Embedded Newtonian Stability Analysis of Flared Reentry Probes

The design process for aerospace vehicles is a lengthy, complicated process. This paper seeks to streamline one such aspect – stability-based design – through the usage of a MATLAB interface built upon an analytical approach to unsteady flow dynamics. This study will explore the creation of this interface as well as outline the direction and scope of future development capabilities. To fully model the stability characteristics of a hypersonic aerospace vehicle, complex computer-aided simulations are required. An analytical model can serve to reduce some of this complexity; this is accomplished through the utilization of an embedded, unsteady modification of Newtonian theory, in which segment contributions are considered on a piece-by-piece basis. The interface takes a user generated profile in conjunction with desired flow data and then calculates the resulting stability profile (for static and dynamic modes) in rapid fashion, allowing for ease of prototype modification. Current iterations of the tool are restricted to axisymmetric body profiles – its key advantages lie in its simplicity and piecewise structure. This allows for a somewhat more complex body to be broken down into smaller, simplified components, while still maintaining the ability to quickly determine full-profile stability properties. In addition, the tool can be adapted to work in tandem with CFD studies, thereby allowing for a higher-fidelity stability analysis of more complex body profiles.

Introduction

Motivation

With an ever-increasing demand for space-related technologies, the importance of spacecraft continues to grow with each passing year. By necessity, these vehicles are subjected to a wide range of often extreme flight conditions. Throughout these varied regimes, vehicle stability must be ensured. To design a stable vehicle, an accurate model of expected flight conditions is required. For modern vehicles, this often includes rigorous simulation-based studies, coupled with extensive experimental testing, both of which can be expensive and time-consuming. One potential method for minimizing the time and effort required for early-phase vehicle design is through the usage of analytical models of vehicle flight dynamics. This work will look to take such an approach and investigate the creation of a design tool capable of analyzing flight characteristics of a reentry vehicle. Early focus will be centered on the analysis of flight surfaces added to an initial design (flaring, finning, etc) and how these additions might affect flight stability.
Literature Review

Given the complex nature of a body in flight and the wide range of flight conditions possible, creating stability models for aerospace vehicles has proved to be a rather challenging topic for engineers. Traveling through a fully three-dimensional space results in six degrees of freedom of movement, each of which must be accounted for and controlled. However, due to the nature of craft design, movement within this space is typically governed by a three degree of freedom control system, focused on the angular movements of pitch, yaw and roll. The stability properties of a body are therefore largely concerned with movements along these three angular axes, with each movement having an associated static and dynamic stabilizing response. Much of the work done previously on this modeling has limited analysis scope to one or two forms of stability, typically those associated with pitching and rolling, to simplify the creating of a stability model. (DeSpirito, et al, 2009) This is often further simplified by treating a body as non-rolling, or to be rolling at a constant rate, as a means to limit a model to solely pitching stability. In addition, by focusing studies on axisymmetric profiles, models can be expanded to capture three-dimensional stability effects.

The construction of stability models will typically fall into some combination of the following three categories:

- Experimental-based models
- Analytical-based models
- Simulation-based models

Often times, experimental results and those obtained through simulation will serve a similar purpose: to provide a solid benchmark for an analytical methodology. Much of the earlier work done with regards to stability modeling would utilize data obtained experimentally, then create an analytical basis to curve fit said data into a reliable extrapolation of associated stability properties. (Ericsson, et al, 1966) Later work done on the topic would utilize similar means, while substituting lab-created data with results obtained through a simulation. (DeSpirito, et al, 2009)

The upcoming analysis will largely base its methodology on the approaches taken by Lars Ericsson during his research into stability modeling of reentry profiles. (Ericsson, 1968) These works, and their progression, will be expanded upon in the following section, with specific regards given to the formulation of those theories to be utilized within this paper.

The Stability Analysis of Lars Ericsson

Ericsson’s work was largely focused on the creation of an analytical methodology to model stability properties of slender-boded reentry profiles. This was primarily centered around the usage of a quasi-steady approach to dynamic analysis, as a means to determine the relationship between static and
dynamic stability properties. (Ericsson, et al, 1965) In general, experimental data was combined with an analytical theory, and qualitative relationships were determined for stability properties associated with a specific profile element. These qualitative trends were then refined into a quantitative model to calculate stability coefficients, while accounting for the effects of body-flaring and body-ablation. (Ericsson, 1968)

The first of his works to be looked at within this paper was a study conducted on the dynamics of separated flow over blunt bodies. Conducted in 1965, this work specifically looked into modeling the effects of flow separation on the Saturn-Apollo launch vehicles, with wind tunnel testing serving as the primary source of experimental data. (Ericsson, et al, 1965) Here, quasi-steady methods were used to determine the analytical relationship between unsteady and static stability characteristics. Initial modeling and testing were specifically focused on the upper stages of the launch vehicle, with additional testing done on a simplified, blunted-cylinder-flare body. Ultimately, it was determined that separated flow produced large and highly non-linear aerodynamic loads on the body, which had a dominant influence on the dynamic properties of the vehicle. (Ericsson, et al, 1965) In addition, it was found that the results obtained by using quasi-steady theory were in good agreement with results obtained experimentally, wherein the study was limited to a transonic regime. While results could be linked qualitatively for supersonic flows, a lack of experimental data within these ranges prevented a more direct link from being established. (Ericsson, et al, 1965) This initial study supported that quasi-steady methodologies could be used to compute aerodynamic damping properties, as well as introducing the inverse link between static and dynamic stability brought about by flow separation in conjunction with a finite-time lag.

(Ericsson, et al, 1965)

The next work observed was an initial analysis conducted on the stability effects associated with an ablating body profile. (Ericsson, et al, 1966) While largely speculative in nature, once again due to the difficulty in obtaining a robust set of experimental data, it produced qualitative relationships on the static and dynamic stability profiles of a slender, blunted cone. This analysis was conducted for both rigid and elastic bodies and cataloged the associated stability response for both body types. For ablation concentrated on the profile nose and trailing edge of the cone, it was noted that ablation produced inverse static and dynamic stability effects. (Ericsson, et al, 1965) For low angles of attack and low pitching rates, static stability was increased (while dynamic stability was reduced), while high angles of attack and high pitching rates would result in dynamically stabilizing, statically destabilizing effects. (Ericsson, et al, 1966) Although this study was limited to modeling single degree of freedom pitching oscillations, it demonstrated that the quasi-steady techniques developed by Ericsson and his team could be directly applied to modeling the vehicle dynamics of an ablating body.

This would serve as the basis for the creation of a combined hypersonic model, as a means to link the effects of body flaring and surface ablation. A study was conducted on a blunted-cylinder-flared profile, seeking to model the
dynamics of an ablating flared reentry body. Building upon the previously supported quasi-steady analysis, this study would utilize generalized unsteady embedded Newtonian flow as the primary analytical basis. (Ericsson, et al, 1968) Initially the effects of flaring and ablation were modeled separately, with a full stability profile created through the addition of the two component parts. As before, qualitative relationships were determined for all contributing effects, specifically those of the nose-induced entropy gradient, local flaring effects, and local ablative effects. For a non-ablating body, entropy effects were found to be statically stabilizing and dynamically stabilizing. (Ericsson, et al, 1968) Locally generated forces on a non-ablating flare, in contrast, were found to be both statically and dynamically stabilizing. Local ablative effects affected stability in a similar manner to those of the nose-induced entropy, increasing static stability while leading to greater dynamic instability. Furthermore, it was found that higher ablation rates, larger flaring, and non-spherical nose profiles would all contribute to decreased dynamic damping. (Ericsson, et al, 1965) In addition to the above qualitative relationships, derivative forms of stability coefficients were analytically determined, allowing for a generalized, three-dimensional modeling of both flaring and ablative stability profiles. These models were in good agreement with experimental data, but only at moderate rates of ablation. For higher ablation rates, the lack of a well-developed boundary layer required a different modeling approach.

In 1975, additional work was conducted towards a more refined and expansive analytical modeling of embedded Newtonian theory. Those methods previously set up in Ericsson’s earlier works were given a more generalized form, as a means to extrapolate stability properties across a wider range of flow Mach numbers. Utilizing a curve fit to experimental data, an analytical relationship was created for post shock properties, with equations produced for both slender, blunted cones and blunted-cylinder-flared body profiles. (Ericsson, 1975) These results were in excellent agreement with experimental data, with higher ranges of Mach numbers providing the best fit. For lower Mach numbers and specific slenderness ranges, additional relationships were determined, with a good experimental agreement extending down to a flow regime of Mach 3. (Ericsson, 1975) In addition to modeling stability characteristics, analytical relationships were created to calculate post-shock flow properties, specifically those of local dynamic pressure ratios and local velocity ratios.

The above methodologies for stability analysis would be later applied to future work done on the modeling of a finned missile profile. Conducted at low hypersonic speeds (from Mach 3 to Mach 6), this study sought to categorize the non-linear aerodynamic effects brought about with the addition of body finning. (Ericsson, 1979) As before, an integrated numerical analytic procedure was created to determine the static and dynamic stability responses of the body. Properties were modeled with specific regards given to the effects of nose bluntness and those induced thermodynamically. It was noted that, while viscid-inviscid interaction effects on the fins were minimal, the viscous
interaction between the fins and the body boundary layer greatly degraded the aeroelastic stability properties of the missile. (Ericsson, 1979) In addition, the divergence velocity was greatly overestimated by classical linearized theory. (Ericsson, 1979) Ultimately, this study expanded upon the numerical methods established within Ericsson’s previous works, allowing for the modeling of body profiles with more complex interaction effects.

Additional Approaches

Aside from the works listed above, additional work has been undertaken with the goal to model stability properties of slender-bodied vehicle profiles. This typically will take one of the following forms: initial supporting studies meant to give validation of those theories used to calculate stability properties or direct studies focused on modeling a specific (or range of specific) stability properties. As previously mentioned, these works either utilize experimentation or simulation as a means of analytical validation. The first of the following sources is representative of an experimental study, while the latter two reflect simulation-based approaches.

In 1970, Monson and Kuehn, in coordination with NASA Ames, performed a series of studies on slender body profiles, with the intent to model the hypersonic flight characteristics on hemispheres and cone-cylinders. (Monson, et al, 1970) These studies were conducted on highly cooled bodies, with separate analyses performed for ablating and non-ablating nose profiles, with test conditions set for Mach 14, and Reynolds numbers of 9000 and 1000. Here, experimental data was collected for shock-wave profiles and post-shock surface pressures, as a means to test the predictions (and range of validity) of viscous-interaction theory. Specific interest was given towards the merged-layer regime, in which the body boundary layer and shock layers interact. Ultimately, viscous interaction theory proved effective at accurately predicting shock wave shapes but was unable to fully predict surface pressures. (Monson, et al, 1970) While the theory qualitatively represented the observed flow behavior, within the merged regime, measured pressure were found to be lower than predicted, although results obtained outside of the merged region remained in good agreement with theory. (Monson, et al, 1970)

Key Topics of Focus

Flare Effects

A flared surface refers to a body configuration in which a portion of the body is “flared,” or expanded, outward, increasing the cross-sectional area of the body aft of the flaring. This flaring can serve a variety of purposes, be it to provide additional structural volume (as is present in launch vehicles) or to alter the flight stability of the body. For the purposes of this report, this second type of flaring will be considered. Initially, this study will focus on the effects of end-of-body flaring but will later be adapted to consider the effects of alternative, non-slender body profiles.
This project seeks to create a stability analysis tool for reentry vehicles. This tool will be focused on slender, flared reentry profiles, and will model the static and dynamic stability associated with said profile.

Methodology

Within the scope of this project, two primary deliverables will be required and they are as follows.

1. A Surface Parameterization Tool
2. An Analytical-Based Stability Analysis Tool
Upon completion of the above requirements, all design tools will be combined to create an all-in-one stability modeling script, with MATLAB serving as the language base.

**Surface Parameterization**

To facilitate future analysis, the reentry vehicle profile must be projected into a usable coordinate system. As the proceeding study will be initially two-dimensional in scope, a two-dimensional representation of the vehicle profile will serve as the basis of parameterization attempts. It is important to note that the proceeding two-dimensional analysis can be modified to encompass a fully three-dimensional system analysis. To ensure an accurate transition from two to three dimensions of study, several restrictions must be placed on the profiles to be analyzed.

These profiles must be quasi-one-dimensional in nature, in that they:

- Have axial symmetry in two dimensions
- Maintain rotational symmetry about the main body axis

Additionally, these profiles must either be geometrically simple in nature, or be given a geometric simplification so that:

- All profile surfaces are linear
- All surface segments are of a similar scale to the main body

By restricting all surfaces to simplified linear profiles, the results obtained will better remain within the scope of the analytical basis of this project. Namely, the analytical methods used by Ericsson are designed to model the effects of a single, conical flare with straight edges. While future work can be taken to model the interaction effects of more complex flare geometries, non-linear geometries will be excluded from the scope of this study.

It is important to note that this does not necessarily apply to the geometries of the nose itself. This is due to the fact that the nose geometry is directly responsible for the characteristics of the shockwave profile. While the full effects of altered nose geometries will not be studied within this report, these effects can be captured through a singular drag coefficient associated with a given nose profile. Although altering a nose profile can be found to alter the boundary layer flow across the vehicle profile, for long enough slender bodies, these effects will tend toward a steady state by the time the boundary layer flow reaches the body flaring. As such, it can be found that nose geometry will not prove detrimental for a sufficiently slender body.

The secondary geometric restriction is to keep all surface features within the scale of size as the main body itself. More specifically, any small features (in which the length of the feature is much smaller than the length of the body) should be simplified to a more linear feature, as specified above. For a true reentry profile, such features would largely be caused by external
instrumentation or connective elements. While any additional surface
modifications would alter the flow within the boundary layer, a collective
simplification can serve as a decent approximation of their overall effects. For
the case of features that are both small and sparsely distributed, effects can be
neglected entirely at this time.

One last geometric restriction must be placed on the body profile for the
initial phase of this study. Namely:

- All body features must not result in mid-body expansion waves

Initially, the scope of this study is only concerned with the effects of flared
elements on craft stability. By definition, any element which can be considered
“flared” will expand outward from the main body profile, increasing the
effective radius of the body segment. While these flaring features will produce
expansion waves within the flow, such waves will predominantly alter the flow
within the wake of the vehicle and have relatively minimal effects on the
upstream body elements. Should such a flared element contain a segment
which reduces the effective radius of the body, it will produce an expansion
wave. Within the current scope of the analysis, any body segments within this
realm of expansion are neglected from the analysis. This is due to the large
decrease in pressure found behind an expansion wave, which would in turn
reduce any force contributions from such segments.

**Figure 3. Simple Flared Element**

![Simple Flared Element](image)

**Figure 4. Inverse-Flared Element**

![Inverse-Flared Element](image)
Future work will be taken into studying the effects of multi-flared bodies, including the impact of mid-body shock or expansion waves, as well as the compounded effects of sequential flared elements.

Profile Segmenting

With these considerations in mind, the profile of any given body can be broken up into several key segments. For the purposes of this study, these segments can be divided into the following categories of elements:

- Blunted/Circular
- Body
- Conical

As mentioned above, blunted (or circular) body segments will be restricted to the nose of the reentry profile. Typically, these segments will reflect a semi-circular nose segment, with the primary restriction being a half-angle of no greater than 90 degrees. This is to prevent the introduction of the aforementioned expansion waves. For the following study, the nose will be initially simplified as a 90-degree semi-circle. This represents the most simplistic blunted nose profile, and should allow for results which better match the researched benchmark.

A body segment is simply a linear extension of the body profile, tangent to the axis of body symmetry. For a simple, slender profile, these body segments
will make up the majority of profile length. For body profiles with sharper flaring, a longer body segment can be used to ensure a proper slenderness ratio is maintained.

Lastly, angled extensions of the body (often in the form of an extended conical segment) will be considered conical elements. These segments will refer to both flared body elements, in addition to any nose profiles featuring a blunted-cone topology.

Typically, a full body profile will feature a nose element, followed by some combination of body elements and conical element, with a flared segment located at the trailing edge of the profile. For the purposes of this study, the main body of review will consist of three consecutive segments: a blunted nose (with an arc angle of 90 degrees), a singular body segment, and lastly a singular flared element. This profile of a blunted-cylinder-flare will allow for the main effects of the flaring to be observed, while removing the compounded effects of any additional body elements.

**Design Variable Specification**

For the aforementioned blunted-cylinder-flare, the following three segments must be given a parameterization:

- Nose
- Body
- Flare

As previously specified, the blunted nose will be restricted (initially) to an arc-angle of 90 degrees. With this restriction, the nose can be fully defined with one measurement: nose radius ($r_n$). It is important to note that, given the profile elements selected, nose radius also corresponds with the radius (or caliber) of the vehicle body. When considering more complex geometries in the future; however, these two measurements will not align for nose arc length that is not angularly restricted. Therefore, the nose radius will specifically refer to the radius of curvature of the blunted segment of the nose. Given a nose made up of additional elements, an additional parameter will be introduced to encompass body radius.

For the body segment, the parameterization is once again relatively straightforward. As body elements have been defined as a purely linear segment, parallel to the primary axis of the vehicle, the body segment can be fully defined by parameterizing the length of the body ($l_b$). For more complex geometries with multiple “body” segments, each segment will be given a similar length parameterization.

The last full segment which requires parameterization is the flared portion of the profile. To fully define the flare, two measurements are required: a length of the flare, and the angle at which the flare extends from the main body. For the flare length, there exists three possible measurements which can be used to describe the flare. First, the ultimate radius of the flare, or the largest radial element of the flare, can be selected. The flare can additionally be
described by the chord length of the flare, or the length at which the flare
extends from the body. Lastly, the horizontal extension of the flare, parallel to
the body, can serve as the final means of parameterization. To maintain
consistency within defining all profile length scales, it is the last of these three
which will be used to model the flare profile, where flare length \( l_f \) is defined
as the horizontal length of the flare, taken with respect to the body’s neutral
axis.

With a length scale defined, all that remains is to give an angular
measurement to the flare. For this purpose, the angular offset of the flare
\( \theta_f \) will be taken from the primary axis of symmetry of the flare. It can be
noted that this method of parameterization – using an angle-length pair – can
additionally be utilized for any similarly conical section of the vehicle. This
can apply to mid-body flaring, sequential flaring, or in defining the conical
section of a blunted nose-cone geometry.

While all body segment parameterizations have been determined, one
additional parameter must be given to fully define the vehicle profile for the
upcoming analysis. As this project looks to model the stability (both static and
dynamic) of a reentry vehicle, it is important to include the location of the
center of gravity of the vehicle. By doing so, all body forces can be properly
defined with respect to the body’s center of gravity, therefore allowing a
dynamic interpretation of how these forces will affect the movement of the
vehicle. While for simplicities sake the center of gravity can be determined
generically, this simplification assumes that mass is evenly distributed
within the body profile. Given the internal complexity of a spacecraft, this
restriction would produce results with limited value. Rather, a more
generalized assumption will be taken, in that the center of gravity of the vehicle
is oriented along the body’s neutral axis. This allows for the center of gravity
to be parameterized by a single length value, corresponding to the linear offset
of the center of gravity \( l_{cg} \). Within the context of this study, the center of
gravity will be treated as the coordinate center of the parameterization. As
such, the linear offset will correspond to not to the location of the center of
gravity itself, but rather the distance between the center of gravity and the
radial center of the nose. This allows for the profile to translate about the origin
of the coordinate system without the need for an additional change of reference
frame, while maintaining that all relevant forces will act within relation to the
center of gravity.

With the above measurements, the full body profile is defined within the
reference frame through the following parameters:

- Nose Radius \( r_n \)
- Body Length \( l_b \)
- Flare Length \( l_f \)
- Flare Angle \( \theta_f \)
- Center of Gravity Offset \( l_{cg} \)
For an axisymmetric representation of the body profile, this parameterization takes the following form in Figure 7.

**Figure 7. Surface Parameterization**

The above methodology can be applied when working with more complex vehicle profiles. For a body with a blunted, conical nose and two flared segments, the parameterized surface is found in Figure 8.

**Figure 8. Bi-Flared-Blunted-Cone-Cylinder Parameterization**

With this, a process for surface parameterization has been created, and will be utilized as described above for all future analysis.

**Outline Generation**

The next requirement for a parameterization tool is the ability to recreate a profile surface within the desired interface. For the purposes of this project, the two interfaces used will be MATLAB and STAR CCM+, with MATLAB serving as the primary interface for all analytical study. The previously described parameterization process will be utilized within a MATLAB script. The goal of this script is two-fold and must do the following:
Create a coordinate representation of all profile surfaces
Represent the full coordinate profile graphically

Of these two, translating the profile into a set of two-dimensional coordinates is the primary concern, as this coordinate system will directly be used for the analytical study.

The graphical representation is largely included as a means to benchmark the accuracy of the parameterization program. While it might not be completely necessary for simplistic profiles, it serves as an easy way to ensure all components are properly included. For more complex profiles, it has the additional benefit of allowing a user to confirm that the proper data is being used, before the analysis process beings.

The script begins by taking a set of the above given parameter values \((r_n, l_b, l_f, \theta_f, l_{cg})\). Then, each body component is constructed geometrically, modeled as one of the three component types previously discussed. The coordinate profile is then plotted within MATLAB, as seen below:

**Figure 9. Axisymmetric Profile Representation**

It is important to note that, at this time, the primary scope of the process will be limited to the simplest described parameterization. Therefore, early iterations of the parameterization tool will only seek to recreate similar three-segmented body profiles and all parameter values must be input manually. For later iterations of the code, additional features will be included to allow for automatic parameterization, allowing for more complex arrangements of body features.

**Flare-Specific Profile**

A secondary feature of the parameterization tool is extracting the coordinates of the specific segment to be analyzed. This separation is required
for the analytical methods to be used within this study, which model the
stability-altering effects of purely the body flare.

Three-Dimensional Considerations
At this time, all body profiles are assumed/expected to have rotational
symmetry. As a result, the three-dimensional effects of the body are treated as
having similar symmetry, and the effects of non-symmetric profiles are to
remain outside the scope of this study.

Analytical Methodology

Analytical Background and Setup

Defining Reference Frames
A reference coordinate frame must now be defined, as to create a
parameterization for the reentry body surface. In general, a reentry body can be
approximated as a semi-cylindrical object, with axial symmetry. Accordingly, a
cylindrical coordinate system will serve as the basis for this report. This will
follow the coordinate system utilized by Ericsson in his findings, as can be
viewed below:

Figure 10. Blunted-Flared Cylinder Coordinate Definitions

Here, the primary axis will be aligned with the central axis of symmetry of
the reentry body. To best observe stability properties, the origin will be set at
the bodies center of gravity. The axial coordinate \( x \) will increase towards the
trailing (flared) edge of the body, with a maximum value of \( x \) represented the
back of the body. Additionally, \( x_N \) will refer to the center of the blunted nose.
For the case of a simple blunted body, this is the radial center of the nose arc.
In the case of more complicated nose geometries, \( x_N \) can be best approximated
by the geometric center of a given nose geometry. It is important to note that
this reference axis is fixed with the body and can rotate about the center of
gravity. This rotation about the system’s y-axis will be represented by a
reference angle of attack, \( \alpha \). Additionally, axial rotational velocity will be
represented by a processional rate, \( q \).

To capture radial geometries, a radial coordinate, \( r \), will be used, and will
refer to the distance between the axial body center and the outermost edge of
the body (representing body radius). A secondary radial coordinate, \( R \), will
additionally be used, and will refer to the distance from the axial center of the
shockwave to the outermost body edge. Although both \( r \) and \( R \) refer to the
same body coordinate location, their magnitudes will only be similar in the
case that the body is aligned axially with shockwave propagation. This occurs
during the 0 angle of attack case, and in all such future observations, \( r \) can be
found to be functionally equivalent to \( R \).

For all other cases (in which \( \alpha \neq 0 \)), two additional variables can be defined
to express the relationship between \( r \) and \( R \). The first of these will be used to
express the vertical offset of the shockwave center, \( \Delta z \). As can be seen above,
this refers to the distance between the shockwave’s axis of propagation and the
body profile’s central axis. As previously defined, all pitching will be limited
to rotational movement about the y-axis, ensuring that \( \Delta z \) is a one-dimensional
offset. It is important to note that the definition of \( \Delta z \) is largely one of
convenience and simplification, rather than as a strict design parameter. For a
rigid body (as all profiles within this study will be limited to), values of \( \Delta z \) are
dependent on two conditions:

- Body angle of attack (\( \alpha \))
- Horizontal translation (\( x \))

For a body rotating about a fixed point, located at \( x=0 \), this relationship
can be observed in the following equation:

\[
\Delta z = (x - x_N) \sin \alpha \quad (1)
\]

Given our earlier assumption that the body profile is symmetric about the
x-axis (namely, that it has revolved symmetry), the secondary defining variable
for \( R \) can be expressed through an angle of rotation about the x-axis, \( \phi \). This
angle will be the same angle used to define axial rotation of \( r \) about the x-axis,
rather than the angular rotation about the shock center.

With the above defined, the coordinate location \( R \) can be used to fully
express all surface coordinates of a given body profile. As such, \( R \) will serve as
the primary “location” indicator for all future operations, and can be expressed
in the following equation:

\[
R = \left[(\Delta z + r^2 \sin \phi)^{1/2} + r^2 \cos^2 \phi \cos^2 \alpha\right]^{1/2} \quad (2)
\]
Nondimensionalization

To allow for easier generation of non-dimensional pressure and force coefficients, the above coordinate variables will be made dimensionless using the gauge of the body profile. For a blunted flared cylinder, this is simply the cylinder radius \( c \), giving the following dimensionless coordinates:

\[
\begin{align*}
\xi &= \frac{x}{c} \\
\zeta &= \frac{z}{c} \\
\eta &= \frac{r}{c} \\
\sigma &= \frac{R}{c}
\end{align*}
\]  #(3)

Where \( x, z, r, \) and \( R \) are defined as above.

Embedded Newtonian Shear Flow

For the following analysis, a concept known as Embedded-Newtonian flow will serve as the basis upon which all analytical equations are derived. Based upon the juncture of Newtonian Flow and Blast Wave theory, Embedded-Newtonian flow can be used to determine the pressure coefficient contributions of the body flare.

\[
C_p = C_{p0} + \left( \frac{\rho U^2}{\rho_\infty U_\infty^2} \right) (C_p)_{Newtonian} \]  #(4)

Wherein \( C_{p0} \) is the local pressure coefficient in the absence of a flare (as described by blast wave theory), and \( (C_p)_{Newtonian} \) is the local pressure coefficient obtained through a Modified Newtonian Flow theory, as corrected by the local dynamic pressure ratio.

Newtonian Flow

Newtonian flow theory represents an additional methodology for approximating local surface properties behind a shockwave. While initially developed as a means to model properties of accuracy in accordance with increasingly hypersonic flow regimes. (Anderson, 2006) This is due to the thinning of boundaries between streamlines within a hypersonic flow region, which places said flow lines nearly parallel to local surface geometry. In this way, the local pressure coefficient can be found through a direct correlation with local surface inclination, through the following relation:

\[
C_p = 2 \sin^2 \theta \]  #(5)

In which \( \theta \) is the local surface inclination with respect to the free stream.
Newtonian theory proves incredibly useful for modeling local pressure coefficients for high Mach flows (with increased accuracy as \( M \to \infty \)), while maintaining such accuracy within high temperature flows. In addition, this equation is independent of flow Mach number, and only relies on surface inclination. It is not without its weaknesses, however, and remains best suited to sharp body profiles, with greatly reduced accuracy for blunt or curved body surfaces. With that said, its simplicity proves to be invaluable for simplifying flow analysis, and the aforementioned inaccuracies can be corrected through modifications to the base theory.

**Modified Newtonian Flow**

As the body profile selected within this study is a blunted-flared cylinder, Newtonian theory must be modified as to include blunted profiles. This can be accomplished by modified the primary coefficient of the above equation, resulting in the following modified form:

\[
C_p = C_{p_{\text{max}}} \sin^2 \theta \tag{6}
\]

Where \( C_{p_{\text{max}}} \) is the maximum pressure coefficient associated with a given profile. For a blunted cylinder, \( C_{p_{\text{max}}} \) is simply the stagnation pressure coefficient, which can be calculated using the normal shock relations for a bow shock. For the spherical nose of our body profile, this is represented by the following equation:

\[
C_{p_{\text{max}}} = \left[ \frac{\gamma + 3}{\gamma + 1} \right] \left[ 1 - \left( \frac{2}{\gamma + 3} \right) \left( 1/M_{\infty}^2 \right) \right] \tag{7}
\]

While this introduces a dependency on the flow Mach number, this modification to Newtonian theory better approximates pressure coefficients for non-flat body profiles. Although it still maintains a level of inaccuracy for curved surfaces, given our profile is made up of blunted and flat body segments, it will prove sufficiently accurate to model surface flow properties at this time.

**Modified Newtonian Flow – Three-Dimensional Corrections**

In order to encompass full flow modeling for a three-dimensional body of revolution, one additional correction must be made to the Modified Newtonian theory. This can be accomplished by generalizing the approximation of local flow inclination. As previously explained, for a hypersonic flow, local flow inclination can be approximated as proportional to local body inclination. In Modified Newtonian theory, this local inclination contribution is represented within \( \sin^2 \theta \). This can be generalized into three-dimensions, by utilizing local flow velocity components in conjunction with trigonometric identities. For two-dimensions, this can be observed in Figure 11.
Figure 11. Axial Velocity Component Relations

Where $U$ represents the axial flow velocity, which can be separated into flow components normal to ($V_\perp$) and parallel to ($V_\parallel$) the body surface, and $\theta$ is the local flow inclination. Here, axial flow and normal flow can be related through a trigonometric identity, in which:

$$\sin \theta = V_\perp / U \tag{8}$$

Combining this identity with Modified Newtonian theory, a new general form is obtained:

$$(C_p)_{Newtonian} = C_{p_{max}} (V_\perp / U)^2 \tag{9}$$

For a three-dimensional body of revolution, the velocity ratio $V_\perp / U$ can be expressed as follows:

$$V_\perp / U = \cos \alpha \sin \theta + \sin \alpha \cos \theta \sin \phi + \{(x + r \tan \theta)q\} / U \cos \theta \sin \phi \tag{10}$$

This expanded general form accounts for a three-dimensional location coordinate (as expressed by $\theta$, $\phi$, $x$, and $r$) in addition to a pitching of the body ($\alpha$) with pitch rate ($q$). For a steady body with no pitching rate, this reduces to a simplified form:

$$V_\perp / U = \cos \alpha \sin \theta + \sin \alpha \cos \theta \sin \phi \tag{11}$$

Here, the normal velocity ratio is purely a function of local surface inclination ($\theta$), pitching angle ($\alpha$), and radial displacement along the revolved surface ($\phi$).
With the Newtonian pressure coefficient defined above, all that remains is to create similar expressions for the remaining two components of equation 4:

- Dynamic pressure ratio
- Pressure coefficient in the absence of a flare

Both of which can be described through blast wave theory.

**Blast Wave Theory**

A variation on the hypersonic equivalency principle, blast wave theory proves an effective method for determining the pressure distribution on axisymmetric blunt-nosed cylinder traveling within the hypersonic regime. (Anderson, 2006) Here, the shockwave is modeled as a mechanical introduction of energy to the system, originating from the blast point. Given that the body profile is axisymmetric, and this symmetry is aligned with the direction of body motion, the shockwave can be expected to have similarly axisymmetric propagation, as can be observed in Figure 12 below.

**Figure 12. Blast Wave Theory for a Blunt-Nosed Cylinder**

These propagation effects are primarily driven by the drag properties of the nose, in addition to the Mach properties of the free stream. As such, shockwave profiles can be “categorized” for specific nose drag profiles, and correlating models can be derived for a given category of profile. For a blunt-
nosed cylinder, surface pressure distribution is described by the following equation (Anderson, 2006):

\[ C_p = \frac{0.096C_D^{1/2}}{(x/d)} \]  

Where \( x \) is the axial coordinate, \( d \) is the cylinder diameter, and \( C_D \) is the nose drag coefficient.

Within his work on Generalized Unsteady Embedded Newtonian Flow, Ericsson combined this methodology with existing experimental data to form such models. (Ericsson, 1975) This included blast wave models for blunt-nosed cylinders, in addition to models for blunted cone and wedge-based profiles. Of these, his model for blunted cones best matches the pressure distribution on blunted-cylinder-flare configuration of this study’s selected profile.

In order to determine pressure distributions, Ericsson’s model is composed of two primary parts. (Ericsson, 1975) These are analytical models for:

- Local Dynamic Pressure Ratio (\( f \))
- Local Velocity Ratio (\( g \))

As represented by the following functions of a similarity parameter (\( x^* \))

\[ f^*(x^*) = \frac{\rho U^2}{\rho_\infty U_\infty^2} \]  

\[ g^*(x^*) = \frac{U}{U_\infty} \]  

This similarity parameter is representative of the combined effects of nose properties and pressure location, as seen in the following equation:

\[ x^* = \left[ \frac{R}{d_N} - \frac{1}{2} \right] / (C_{D_N})^{1/2} \left[ \frac{x}{d_N} - \frac{x_N}{d_N} \right] \]  

Where \( d_N \) and \( C_{D_N} \) are nose diameter and drag coefficient respectively; \( x \) and \( x_N \) are axial coordinates as described previously; and \( R \) is the location coordinate as described by Equation 2.

For a blunted cylinder body, the dynamic pressure ratio is expressed as follows:

\[ f^* = 0.17 + 2.75x^* + 4x^{*2} \]  

The corresponding velocity ratio is derived as follows:

\[ g^*(x^*) = \begin{cases} g_0 + k_g x^{*2} & x^* < x_{crit}^* \\ \frac{1}{x_{crit}^*} & x_{crit}^* \leq x^* \end{cases} \]  

Where coefficients \( g_0 \) and \( k_g \) are described through the following equations:
$$g_0^* = 1 - k_g x_{crit}^* \text{(18)}$$
$$k_g = \{0.304B/[1 + (3.33 - A)/2.33]\}^{1/2} \text{ (19)}$$

Which in turn present three additional coefficients ($A$, $B$, and $x_{crit}^*$), which are analytically modeled through the following three equations:

$$A = \begin{cases} 
3.33 & \frac{\theta_c^*}{\mu} \leq 0.23 \\
3.33 - 3.03[(\frac{\theta_c^*}{\mu}) - 0.23] & 0.23 < \frac{\theta_c^*}{\mu} < 1 \\
1 & 1 \leq \frac{\theta_c^*}{\mu}
\end{cases} \text{ (20a)}$$

$$B = \begin{cases} 
1.375 - 0.5[0.17(10 - M_{\infty})]^3 & 3 \leq M_{\infty} < 10 \\
1.375 & M_{\infty} \geq 10
\end{cases} \text{ (20b)}$$

$$x_{crit}^* = (1 - f_0^*)/AB \text{ (21)}$$

Where $f_0^*$ is expressed through the following relation:

$$f_0^* = 0.165 + [9.65/(M_{\infty} + 8.7)]^3: M \geq 3 \text{ (22)}$$

As these coefficients are representative of a curve fit to experimental data, their values are dependent on two flow-defining regimes:

- Mach Number ($M$)
- Shock Angle Sensitivity to Body Angle ($\frac{\theta_c^*}{\mu}$)

As with the case of Newtonian theory modeling, the above analytical equations are best suited to modeling hypersonic flows. They begin to diverge rather rapidly from experimental data, of which is only modeled for flows of at least Mach 3. For all flows greater than Mach 10 (and less than Mach 20), the equations provide a better fit to flow conditions. Given that this paper seeks to model flows within reentry Mach regimes, this remains well suited.

The secondary parameter, parameter ($\frac{\theta_c^*}{\mu}$), is used to account for a limit for shock sensitivity. Here, $\theta_c^*$ stands in for an effective cone angle for a body segment, where:

$$\theta_c^* = \sin^{-1}(V_\perp/U) \text{ (23)}$$

Where $V_\perp/U$ is given through Equation 10.

The parameter $\mu$ is used to express a limiting body angle. When the effective cone angle of the body goes below $\mu$, shock angle is no longer dependent on body angle, requiring a separate analytical relationship.

It is important to note that the above analytical equations are only valid for blunted cylinder flare bodies. While additional models exist for alternate body profiles, these too are limited to one specific profile type, minimizing the range at which said equations can operate. Given the complexity of hypersonic...
interaction effects, it is quite difficult to fully generalize the above relations for any given body type. To model these interactions, later work will rely on the creation of a catalog of additional profile modeling data.

**Shaded Body Considerations**

With dynamic pressure and velocities ratios now defined through the previously described analytical equations, one additional consideration is required before deriving stability parameters. This comes from the basis of Embedded Newtonian flow theory as a local inclination method. As previously explained, for a hypersonic flow, local surface pressure distributions are heavily reliant on local surface inclination, given the nature of thin shock layers which form near the body profile. These effects, however, are largely limited to profiles (and resulting flows) in which the body profile is angled toward the flow field in a manner to produce shockwave effects. For such positive angles, local flow fields can directly impart pressure-driven forcing effects on the body surface, contributing to the aerodynamic forces experienced by the profile. In contrast, for a body angled away from the flow field (in a manner consistent with the production of an expansion wave), these pressure-driven effects are greatly minimized due to the resulting vorticity created by the “curving” of the flow around a body edge. This minimization is only increased further for higher Mach flows, in which post expansion pressures are further reduced. These effects become more pronounced the sharper the boundary between two profile segments, in which secondary segments are angled further from the flow.

For a flow aligned blunted flared cylinder, this shaded region is limited to the trailing edge of the flare. However, as the body is angled within the flow (as denoted by an increased angle of attack $\alpha$), after a critical $\alpha$ is reached, additional regions of the profile are shaded from the flow. This is both due to the angling of the body, in which surface regions are fully angled away from the flow, in addition to the shading effects imparted by the cylindrical body segment on the body flare. These combined effects will be modeled within the following parameter, $\phi_1$, which represents the angular profile boundary between shaded and unshaded flow. This will allow for future surface integration to be limited to only non-shaded regions.

The exact value of $\phi_1$ will be determined geometrically and is heavily dependent on the overall profile configuration. In addition, for certain angled configurations and body profiles, $\phi_1$ will not be constant along the length of the body region. For the flared profile considered within this study, this discrepancy occurs when the body is angled in such a manner that the blunted cylinder shades the flared region from the flow. This gives $\phi_1$ the following dependencies:
Profile Configuration
- Body Cylinder
  - Length ($l_b$)
  - Radius ($r$)
- Body Flare
  - Length ($l_f$)
  - Angle ($\theta$)
- Angle of Attack ($\alpha$)
- Axial Coordinate Location ($x$)

Flaring Effects

With a methodology established for modeling local surface pressure coefficients, stability effects of flaring can now be determined. These effects can be broken up into two categories:

- Static Stability
- Dynamic Stability

Both of these refer to the innate flight characteristics of a design, specifically in regards to how a particular design is affected by outside perturbations.

Static Stability

Within aerodynamics, static stability refers to the instantaneous flight response of a body when it is acted upon by a force. This response will typically fall into one of three categories:

- Positive
- Negative
- Neutral

A positive response is marked by the tendency of a body to return to stable flight after perturbation, while a negative response results in decreased trajectory stability. For a neutral response, flight trajectory remains altered as a result of a perturbation, with no further induced effects. For a non-neutral profile, this response is typically proportional to the magnitude of the outside force, with larger forces resulting in more extreme response patterns. As such, the previously modeled pressure data will be used to derive local force coefficients.

Static Force Coefficients

An aerodynamic force coefficient is a nondimensional metric for defining a surface force, typically resulting from a pressure distribution on a body. This can be split up into axial and normal force components and solved piecewise
across the body as a function of a local area segment \((\Delta A)\). For a three-dimensional axisymmetric body of revolution, axial and normal forces per area element are as follows:

\[
\Delta C_A = (4/\pi)C_p\eta d\eta d\phi \tag{24}
\]

\[
\Delta C_N = (4/\pi)C_p\eta d\xi d\phi \sin \phi \tag{25}
\]

Where \(\Delta C_A\) and \(\Delta C_N\) refer to axial and normal components respectively, and \(C_p\) is as defined in Equation 4.

For a body in flight with initial angle of attack \(\alpha\), the resulting static response takes the following form:

\[
\Delta C_{N\alpha} = (4/\pi)\left[\partial C_p/\partial \alpha + \partial \Delta^i C_p/\partial \alpha\right] \eta \sin \phi d\xi d\phi \tag{26}
\]

Where \(\partial C_p/\partial \alpha\) and \(\partial \Delta^i C_p/\partial \alpha\) are the local and induced pressure coefficients for a given area segment.

Nose induced pressure effects can be found through the following equation:

\[
\partial \Delta^i C_p/\partial \alpha = \left(\partial C_p/\partial \zeta\right) \left(\partial \zeta/\partial \alpha\right) \tag{27}
\]

Where:

\[
\partial \zeta/\partial \alpha = (\xi - \xi_N) \cos \alpha \tag{28}
\]

The resulting pitching response can be found by solving for the moment produced by the normal force defined in Equation 26, as expressed below:

\[
\Delta C_{m\alpha} = -\Delta C_{N\alpha} [\xi + \eta (\partial \eta/\partial \xi)] \tag{29}
\]

**Dynamic Stability**

While static stability refers to an instantaneous response, dynamic stability refers to the effects of this response over time. As with static stability, dynamic stability can take three forms:

- Positive
- Negative
- Neutral

For dynamic stability, however, these three categories refer to the motion damping properties of a profile. Positive dynamic stability qualities represent dampened motion, in which the oscillations of a perturbed body will dampen out over time. Negative dynamic stability, in contrast, results in oscillations which are amplified over time. Lastly, for neutral dynamic stability, these oscillations remain constant and undampened.
Dynamic Force Coefficients

Normal force and moment coefficient can be derived for dynamic considerations in a similar manner to the above static derivations. The dynamic normal force corollary to equation 26 is as follows:

$$\Delta \left( C_{Nq} + C_{Na} \right) = \frac{4}{\pi} \left[ \frac{\partial C_p}{\partial (c q / U_\infty)} + \frac{\partial \Delta C_p}{\partial \xi} \frac{\partial \zeta}{\partial (c \alpha / U_\infty)} \right] \eta \sin \phi d \xi d \phi \#(30)$$

Where $$\frac{\partial C_p}{\partial (c q / U_\infty)}$$ refers to pressure contributions due to pitching rate, and $$\frac{\partial \Delta C_p}{\partial \xi} \frac{\partial \zeta}{\partial (c \alpha / U_\infty)}$$ refers to nose induced pressure contributions. Pitching rate contributions can be found through equation 5, resulting in the following equation:

$$\frac{\partial C_p}{\partial (c q / U_\infty)} = 2C_{p_{\text{max}}} \frac{f(\sigma) V}{g(\sigma) U} (\xi + \eta \tan \theta) \cos \theta \sin \phi \#(31)$$

Where $$C_{p_{\text{max}}}, f(\sigma), g(\sigma), \text{and } V / U$$ are given by Equations 7, 15, 16, and 11 respectively. The secondary component of Equation 30, namely nose induced flow perturbations, can be modeled by considering the time lag between nose shear flow and local flow effects. Assuming velocity ratio remains constant along a streamline, this can be approximated by the following equation:

$$\Delta \tau \approx c(\xi - \xi_N) \cos \alpha / U_\infty g(\sigma) \#(32)$$

The relative position of the flare within the nose induced flow, as a function of time, can be represented by the following equation of motion:

$$\Delta \zeta = \zeta(\xi, t) - \zeta(\xi_N, t) + \Delta t \zeta(\xi_N, t) \#(33)$$

By combining Equations 32 and 33, the following relation is obtained:

$$\frac{\partial \zeta}{\partial (c \alpha / U_\infty)} = \xi_N(\xi - \xi_N) \cos \alpha / g(\sigma) \#(34)$$

Thereby fully describing all components of Equation 30.

As with the normal force coefficient, a similar correlation can be found for the dynamic moment coefficient. This corresponding derivative is given by the following:

$$\Delta \left( C_{mq} + C_{mA} \right) = -\Delta \left( C_{Nq} + C_{Na} \right) \left[ \xi + \eta (\partial \eta / \partial \xi) \right] \#(35)$$
Total Force Coefficients

General Form
In order to determine the full force contributions acting upon a body profile, the above force coefficients derivatives must be combined. This is accomplished by integrating said derivatives across the body profile. As the profile is axisymmetric, this requires two integration steps.

- Summation across the direction of revolution ($d\phi$)
- Summation along the primary body axis ($d\xi$)

Given that the profile does not have a constant radius for along its length, integration will first be performed in the direction of revolution, followed by integration along the primary body axis.

Static Force Coefficient Integral Form

The axial force coefficient for a given body segment ($d\xi$) is described by the following integral equation:

$$
\frac{dC_A}{d\xi} = \frac{8}{\pi} \eta \tan \theta \int_{-\phi_1}^{\pi/2} C_{p_0} d\phi + \left( C_{p_{max}} \cos^2 \alpha \right) \left( \eta \sin 2\theta \right) \int_{-\phi_1}^{\pi/2} f(\sigma)(\tan \theta + \tan \alpha \sin \phi)^2 d\phi \tag{36}$$

Where all equations components are given in the preceding sections.

A similar integral form can be obtained for the normal force coefficient. For ease of integration, normal force contributions will be split up into two parts – local effects and induced effects – as seen in the following equation:

$$
\frac{dC_{N\alpha}}{d\xi} = \frac{dC_{N\alpha L}}{d\xi} + \frac{d\Delta C_{N\alpha}}{d\xi} \tag{37}$$

For local contributions, the general, two-dimensional axisymmetric form of the normal force integral is as follows:

$$
\frac{dC_{N\alpha L}}{d\xi} = \frac{16}{\pi} C_{p_{max}} \cos^2 \alpha \left( \eta \cos^2 \theta \right) \times \int_{-\phi}^{\pi/2} f(\sigma)\left[ \tan \alpha \sin^3 \phi + (1 - \tan^2 \alpha) \tan \theta \sin^2 \phi - \tan \alpha \sin^2 \theta \sin \phi \right] d\phi \tag{38}$$

While the induced force contributions are given by equation 8.4.
This integral contains one addition component, \( \frac{\partial \sigma}{\partial \xi} \), which relates the dimensionless form of radial position (with respect to shock center) to the vertical displacement of said position from the shock center, as expressed in the following equation:
\[
\frac{\partial \sigma}{\partial \xi} = \frac{(\xi - \xi_N) \sin \alpha + \eta \sin \phi}{\{[(\xi - \xi_N) \sin \alpha + \eta \sin \phi \cos \alpha]^2 + \eta^2 \cos^2 \phi\}^{1/2}} \tag{40}
\]

The general form of the moment equation can be found through the following relation:
\[
\frac{dC_m}{d\xi} = - (\xi + \eta \tan \theta) (dC_N/d\xi) \tag{41}
\]

Where \( dC_N/d\xi \) is given through equation 37.

**Dynamic Force Coefficient Integral Form**

Through a similar methodology as above, an integral form of the dynamic force coefficients can be obtained. First, the normal force coefficient is split into two components:
\[
\frac{d}{d\xi} \left( C_N + C_{N\dot{}} \right) = \frac{dC_N}{d\xi} + \frac{dC_{N\dot{}}}{d\xi} \tag{42}
\]

The integral form of the pitching rate contributions is as follows:
\[
\frac{dC_{N\dot{}}}{d\xi} = \frac{16}{\pi} C_{p_{max}} \cos \alpha (\xi + \eta \tan \theta) \eta \cos^2 \theta \times \\
\int_{-\phi_1}^{\pi/2} \left[ f'(\sigma) \right] (\tan \theta + \tan \alpha \sin \phi) \sin^2 \phi d\phi \tag{43}
\]

While the integral form of the nose induced contributions is expressed below:
Lastly, the dynamic moment coefficient can be obtained through equation 41, with normal force taken as expressed in equation 42.

**Quasi-1D Equations**

For a non-angled body \((\alpha = 0)\), the above equations, and resulting analysis, can be further simplified. With a zero-degree angle of attack, radial coordinates \(R\) and \(r\) are equivalent. In nondimensional terms, this means that \(\sigma \equiv \eta\), further reducing the equations.

**Static Force Coefficients**

At zero angle of attack, equations 37, 38, and 39 reduce as follows, with the total normal coefficient expressed below:

\[
\frac{dC_{Na}}{d\xi} \bigg|_{\alpha=0} = \frac{dC_{NaL}}{d\xi} \bigg|_{\alpha=0} + \frac{d\Delta^iC_{Na}}{d\xi} \bigg|_{\alpha=0} \tag{45}
\]

Simplified local effects are given by equation 46:

\[
\frac{dC_{NaL}}{d\xi} \bigg|_{\alpha=0} = 8C_{p_{max}} f(\eta) \eta \cos^2 \theta \tan \theta \tag{46}
\]

Likewise, simplified induced effects are given by equation 47.

\[
\frac{d\Delta^iC_{Na}}{d\xi} \bigg|_{\alpha=0} = 4 \frac{dC_{p_0}}{d\eta} (\xi - \xi_n) \eta + 4C_{p_{max}} \frac{\partial f}{\partial \eta} (\xi - \xi_n) \eta \sin^2 \theta \tag{47}
\]

**Dynamic Force Coefficients**

Dynamic coefficient equations reduce in a similar fashion, with local contributions expressed in equation 48 and induced effects expressed in equation 49:
Lastly, all moment coefficients are given through a zero-angle of attack corollary to equation 41.

**Total Force and Moment Coefficient Contributions**

With a means to determine all relevant force coefficients as a function of piecewise length \( d\xi \), the total force coefficients for the flare can be derived by integration all radial component effects along the length of the flare. For the normal force coefficient, this is expressed in the following equation:

\[
C_N = \int_{\xi_0}^{\xi_{\text{max}}} \frac{dC_N}{d\xi} \, d\xi \quad #(50)
\]

Where \( \xi_0 \) and \( \xi_{\text{max}} \) represent the axial boundaries of the flare in \( \xi \).

This process can additionally be used to find the total moment coefficient attributed to the flare, for both static and dynamic coefficients.

**Findings and Future Work**

**Improving Modeling Methods**

When translating the above model into a user-friendly MATLAB format, it was found that the model itself was not without its limitations. While stability coefficients could be generated for the selected profile, additional work is required to create a robust stability modeling tool capable of capturing more varied profiles. Two key areas of future focus are: the utilization of simulation analysis to benchmark the analytical method and the expansion of Embedded Newtonian theory to capture modified geometries.

**Generating High-Fidelity Coefficients**

While the above work presents one methodology (analytical) for determining flow properties on a body, this approach can be combined with a simulation-based method as a means to improve both. By creating a blend of simulation and analytics, higher fidelity stability coefficients can be obtained.

As previously described, using an analytical model to study stability properties of a profile is a much more direct and simple process than performing an in-depth simulation. By using a previously formed model, desired results can be obtained much cheaper and faster than those found
through a similar simulative approach. With a robust enough model, these analytical results can hold a similar order of accuracy to more complex simulations. However, it is this last point, the robustness of a model, which is one of the primary limitations of an analytical model. As outlined when discussing blast-wave models, many analytical models are built around existing experimental data. This data is often limited in scope, minimizing the breadth of its applications. For the case of blast wave theory, the analytical models formed are limited to very specific profile configurations at additional specific Mach ranges. In order to utilize such a model outside the realm of previously found data, new experimentation or an entirely new model is required. However, this process can be somewhat simplified and streamlined by using simulation as a means to both verify existing models and expand the bounds in which a model can operate.

Additional Geometries

Currently, the preceding analysis is rather limited in scope, with the primary analytical model only able to accurately model stability properties of a singular profile type (being the blunted-flared cylinder). By creating a more robust model, which is able to accurately predict stability properties for a range of body profiles, the overall applications of said model can increase in scope.

The simplest means to expand upon the modeling capabilities of the above analysis lies in the way in which the profile nose is modeled. As the primary source of shockwave shape and properties, having an accurate model of nose properties will greatly serve the overall range of the model. In addition, many of the equations used within the above analysis only depend on nose drag coefficient and effective center, little changes need to be made to the existing model. Rather, all that would be required would be to create a catalog of drag and shockwave profiles associated with a range of nose configurations. This would still limit the analysis to blunted profiles, however, as a sharp nose would result in the need for an altered shock model.

Some advanced modifications to the model are as follows. In its current form, the model is rather limited in its ability for modeling expansion regions, as it neglects the effects of such shaded body regions. To fully account for these effects, or to model additional body regions which exist post expansion, an additional model would be required. Depending on the complexity of the body configuration, this could take the form of researching or deriving an expansion corollary to the models presented within blast-wave theory and embedded Newtonian shear flow theory.

These above modifications could prove useful for modeling non-slender geometries, such as the modeling of a “capsule” based body profile. By modeling the profile as a cone/flare, a blast-wave analysis can be used to model the stability properties of said profile using the previously outline analytical model. It is important to note that this would require additional modifications to ablation modeling, given that the heat flux would no longer be constant across the full profile surface, given the stagnation point centered at the tip of the cone.
For increasingly complex, combined profiles (as outlined in figure 8), a full revamp of the model base would be required. This could potentially be accomplished by separating the profile into regions, and computing stability effects in a consecutive, piecewise fashion. This, however, would require additional alterations to the induced and time-lag effects on stability, as each profile segment would be rigidly connected.

Lastly, model effects can potentially be expanded to include non-rigid bodies. Of the proposed modifications, this would be the most complex addition, as all stability modeling components are defined for a rigid profile.

**Conclusion**

Ultimately, while a fully robust model was not produced at this time, this study has served to create the groundwork for future model creation. The analytical model created by Ericsson was researched and expanded upon, leading to the generation of stability coefficients for ablating and non-ablating flared profiles. This model was then integrated within a MATLAB code capable of force coefficient determination for a given flared-slender body. Furthermore, a catalog was created containing modeled flow properties for slender conical and non-flared bodies, for use in the creation of a more generalized stability model. By creating simulation models, flow property profiles (including dynamic pressure and velocity ratios) can be produced for a wider range of profile types. These, in turn, can be incorporated within the MATLAB model to create a more robust and accurate stability modeling tool.

**References**


