tuned their instruments empirically.[13]

Controversy continued to rage between those who would advocate the use of proportions and those who preferred an empirical method of tempered tuning.

[11] Edwin M. Ripin, *Early Keyboard Instruments*, (New York: W. W. Norton, 1989), 243.

[12] Jon Fonville, "Ben Johnston's Extended Just Intonation—A Guide for Interpreters" in *Perspectives of New Music*, Vol. 29, No. 2 (Summer, 1991), 106–137.

[13] Huygens, "Musique et mathématique." 76-77.

Gioseffo Zarlino (1517-1590) knew of the practical usage of temperament and believed that the inventor of the keyboard searched for and discovered temperament at the same time, even though his scientific scale did not coincide with it.[14] Others engaged in this controversy included Arnold Schlick (*Spiegel du Orgelmacher und Organisten*, 1511) and Pietro Aron (*Il toscanello in musica*, 1523).[15] Francisco Salinas (*De musica*, 1577) supposedly adjusted his tones and semitones by ear, correcting them by some 1/12 of a *Pythagorean comma,* more or less approximating *equal temperament*; Girolama Frescobaldi (1583-1643) tuned his organ by ear in equal semitones; and both Marin Mersenne (1588-1648) and Christiaan Huygens (1596-1687) called for *equal temperament* in the seventeenth century.[16] Huygens did not stop with the 12 tones of the chromatic scale; he endorsed a system that divided the octave into 31 parts, a system that had been rejected by both Salinas and Mersenne earlier.[17]

Huygens' proposal was seemingly overshadowed by the works of Andreas Werckmeister (1645-1708) in Germany, and Joseph Sauveur (1653-1716) in France. Werckmeister never accurately stated *equal temperament*, but he was probably the first to suggest a system of tuning that did not have a *wolf note* in 1681.[18] Sauveur was a scientist and not a musician and, therefore, stated his tuning systems only in scientific fashion with little or no practical application to music. He proposed three systems of temperament in which the octave was divided into 31, 43 or 55 degrees, the first system being the one supported by Huygens earlier. According to Sauveur the 55-degree system was universally employed by musicians by

[14] Gioseffo Zarlino, *Istituzioni harmoniche* (Venice: Senese, 1588), 160 (Supplement).

[15] Hugo Riemann, *History of Music Theory*, translated by Raymond H. Haggh (Lincoln: University of Nebraska Press, 1962), 287.

[16] Machabey, "Rameau et le tempérament égal." 115.

[17] Huygens, "Musique et mathématique." 147.

[18] Thomas Donahue, *A Guide to Musical Temperament*, (Lanham, Maryland: Scarecrow Press, Inc., 2005), 26.

1700.[19] Rameau cited the work of Sauveur in his *Nouveau système,* giving him credit …*for the establishment of a system that gives all the possible temperaments.*[20]

The most common system of tuning in use by the beginning of the eighteenth century was known as *mean-tone* tuning, although it was in use to some extent as early as the beginning of the sixteenth century.[21] It was based on tuning descriptions of Bartolomeo Ramos de Pareja (1440-1522) and is predicated on slightly tempered fifths while retaining pure 2/1 octaves and 5/4 major thirds.[22] These tempered fifths are necessary in order to obtain pure 5/4 major thirds because of the difference between the Pythagorean third, achieved by stacking four fifths: $(3/2)^4$, or 81/64, and a pure 5/4 third, which may be expressed as 80/64, with the difference in the two fifths being 81:80, known as the *syntonic comma*. Therefore, fifths in *mean-tone tuning* are generally diminished by a quarter of a *syntonic comma* so as to produce a pure 5/4 major third while resulting in acceptable fifths. The slightly diminished fifths also allow the distance between C and E to be divided evenly, thereby doing away with the need for major and minor whole tones between C-D and D-E respectively in *just intonation*. Consequently, this *mean-tone* D is what gave this tuning system its name.[23]

There is, however, a significant problem with this system of tuning. While the tempered fifths are necessary in the first four fifths in order to achieve a pure third, when these tempered fifths are continued there is an excess of compensation that is accrued by the eleventh fifth so that the twelfth fifth must be much larger in order to return to the octave C. This means the major third

[19] Joseph Sauveur, "Méthode générale pour former des systèmes tempérés de musique" *in Mémoires de l'Académie Royale des Sciences*, Année 1701, 203-22.

[20] Rameau, *Nouveau système*, 107.

[21] Jeremy Montagu, "Temperament" in *Oxford Companion to Music,* Oxford online, http://www.oxfordmusiconline.com/subscriber/article/grove/music/00253?q=theory+of+affects&search=quick&pos=1&_start=1#firsthit, *a*ccessed 30 June 2017.

[22] Bartolomeo Ramos de Pareja, *Musica practica* (Bologna: Baltasar de Hiriberia, 1482).

[23] Montagu, "Temperament."

between A-flat and C is much too large, creating the infamous *wolf note* associated with *mean-tone* tuning.[24] Table 5 compares the tuning systems described thus far with equal temperament using *cents,* in which the octave is arithmetically divided into 1200 equal parts so that each *equal-tempered* semi-tone is equal to 100 cents.

Table 5. *Comparison of Tuning Systems in Cents*

	C	D	E	F	G	A	B	C_1
Pythagorean	0	204	408	498	702	906	1110	1200
Just intonation	0	204	386	498	702	884	1088	1200
Mean-tone	0	193	386	503	697	890	1083	1200
Equal temperament	0	200	400	500	700	900	1100	1200

By the eighteenth century composers were pressing the limits of *mean-tone* tuning by exploring tonalities with more and more sharps and flats. For instance, J. K. F. Fischer published his *Ariadna musica* in 1702, a collection of 20 preludes and fugues in different keys, which served as a model for Bach's *Well-Tempered Clavier*.[25] In order to accommodate these and the many other works that were appearing around this time, musicians experimented with numerous variants of *mean-tone tuning.*

Andreas Werckmeister's tuning system mentioned earlier, for instance, contained four fifths diminished by a quarter of the *Pythagorean comma* with the remaining fifths all being pure 3/2 Pythagorean fifths, thereby avoiding the *wolf tone*. Others experimented with such temperaments as *1/3-, 1/5-, 1/6-, 1/7-, 2/7- and 2/9-comma mean-tone* tuning in an attempt to resolve the issue of the *wolf-note.* These tuning systems with no *wolf note* are generally referred to as *well-tempered mean-tone tuning* systems, as opposed to the system proposed by Ramos de Pareja

[24] Ibid.

[25] J. K. F. Fischer, *Ariadna musica*, (Schlackenwerth, 1702). (Bach paid tribute to Fischer by using the subject of Fischer's e-minor fugue in his e-minor fugue in the *Well-Tempered Clavier*.)

with its cycle of eleven slightly diminished fifths followed by an augmented fifth, which we will distinguish from now on as *quarter-comma mean-tone tuning*.[26]

It is reported that Bach did his own tuning and may have tempered his major thirds in order to significantly reduce the dissonance of the *wolf note*.[27] Although his *Well-Tempered Clavier* was a collection of preludes and fugues in all twelve keys, it was not intended as an endorsement of *equal temperament*; on the contrary, if that had that been the case he likely would have entitled it *Das Gleichschwebende Klavier*.[28] Baroque composers, including Bach, were known to exploit certain keys for the sake of their inherent dissonance in accordance with the doctrine of *Affektenlehre*.[29] This phenomenon actually dates back to the Greeks, who ascribed certain moods or *affects* to certain modes.[30] Rameau was well versed in this doctrine and not only indulged in it as a composer but also described the *affections* associated with various tonalities in his *Traité de l'harmonie:*

In major keys:

C, D and A are suitable for songs of mirth and rejoicing;
F and B-flat are best suited for tempests and furies, etc.;
G and E are best suited for tender and gay songs;
D, A and E are also suited for grandeur and magnificence;

[26] Owen Jorgensen, *Tuning the Historical Temperaments by Ear* (Marquette: Northern Michigan University Press, 1977), 423.

[27] A Mathematical Approach Reconstituting J.S. Bach's Keyboard-Temperament, in *The Quarterly Journal of the Riemenschneider*, Bach Institute, Berea, OH, Ed. by E. Barber, October 1979, 22, 2-8.

[28] Jeremy Montagu, "Temperament."

[29] George J. Buelow, "Theory of Affects" in Groves Music Online, Oxford Music Online, http://www.oxfordmusiconline.com/subscriber/article/grove/music/00253?q=theory+of+affects&search=quick&pos=1&_start=1#firsthit, accessed 28 June 2017.

[30] Ibid.

and in Minor keys:

D, G, B and E are suited for sweetness and tenderness;
C and F are suited for tenderness and plaints. [31]

The other keys, he indicates, were not in general use because their thirds were not of the correct proportions. He explains:

… the major third, which naturally excites us to joy according to that which we feel in it, drives us as far as ideas of rage whenever it is too large, and the minor third which naturally brings us to sweetness and tenderness, saddens us whenever it is too small.[32]

He explained that the use of these inaccuracies could be exploited in order to create certain desired *affects*, and that …*experience is the best way to learn their properties*.[33]

Rameau's *Adjusted Mean-Tone* Tuning

This was the state of affairs in regards to tuning when Rameau decided to enter the foray in 1726. As a musician he was well aware of the infamous *wolf note* associated with *quarter-comma mean-tone* tuning and of the multitude of efforts being made to address it. As a philosopher he first states the problem:

… temperament is absolutely necessary in the tuning of organs and

[31] Rameau, *Traité*, 164.
[32] Ibid.
[33] Ibid.

clavecins. Musicians observe it in practice quite regularly without any other help than that of the ear...[34]

He felt it his duty to fix the true temperament and to ground it on convincing reason. He based his system on two principles: 1) that the octave is an absolute 2/1 consonance, and 2) that the major third must remain in a perfect 5/4 proportion.[35] Both principles had been well established by Ptolemy in the second century, and were fundamental to both *just intonation* and *quarter-comma mean-tone* tuning. Rameau based his decision to retain pure thirds while accepting tempered fifths on his own experimentation with sympathetic vibrations in strings:

The experiment with strings teaches us that the fifth still trembles with its tuning slightly diminished and the major third does not...[36]

Although this was a mistaken interpretation that he would later correct, at this point the tradition of maintaining pure thirds was so embedded in Rameau's mind that he misread the results of his experimentation. Therefore, having committed to maintaining pure thirds, he sets about to address the problem of over-compensation inherent in *quarter-comma mean-tone* tuning:

When one has arrived in the middle of the tuning, the fifths should be rendered a little more perfectly...down to the last...[37]

[34] Rameau, *Nouveau système*, 107.
[35] Ibid.
[36] Ibid.
[37] Ibid. 108.

This, he explains, is so that distant keys may be in relatively good tune. He then summarizes his proposal step by step, sighting a chart of mathematical ratios used to demonstrate progressions by fifths (1:3:9, etc.) and progressions by thirds (1:5:25, etc.) by which he demonstrates the *Pythagorean comma* and, therefore, the need for temperament.[38]

At this point Rameau simply excuses himself from the necessity of providing a scientific basis for his process of tuning.

> *This is not within the jurisdiction of our system because our system is based on harmonic divisions, that is to say, on unequal parts…one could never find an interval that is exactly the fourth of another.*[39]

The fact that he does not attempt to demonstrate his system of temperament using mathematics reflects the fact that the separation of theory from practice had been the tradition for centuries. However, Rameau is a practitioner as well as theorist, and at this point switches hats and proceeds to give a step-by-step empirical description of how to implement his *adjusted mean-tone tuning*.

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[38] Ibid. The *Table of Progressions* is a foldout inserted between pages 24-25 of the *Nouveau système* containing eight columns of progressions by the interval of a 3/2 fifth (stacked fifths, althrought shown here in descending fashion) to si#### and progressions by the interval of a 5/4 third (stacked thirds shown from left to right) all the way to ut####:

Column 1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
ut 1	mi 5	sol#25	si# 125	re##625	fa### 3125	la###15625	ut####76125
sol 3	si 15	ri# 75	(etc.)				
re 9	fa# 45	(etc.)					
mi 81	(etc.)						
si 243							
fa# 729							
ut# 2187							
sol# 6561							
ri# 19683							
la# 59029							
mi#							
si# 531441							
(etc.)							

[39] Ibid. 111.

Although Rameau does not demonstrate his proposed tuning system in mathematical terms, by applying his own intervallic ratios given in the preface of the *Nouveau système*, it is not difficult to demonstrate his *adjusted mean-tone* system mathematically.[40] He begins his verbal description of his system:

> *There is only a maximum comma too much between the ut and the si# in the first column.* [41]

This is a reference to the *Table of Progressions* cited above in which the B-sharp (si# 531441) in the first column is the result of the cycle of twelve 3/2 fifths. When this B-sharp is compared to the C that results from the stacking of seven 2/1 octaves (not in the Table of Progressions), it produces the *Pythagorean comma* (Rameau's *maximum coma*), which exceeds the C by 24 cents, almost a quarter of a semitone. This comparison was expressed mathematically by Pythagoras as follows: $(3/2)^{12} \times (2/1)^{7} = 531441/524144$. Rameau continues:

> *...there is a major comma and a minor comma too little between the same ut and si# of the fourth column.*[42]

This is the deficit between the B-sharp ($si^{\#}$ 125) resulting from the stacking of three major thirds (C-E-$G^{\#}$-$B^{\#}$) as found in *mean-tone tuning*, and the C. In the preface to the *Nouveau système* Rameau also gives us the value of the *major comma* (*syntonic comma*) as 81/80, and the value of the *minor coma* as 2048/2025. The addition of these two *commas* may be expressed mathematically as: 81/80 x

[40] Armand Machabey presented his own calculations for Rameau's *adjusted mean-tone tuning* in his article cited earlier entitled "Jean-Philippe Rameau et le tempérament égal," 117-18.

[41] Rameau, *Nouveau système*, 111.

[42] Ibid.

2048/2025 = 128/125. Likewise, the *mean-tone* B-sharp may be calculated by stacking three major thirds: $(5/4)^3 = 125/64$; and when compared to the octave, C, the result is the same as above: 125/64 ÷ 2/1 = 128/125.

> *Now by diminishing the fifths by a quarter of a major comma, when one arrives at G-sharp… there will already be a maximum comma and a minor comma less.*[43]

Since four of these "diminished fifths" equals one 5/4 third (C-E), eight of these diminished fifths equal two 5/4 thirds (C-E-$G^{\#}$), which stated mathematically is: $(5/4)^2 = 25/16$. The difference between eight perfect fifths: $(3/2)^8 = 6561/4096$, and the eight diminished fifths as shown above may be calculated thusly: 6561/4096 ÷ 25/16 = 6561/6400. A *maximum comma* plus a *minor comma* is calculated as: 531441/524144 x 2048/2025 = 6561/6400. Therefore, Rameau concludes that:

> *...one cannot dispense with rendering the fifths a little more in tune from this G-sharp to the end in order to recover the minor comma that was lost.*[44]

The difference in Rameau's *adjusted mean-tone tuning* and *quarter-comma mean-tone tuning* is that he proposes steps to recover the *minor comma* lost after eight of his diminished fifths rather than create an even larger deficit by diminishing the four remaining fifths. He designed his system to achieve a true octave by the twelfth fifth so as to avoid the infamous *wolf tone* of *quarter-comma mean-tone tuning*. At this point the formula for each of these last four fifths may be expressed mathematically as: $3/2 \div \sqrt[4]{81/80} \times \sqrt[4]{2048/2025}$.

[43] Ibid.
[44] Ibid.

Although his proposal addresses the issue of the *wolf tone* had he stopped at this point, Rameau was apparently not yet satisfied. He then suggests that it is not necessary to wait until one has arrived at the G-sharp before rendering the fifths larger:

> *This task should commence with the fifth from C-sharp to G-sharp so as to have less to recover with the remaining fifths. By doing so the last major thirds suffer much less from it, although it is necessary in this case to render them somewhat augmented but no more than the last two fifths.*[45]

By beginning the recovery process one fifth earlier, the accumulated deficit will then be one quarter of a *syntonic comma* smaller, and this smaller deficit will be recovered over five fifths instead of four. Although his last sentence above may sound a bit confusing, by suggesting that only the last two fifths should be somewhat augmented, he is implying that the three fifths prior the last two should be pure 3/2 Pythagorean fifths. Therefore, if after seven of Rameau's "diminished fifths" he begins to reclaim the deficit with three pure fifths, followed by two fifths each augmented by 1/3 of a *syntonic comma,* the octave C will then be in tune. This adjustment would leave the most out-of-tune intervals until the last, yet they would not rise to the level of a *wolf tone*. His entire *adjusted mean-tone tuning* system may be mathematically expressed by the following equation: $[(3/2)^7 \div \sqrt[4]{81/80}] \times (3/2)^3 \times [(3/2)^2 \times \sqrt[3]{81/80}] = 2$.

Rameau's final suggests is that the tuning process begin on B-flat, which would not change any of the calculation but would shift the most in-tune portion of the tonality to the most commonly used keys. This was, in fact, quite normal since most tuning systems at the time began the tuning either two or three fifths prior to C.

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[45] Ibid.

Using cents to express the results of his instructions, Rameau's *adjusted mean-tone* system would consist of seven fifths of 696.5 cents each, three pure fifths of 702 cents each, followed by two augmented fifths of 709.25 cents each. In Table 6 Rameau's *adjusted mean-tone tuning* scale is compared to *quarter-comma mean-tone* tuning and *equal temperament*, shown rounded to whole cents. This comparison demonstrates how Rameau's system significantly improves the most severely out-of-tune pitches of the *quarter-comma mean-tone* system, namely C-sharp, D-sharp, F, G-sharp and A-sharp.

Table 6. *Comparison of Tuning Systems*

	C	**C#**	**D**	**D#**	**E**	**F**	**F#**	**G**	**G#**	**A**	**A#**	**B**	**C'**
Quarter-Comma Mean-Tone:	0	79	194	273	386	467	582	697	776	891	970	1085	1200
Rameau's Adjusted Mean-Tone:	0	87	193	298	386	503	585	697	789	890	1007	1083	1200
Equal Temperament:	0	100	200	300	400	500	600	700	800	900	1000	1100	1200

Tuning Systems by Mid-Eighteenth Century

In the eleven years between the publication of the *Nouveau système* and his *Génération harmonique* in 1737 Rameau came to the conclusion that *equal temperament* was the best system of tuning, in which all twelve fifths are diminished by 1/12 of the *Pythagorean comma*. He based his decision on four realizations: 1) that he was mistaken in his experiment with strings in the *Nouveau système* and that thirds, like fifths, may be tempered slightly and still resonate sympathetically; 2) that the ear could adjust to tempered pitches by refocusing on the proper ratios without being confused by the slightly altered interval; 3) that the advantage of having all keys in relatively good tune was more important than the benefits of the *affections* associated with the inherent dissonances of *quarter-comma mean-tone* tuning, and 4) that it could be expressed mathematically.[46]

Table 7. *Rameau's Equal Temperament*

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
1	$\sqrt[12]{2}$	$\sqrt[12]{2^{2}}$	$\sqrt[12]{2^{3}}$	$\sqrt[12]{2^{4}}$	$\sqrt[12]{2^{5}}$	$\sqrt[12]{2^{6}}$	$\sqrt[12]{2^{7}}$	$\sqrt[12]{2^{8}}$	$\sqrt[12]{2^{9}}$	$\sqrt[12]{2^{10}}$	$\sqrt[12]{2^{11}}$	2

By the end of the eighteenth century *equal temperament* had become wide spread throughout most of Europe, although some continued to use one or more versions of *well-tempered mean-tone tuning* for decades, some even as late as the mid-nineteenth century.[47] Those who adhered to the old system continued to strive for pure—or nearly pure—thirds in the early portion of the cycle of fifths and then compensated for any deficit that might have occurred in varying ways during the remainder of the tuning. Some examples of *well-tempered mean-tone tuning* are summarized here in order to better understand the context of Rameau's *adjusted mean-tone* system.

Andreas Werckmeister, mentioned earlier, is considered by many to have proposed the first *well-tempered mean-tone* system, although it is generally considered a theoretical scale. It is included here because it was quite influential. In his lifetime he proposed several versions, of which the one referred to as *Werckmeister III* is commonly cited. In it he uses four fifths tempered by a quarter of the *Pythagorean comma* leaving the remaining fifths pure, and resulting in no *wolf tone*. Interestingly, he uses three tempered fifths beginning on C, followed by two pure fifths, then one tempered fifth followed by six more pure fifths.[48]

[46] Rameau, *Génération harmonique,* 94-104.
[47] Donahue, *A Guide to Musical Temperament,* 25-26.
[48] Ibid. 31.

Johann Georg Neidhardt (ca. 1685-1739) proposed a system that consisted of four fifths tempered by 1/6 of a *Pythagorean comma*, four fifths tempered by 1/12 of a *Pythagorean comma* and four pure fifths; however, he avoided using four consecutive pure fifths, which ruled out the possibility of any Pythagorean thirds, and which kept all his thirds within a range of 392-404 cents. This type of temperament that uses multiple sizes of fifths is generally referred to as a *subtle* temperament.[49] Over his lifetime Neidhardt developed a number of temperaments that he would suggest according to the needs of the user. There is some indication that Bach was aware of Neidhardt's system and may have been influenced by him.[50]

In 1751 Jean-Jacque Rousseau (1712-1778) published an article in *Encyclopédie* defining a tuning procedure in common use at the time in France generally referred to as *Tempérament Ordinaire*.[51] Unlike other systems discussed here, this is not one person's invention but rather a general process that was evidently well known in France in the mid-eighteenth century, and as such there are many variations on this general process. Thomas Donohue includes one such example in his book on temperament that consists of six fifths tempered by 1/5 of the *Pythagorean comma*, four pure fifths, and two fifths augmented by 1/10 of a *Pythagorean comma*.[52] It is different from most *mean-tone temperaments* due to the fact that he uses two fifths larger than a Pythagorean fifth. Perhaps it is not coincidence Rameau's system also used two fifths larger than a Pythagorean fifth, but it is not clear if Rameau influenced *Tempérament Ordinaire*, or if *Tempérament Ordinaire* influenced Rameau. It is a fact that Rameau's system was penned in the

[49] Ibid. 29-30.

[50] Mark Lindsey, "Tuning and Intonation" in *Performance Practice: Music after 1600,* ed. by Howard M. Brown and Stanley Sadie (New York: W. W. W. Norton & Co., 1989), 180

[51] Jean-Jacque Rousseau, "Tempérament Ordinaire" in *Encyclopédie, ou dictionnaire raisonné des sciences, des arts et des métiers*, ed. Denis Diderot et Jean le Rond d'Alembert, Tome 16, (Paris: Breton, et al, 1751), 56-58.

[52] Donahue, *A Guide to Musical Temperament*, 24.

year 1726 while Rousseau's was penned a quarter century later. On the other hand, it is doubtful if Rousseau would give Rameau credit even if it were due since the two men had a falling out in the early 1740's when Rameau was openly critical of an opera that Rousseau had composed.[53]

Johann Philipp Kirnberger (1721-1783) a composer and music theorist influenced by his teacher, J. S. Bach, worked on his own systems of tuning throughout his lifetime. His systems known as Kirnberger II and III are still used by some early music specialists today.[54] In system III he tempered the first four fifths by ¼ of the *syntonic comma*, but after that all the remaining fifths are pure except for F#-C#, which is tempered by 1/12 of a Pythagorean comma. There is no *wolf tone* in this case, but two of his thirds are Pythagorean thirds of 708 cents while three others are at 706 cents.

In summary, Table 8 demonstrates quarter-comma mean-tone tuning in comparison with the *well-tempered mean-tone* systems described above, including Rameau's *adjusted mean-tone* tuning, with their different sizes of fifths.

Table 8. *'Quarter-comma mean-tone' compared to 'well-tempered mean-tone' systems*

	C	**C#**	**D**	**D#**	**E**	**F**	**F#**	**G**	**G#**	**A**	**A#**	**B**	**C_1**
Quarter-Comma Mean-tone: 1490	0	79	194	273	386	467	582	697	776	891	970	1085	1200
Werckmeister: 1681	0	90	192	294	390	498	588	696	792	888	996	1092	1200
Neidhardt: 1724	0	94	196	296	392	488	592	698	796	892	996	1092	1200
Rameau: 1726	0	87	193	298	386	503	585	697	789	890	1007	1083	1200
Tempérament Ordinaire: 1751	0	90	194	294	398	503	588	697	792	892	998	1086	1200
Kirnberger III: 1776	0	90	193	294	386	498	590	697	792	890	996	1088	1200

[53] Cuthbert Girdlestone, *Jean Philippe Rameau: His Live and Works*, (New York: Dover Publications, 1969), 478-79.

[54] David Ledbetter, *Bach's Well-Tempered Clavier: The 48 Preludes and Fugues*, (New Haven: Yale University Press, 2002), 48.

Conclusion

Prior to Rameau, tuning was considered the domain of practitioners rather than theorists, generally speaking, so the concept of treating temperament in a theoretical treatise was a bold step for a young musician set on establishing a revolutionary new theoretical premise like his theory of the fundamental bass. On the other hand, Rameau had already broken with tradition by including accompaniment in his *Traité de l'harmonie* of 1722; therefore, proposing a new system of tuning in his second treatise was just another effort on his part to further enhance musical practice.

Rameau grew up in the middle baroque, just as functional harmony was reaching its maturity, and music theory had not kept up with the harmonic developments of the time. His ear had led him to hear and understand the principle of the fundamental bass, and it was his ear that led him to address the issue of tuning. The world into which he was born held that the major third must be of a pure 5/4 ratio, and some traditions do not die easily. He was so entrenched in this tradition that he misread his experiment with strings as outlined in the *Nouveau système*. Yet the *Nouveau système* was the first time a music theorist ever proposed acoustical science as a basis for harmony. Rameau was, in fact, challenging a long-standing tradition, and many aspects of his new theory would prove difficult, some for the remainder of his life. Yet Rameau was always searching for understanding, and a decade later he admitted his error in regards to the experiment with strings and accepted the fact that thirds, like fifths, could be tempered slightly and still vibrate sympathetically. At that point he adopted *equal temperament* because, as he explained in his *Génération harmonique*, temperament was necessary, it was possible, it could be mathematically achieved, and it was musically superior because all keys would sound alike.[55]

His proposed *adjusted mean-tone* system, much like the *Nouveau système* itself, provides an excellent glimpse of a great mind at work, searching, analyzing, codifying, and developing new terminology as needed, always with a goal of benefiting musical practice. With the publication of the *Traité* he sought to establish the triad as the building block of music with his principle of the fundamental bass, and with the *Nouveau système* he attempted to undergird his principle of the fundamental bass with the principle of the *corps sonore.* With his *adjusted mean-tone* tuning he sought to rectify the wolf-note of *mean-tone* tuning and to clarify the confusing over temperament that surrounded him. And although his new tuning system did not, in and of itself, solve the issue of tuning, it was a step along the way. The problem was that the evolution of music was outpacing the evolution of tuning systems. Rameau's *adjusted mean-tone* system has been largely forgotten, yet by considering it in light of his evolution as a theorist, it may be perceived as an essential step along the way toward *equal temperament*, and, perhaps more importantly, as a significant step in bringing the subject of temperament into the mainstream of music theory.

Rameau is remembered today primarily for his operas and for his theory of harmony, and perhaps some will even remember him for his early endorsement of *equal temperament*. Most historians today consider that his greatest legacy was to move music from a basis in medieval numerology to a foundation in acoustical science. Yet, those who have studied his methodology understand that one of his greatest assets was his ability to think aloud. He was not afraid to admit when he was wrong, to learn from his mistakes, and to keep moving toward his goal. His first efforts to explain what was happening in terms of temperament were

[55] Rameau, *Génération harmonique,* 104.

indicative of these characteristics and provide a snapshot into the workings of a great mind. It was not the end, but rather the means toward an end. Looking back now, Rameau can be viewed in the middle of his career as a giant colossus standing with one foot in the old metaphysical tradition and one foot in the new scientific world. These two disparate worlds separate the *Traité* and the *Génération harmonique*, with the *Nouveau system*—and his *adjusted mean-tone tuning* system—positioned squarely in the middle.

Bibliography

"A Mathematical Approach Reconstituting J.S. Bach's Keyboard-Temperament" in *The Quarterly Journal of the Riemenschneider.* Bach Institute, Berea, OH. Ed. by E. Barber. 22 (October 1979).

Bach, J. S. *Das Wohltemperierte Klavier*. 1722.

Buelow, G. "Theory of Affects" in Groves Music Online, Oxford Music Online. Accessed 28 June 2017. http://www.oxfordmusiconline.com/subscriber/article/grove/music/00253?q=theory+of+affects&search=quick&pos=1&_start=1#firsthit.

Christensen, T. *Rameau and Musical Thought in the Enlightenment*. Cambridge: Cambridge University Press. 2006.

Donahue, T. *A Guide to Musical Temperament*. Lanham, Maryland: Scarecrow Press, Inc., 2005.

Fischer, J. K. F. *Ariadna musica*. Schlackenwerth. 1702.

Fonville, J. "Ben Johnston's Extended Just Intonation—A Guide for Interpreters" in *Perspectives of New Music*. Vol. 29, no. 2 (Summer, 1991).

Girdlestone, C. *Jean-Philippe Rameau: His Live and Works*. New York: Dover Publications, 1969)

Huygens, C. "Musique et mathématique" in *Oeuvres complétes*. Vol. XX. The Hague: Nijhoff. 1940.

Johnson, B. *"Maximum Clarity" and other writings on music*. Ed. by B. Gilmore. Urbana and Chicago: University of Illinois Press. 2006.

Jorgensen, O. *Tuning the Historical Temperaments by Ear.* Marquette: Northern Michigan University Press. 1977.

Ledbetter, D. *Bach's Well-Tempered Clavier: The 48 Preludes and Fugues*. New Haven: Yale University Press. 2002.

Lindsey, M, "Tuning and Intonation" in *Performance Practice: Music after 1600.* Ed. by H. Brown and S. Sadie. New York: W. W. Norton & Co., 1989.

Machabey, A. "Jean-Philippe Rameau et le tempérament égal" in *La revue musicale*. numero special 260 (1964).

Montagu, J. "Temperament" in *Oxford Companion to Music.* Oxford Music Online. Accessed 30 June 2017.http://www.oxfordmusiconline.com/subscriber/article/grove/music/00253?q=theory+of+affects&search=quick&pos=1&_start=1#firsthit.

Murray C. and Clive, G. *The Musician's Guide to Acoustics*. Oxford: Oxford University Press. 1994.

Pipin. E. *Early Keyboard Instruments*. New York: W.W. Norton, 1989.

Rameau, J. P. *Génération harmonique, ou Traité de musique théorique et pratique.* Paris: Prault. 1737.

_______ *Nouveau système de musique théorique*. Paris: J. B. C. Ballard. 1726.

_______ *Traité de l'harmonie réduite à ses principes naturels.* Paris: J. B. C. Ballard, 1722.

Ramos de Pareja, B. *Musica practica*. Bologna: Baltasar de Hiriberia. 1482.

Riemann, H. *History of Music Theory*. Trans. by R. Haggh. Lincoln: University of Nebraska Press. 1962.

Rousseau, J. "Tempérament Ordinaire" in *Encyclopédie, ou dictionnaire raisonné des sciences, des arts et des métiers*. Ed. by D. Diderot et J. d'Alembert. Tome 16. Paris: Breton, et al. 1751.

Sauveur, J. "Méthode générale pour former des systèmes tempérés de musique" in *Mémoires de l'Académie Royale des Sciences* (Année 1701).

Zarlino, G. *Istituzioni harmoniche.* Venice: Senese. 1588.