The World Formula

How to solve (almost all) problems of the world

Transfer Functions relate the output or response $\boldsymbol{y}$ *of a system such as a filter circuit* $\boldsymbol{A}$ *to the input or stimulus* $\boldsymbol{x}$*. They address the World Formula* $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$*. Explaining transfer functions to students of the 21st century is paramount for understanding what constitutes the Internet, and many methods and techniques such as Big Data and Artificial Intelligence. For linear functions between vector spaces, the Eigenvector method makes calculating a solution* $\boldsymbol{x}$ *easy, if it exists. Numerical algorithms are available for solving. Thus, the method is suitable for teaching students who are interested in the foundations of technics. However, the system* $\boldsymbol{A}$ *must meet certain conditions to make the eigenvector method applicable. The Theorem of Perron-Frobenius defines these conditions.*

Keywords: *Problem Solving, Transfer Functions, Cause/Effect Analysis, Customer Needs, Quality Function Deployment, Artificial Intelligence*

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Introduction

For decennials, *Quality Function Deployment* (QFD) is the discipline to uncover hidden customer needs for creating successful products (ISO 16355-1, 2015). The main task is to capture the *Voice of the Customer* (VoC). Many proven methods and tools exist to understand the VoC and turn it into a prioritization profile.

QFD uses the concept of *Six Sigma Transfer Functions* (SSTF). These functions are linear *Transfer Functions* in the form $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{y}$ is the vector representing qualitative or quantitative user needs, and $\boldsymbol{x}$ the vector of quantitative parameters related to the technical solution characteristics. Since $\boldsymbol{A}$ is linear, it can be represented as a matrix (Fehlmann, 2016, p. 65ff). It has many similarities to Six Sigma root cause analysis, where $\boldsymbol{y}$ is the observable response and $\boldsymbol{A}$ the matrix of measurements that correlate each vector dimension of $\boldsymbol{x}$ with each vector dimension of $\boldsymbol{y}$. For measuring these correlations in Six Sigma, the *Design of Experiments* technique (Creveling, et al., 2003, p. 549) provides guidance how to get a sufficiently well-defined transfer function matrix for identifying main causes for an observed effect.

In both QFD and Six Sigma for manufacturing, finding the right controls for the vector $\boldsymbol{x}$ is the difficult part. Because of the non-decidability of first-order logic (Turing, 1937), there is no automated method possible to devise the “correct” instances of $\boldsymbol{x}$**,** not even its dimensions – otherwise we would have a general problem solver and could let computers develop new technologies and new products.

The main difference between Six Sigma in manufacturing and QFD is that, in QFD, proper measurements are often not possible. Classical QFD for product design replaces measurements by team consensus; thus, measuring expert judgment rather than physical evidence.

Literature Review

Measuring the response $\boldsymbol{y}$ in QFD involves techniques to understand the VoC that often rely on social science or involve not only mathematics but also psychology such as Saaty's *Analytic Hierarchy Process* (AHP) (Saaty & Alexander, 1989). Methods and techniques for the acquisition of the voice of the customer make up for the larger part of the ISO 16355 series of standards.

Finding the SSTF and assessing the right topics and dimension of $\boldsymbol{x}$ requires a very creative but disciplined process. This is the essence of QFD. As for any SSTF, it is possible to validate any pair of $\boldsymbol{A}$ and $\boldsymbol{x}$ by applying $\boldsymbol{A}$ to $\boldsymbol{x}$. The result, $\boldsymbol{Ax}$ is a vector with the dimensions of the original response $\boldsymbol{y}$, in QFD typically the voice of the customer. Because of the measurement errors and the uncertainty of expert judgements, $\boldsymbol{y} \cong \boldsymbol{Ax}$ but not equal.

The vector difference between $\boldsymbol{Ax}$ and $\boldsymbol{y}$ is called the *Convergence Gap*. This is an indication how well $\boldsymbol{A}$ and $\boldsymbol{x}$ together explain the response $\boldsymbol{y}$, or in other words, whether a product or technology based on the quantitative parameters $\boldsymbol{x}$ and providing the transfer function $\boldsymbol{A}$ are capable to deliver the qualitative requested user needs $\boldsymbol{y}$, thus validating the approach but not able to exclude the existence of other approaches.

Let $\boldsymbol{x} = \langle x_1, x_2, \dots, x_n \rangle$ and $\boldsymbol{y} = \langle y_1, y_2, \dots, y_m \rangle$ be vectors in two respective linear vector spaces, and let the matrix $\boldsymbol{A} = (a_{ij})$ be a linear transfer function, then the convergence gap is defined as the Euclidean distance between the m-dimensional vectors $\boldsymbol{y}$ and $\boldsymbol{Ax} = \langle \sum a_{i1}x_i, \sum a_{i2}x_i, \dots, \sum a_{im}x_i \rangle$:

$$\|\boldsymbol{y} - \boldsymbol{Ax}\| = \sqrt{\sum_{\boldsymbol{j=1}}^{\boldsymbol{m}} \left(y_j - \sum_{i=1}^{n} a_{ij} x_i \right)^2} \tag{1}$$

The convergence gap can be used to optimize the SSTF $\boldsymbol{A}$, and thus the solution $\boldsymbol{x}$, by using domain expertise, or by numerical optimization. The preferred method is the *Eigenvector Method* because it settles and flattens variations that originate from measurement errors or opinion blur. This was first observed by Saaty and applied for the *Analytic Hierarchy Process* (AHP) (Saaty, 2003). For more details, including limitations of the AHP approach, see for instance Hontoria and Munier (Hontoria & Munier, 2021).

For literature about QFD, consult the ISO series of standards 16355, explaining its statistical methods (ISO 16355-1, 2015). For Six Sigma transfer functions, consult Fehlmann (Fehlmann, 2016), and for the matrix calculations behind some textbook about linear algebra, e.g., Meyer (Meyer, 2000).

However, we should mention the Foxes Team of Volpi. This work became famous and often referenced and used because it extended the use of

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Microsoft Excel for scientific calculations by Linear Algebra. Volpi's team created the add-on to Microsoft Excel called *Matrix.xla* (Volpi & Team, 2007), the Tutorial (Volpi & Team, 2004) and the Reference (Volpi & Team, 2006). The authors also rely on their work to calculate Six Sigma transfer functions by our own tools in Excel.

De Levie's book (Levie, 2012) explains how to use Excel for scientific calculations, as a textbook for most scientific disciplines. He also maintains a web site with many useful links (Levie, 2012ff). However, commercial but expensive tools such as MATLAB (MathWorks, Inc., 2021) provide such functionality more intuitively but less easy to access. The open-source tool R (The R Foundation, 1993) is probably better suited for educational purposes. Nevertheless, Excel is widely used for statistics and provides a simple approach to basic mathematical programming for scientists and authors.

While De Levie builds on the work of Volpi's team, he also extends the use to higher precision, adding more stability for numerical calculations.

Such an approach is especially useful if teachers want to help students understanding the roots of the technology that dominates our century. Knowing how to use technology sometimes is not good enough; it does not allow people to distinguish fakes from reality. Therefore, they start believing unscientific claims. To educate people to freedom and self-determination they must be empowered to understand the world they are living in. A good approach to achieve this is explaining them the World Formula and thus demonstrating what it means to distinguish cause and effect.

The Problem with the World Formula

Obviously, it is not always possible to solve the world formula. If it were, we would have solutions to all possible problems. Normally, the challenge is less finding the solution profile $\boldsymbol{x}$, but defining the transfer function $\boldsymbol{A}$ that describes the problem accurately.

Famous sample solutions exist; the best known probably is the analog-digital conversion used for audio and video – thus, incidentally, the foundation of the Internet as we know it today. Here, $\boldsymbol{y}$ is the audio wave that we can hear with our ears, while $\boldsymbol{x}$ is the digital representation of the audio stream as frequency ranges. The *Fast Fourier Transform* (FFT) is the algorithm that defines $\boldsymbol{A}$ in limited time frames (Cooley & Tukey, 1965).

However, in general, the existence of solutions is not guaranteed. Even if the goal $\boldsymbol{y}$ is known, the transfer function $\boldsymbol{A}$ is not and thus no solution $\boldsymbol{x}$ exists.

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The Eigenvector Method

The authors are mainly concerned with world formula solutions in the domain of *Quality Function Deployment* (QFD) and *Six Sigma* (6σ). Both use matrices and Linear Algebra for correlations and statistics. These SSTF matrices in a real vector space $\mathbb{R}^{n \times m}$, $n, m \in \mathbb{N}$, are in most cases positively definite. In QFD, such matrices are professionally guessed by expert teams; in Six Sigma, their cell values are measured by some suitable process measurement method (Fehlmann, 2016).

Correlations or Cause-Effect?

The first misunderstanding must be to clarify that an SSTF describes a cause-effect relationship, not a statistical correlation. $\boldsymbol{x}$ is the cause for $\boldsymbol{y}$, and $\boldsymbol{x}$ controls the outcome of $\boldsymbol{y}$ when applying the SSTF $\boldsymbol{A}$. In QFD, it is a common problem that cause-effects are messed up. If $\boldsymbol{x}$ describes the solution, and $\boldsymbol{y}$ the needs of the customer that should be satisfied with the solution $\boldsymbol{x}$, then it is common when asking the customer for its needs that the customer responds with some solution idea. Such QFD attempts are quite likely to fail.

Nevertheless, in the QFD literature quite often the statistical notion "correlation" is used when cause-effect would be correct. Correlations in statistics never tell you the direction of what causes which outcome. Correlations are useful observations for systematically exploring relationships for later determination what is the cause producing which effect. But correlations never prove anything.

On the other hand, the notion of cause-effect does not necessarily imply a quantification, how much cause is needed to produce which effect. With Design for Six Sigma measurement strategies this quantification is always included (Creveling, et al., 2003). Therefore, we prefer the notion of "transfer function" to "cause-effect", rather pointing at the need for quantification than at the quality of causality, but as well at the need to define the direction from cause to effect.

The Existence of a Solution

If $\boldsymbol{A}$ is a SSTF between profile vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, and if $\boldsymbol{y}$ is close to some eigenvector of $\boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}$, where $\boldsymbol{A}^{\mathsf{T}}$ denotes the transpose of $\boldsymbol{A}$, then an approximate solution $\boldsymbol{x}$ exists such that $\boldsymbol{y} \cong \boldsymbol{A}\boldsymbol{x}$ up to the convergence gap.

Let $\boldsymbol{\tau}$ be some eigenvector of $\boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}$ close to $\boldsymbol{y}$. Then there is an eigenvalue $\lambda \in \mathbb{R}$ such that $\boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}\boldsymbol{\tau} = \lambda\boldsymbol{\tau}$. Norming $\boldsymbol{A}\boldsymbol{A}^{\mathsf{T}}$ allows setting $\lambda = 1$.

Then, by setting $\boldsymbol{x} = \boldsymbol{A}^{\mathsf{T}}\boldsymbol{\tau}$, $\boldsymbol{x}$ solves $\boldsymbol{\tau} = \boldsymbol{A}\boldsymbol{x}$. Thus, our world formula has a solution if the convergence gap between $\boldsymbol{y}$ and $\boldsymbol{\tau}$ is zero, respectively an approximate solution if the gap is small. In QFD and 6σ, we are usually

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satisfied with an approximate solution, since neither our guesses nor the measurements provide exact numbers.

Solving the World Formula

Solving the world formula $\boldsymbol{y} = \boldsymbol{Ax}$ involves finding a SSTF $\boldsymbol{A} \in \mathbb{R}^{n \times m}$ whose squared matrix $\boldsymbol{AA}^\top \in \mathbb{R}^{n \times n}$ has an eigenvector $\boldsymbol{\tau}$ close to $\boldsymbol{y} = \langle y_1, y_2, \cdots y_n \rangle$. The solution is $\boldsymbol{x} = \langle x_1, x_2, \cdots, x_m \rangle$. Thus, solving $\boldsymbol{y} = \boldsymbol{Ax}$ involves he ability to rapidly check eigenvectors for $\boldsymbol{AA}^\top$.

Since the cell values of an SSTF consists of either expert choices or measurements for the transfer of cause to effect, and since, according to ISO/IEC 16355, ratio scales are used to quantify such a transfer, we can use linear algebra to calculate the effects $\boldsymbol{y} = \boldsymbol{Ax}$. Thus, solving the world formula effectively solves problems in QFD or Six Sigma.

However, there is a caveat. An n-dimensional matrix $\boldsymbol{AA}^\top \in \mathbb{R}^{n \times n}$ has up to n eigenvectors, and we only need one. Incidentally, the theorem of *Perron-Frobenius* guarantees that for the class of positive-definite symmetric square matrices there exists a distinguished *Principal Eigenvector* that dominates all others in the following sense:

- It corresponds to the highest eigenvector;
- Its components are equally signed; i.e., there is no mix of positive and negative vector components.

Obviously, $\boldsymbol{AA}^\top = (c_{i,k})$ is symmetric for $i, k = 1, \ldots, n$. When $\boldsymbol{A} = (a_{i,j})$ for $i = 1, \ldots, n$, $j = 1, \ldots, m$, the coefficients of the associated matrix $\boldsymbol{AA}^\top$are

$$c_{i,k} = \sum_{j=1 \ldots m} a_{i,j} a_{j,k} = c_{k,i} \qquad (2)$$

Moreover, if $\boldsymbol{A}$ is positive definite, this holds as well for $\boldsymbol{AA}^\top$. However, a few negative coefficients in $\boldsymbol{A}$ do not necessarily affect $\boldsymbol{AA}^\top$; therefore, cause-effects in QFD sometimes can become negative. The theorem of Perron-Frobenius in this context is discussed in the author's book about SSTF (Fehlmann, 2016, p. 359), as well as how to proof the theorem.

A Model for Solutions of the World Formula

The aim of this section is to clarify that the existence of a solution for some given world formula remains unknown.

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The Question whether a Solution Exists is Undecidable

Trivial solutions always exist. Set the dimensions m and n equal, the SSTF is the identity transfer function and set $\boldsymbol{y} = \boldsymbol{x}$. However, it is not obvious what the existence of a non-trivial solution exactly means. Given a goal profile $\boldsymbol{y}$, does a SSTF $\boldsymbol{A}$ and a solution profile $\boldsymbol{x}$ exist, of any dimension, such that $\boldsymbol{y} = \boldsymbol{Ax}$ holds? For which problems can $\boldsymbol{x}$ be considered a solution? How shall problems be stated?

Following the methods of mathematical logic, it is necessary to construct a non-empty model for the problems that the world formula shall possibly address. The *Graph Model* of *Combinatory Logic* (Engeler, 1995) is a model of Combinatory Logic with explains how to combine topics in areas of knowledge. An excellent example for a graph model is the Neural Algebra described by Engeler (Engeler, 2019). The model is explained further in last years' ATINER paper of the authors in a version targeted at software testing (Fehlmann & Kranich, 2021) and intuitionism (Fehlmann & Kranich, May 2020). From the construction of the model, it will turn out that the question whether a solution exists remains undecidable.

The Graph Model of Combinatorial Logic Adapted to SSTF

It is necessary to add a few properties to the graph model such that it can serve as a general model for what a SSTF can solve.

A graph model is recursively defined over a set $\mathcal{L}$ of assertions. An *Arrow Term* is recursively defined as follows:

- Every element of $\mathcal{L}$ is an arrow term.
- Let $\alpha_1, \dots, \alpha_{\hat{m}}, \beta$ be arrow terms. Then

$$\{\alpha_1, \dots, \alpha_{\hat{m}}\} \to \beta \tag{3}$$

  is also an arrow term.

Thus, arrow terms are relations between finite subsets of arrow terms and another arrow term, emphasized as successor. Arrow terms constitute a *Combinatorial Algebra* under composition

$$M \bullet N = \{\gamma \mid \exists\{\beta_1, \beta_2, \dots, \beta_m\} \to \gamma \in M;\ \beta_i \in N\} \tag{4}$$

For extending the graph model to SSTF, called *SSTF-Model*, two more notions are needed. First, a set of *Categories* $\mathcal{C}$ must exist such that every assertion has one or more categories assigned. The categories correspond to the rows and columns in SSTF matrices. They reflect the kind of assertion that an element of $\mathcal{L}$ is referring to.

The categories of an arrow term are the union of the categories of its subterms. The categories of $M \bullet N$ are the categories of its elements. Denote the categories of an arrow term α by $\mathcal{C}(\alpha)$ and the category of a set of arrow

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terms M by $\mathcal{C}(M)$. A term, or a set, corresponds to more than one category.

Second, every arrow term needs a *Size*; a scalar that reflects the wight, or functional size, or cost that occur when the item described by the arrow term is realized. For an arrow term α, denote its size by $\|\alpha\|$, for a set of arrow terms M, by $\|M\|$. The needed properties are

$$\begin{gathered} \|\emptyset\| = 0 \\ \|\{\alpha_1, \dots, \alpha_{\hat{m}}\} \to \beta\| \geq \sum_{i=1}^{m} \|\alpha_{\mathrm{i}}\| + \|\beta\| \\ \text{if } M = \{\alpha_1, \dots, \alpha_{\hat{m}}\}, \text{then } \|M\| = \sum_{i=1}^{m} \|\alpha_{\mathrm{i}}\| \\ \|M \bullet N\| \leq \|M\| + \|N\| \end{gathered} \tag{5}$$

Every type of arrow term size is a *Ratio Scale* (ISO 16355-1, 2015). It cannot fall below zero, and it has no upper limit.

It is left to the reader to verify that it is possible to assign sizes such as functional sizes, cost, or importance weight for customers to arrow terms, fulfilling (5). However, these properties do not define size in full; they leave room for specificities. If the size is functional size, most likely the base assertions in $\mathcal{L}$ have size 0, because they just specify the program state before execution of a test, or function, while if size is weight, or cost, even base assertions add size, and the size of higher-level arrow terms is not even largen than the size sum of its terms.

Construction of an SSTF-Model

Assume a collection of finite arrow term sets $\boldsymbol{\mathcal{M}} = \{\mathcal{M}_i | i = 1, \dots, n\}$ arranged as a $n \times m$ matrix, with

$$\mathcal{M}_i = \left\{\{\alpha_{i,1,k}, \dots, \alpha_{i,m,k}\} \to \beta_{i,k} \middle| k \in \mathbb{N}\right\} \tag{6}$$

One can associate an SSTF $\boldsymbol{A} = \left(a_{i,j}\right)$ with a goal profile $\boldsymbol{y} = \langle y_1, y_2, \cdots, y_n\rangle$ using the total size of the arrow terms that refer to the matrix cell i, j

$$a_{i,j} = \sum_{k \in \mathbb{N}} \left\|\alpha_{i,j,k}\right\| \tag{7}$$

Thus, an approximate solution $\boldsymbol{x} = \langle x_1, x_2, \cdots, x_m\rangle$ may exist for the equation $\boldsymbol{y} \cong \boldsymbol{Ax}$. The model (6) for the SSTF $\boldsymbol{A}$ consists of the arrow terms $\mathcal{M}_i$ filling the row with index i in the matrix. If such a model exists, the SSTF $\boldsymbol{A}$ with cell values (7) might have a solution. Using the convergence gap, it is decidable whether a solution exists. It is left to the reader to argue why it remains undecidable whether such a model $\boldsymbol{\mathcal{M}}$ exists, given some SSTF $\boldsymbol{A}$.

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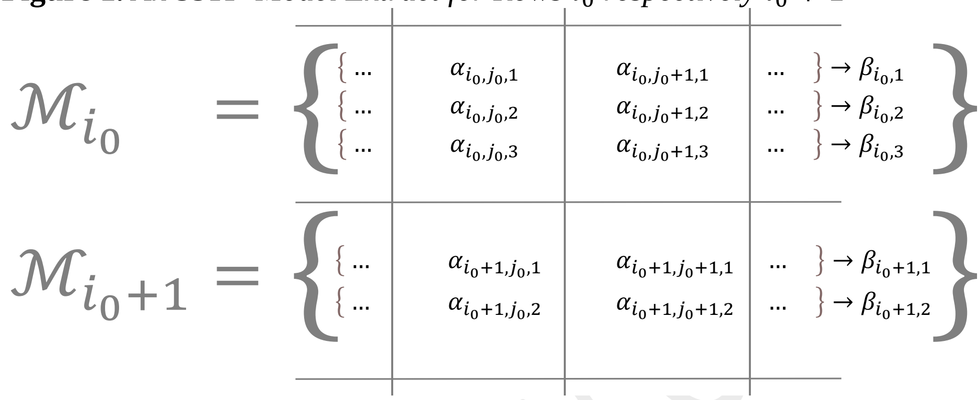
Detailed SSTF-Model

The following figure might help understanding how SSTF and arrow terms relate to each other. Assume a matrix cell on the i_0 th row and the j_0 th column, with its neighboring cells indexed $i_0 + 1$, respectively $j_0 + 1$. These four cells are shown in Figure 1 together with the corresponding arrow terms.

Figure 1. *An SSTF-Model Extract for Rows* i_0 *respectively* $i_0 + 1$



The same category, corresponding to the j_0 th and the $j_0 + 1$ st column are shared by all terms $\alpha_{i,j_0,k}$ and $\alpha_{i,j_0+1,k}$ respectively. The size of the cells is the sum of the $\|\alpha_{i,j,k}\|$, as in equation (7).

Improving the SST-Model

When an SSTF has no solution, that is, the convergence gap does not close, adding or removing arrow terms to the model $\boldsymbol{\mathcal{M}}$ adds or decreases size in a cell and thus might solve $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$ by modifying the SSTF $\boldsymbol{A}$. Note that any such change means that additional or removed arrow terms in the model means that the transfer function is improved by adding or removing actions connected to each cell in the matrix.

Numerical Methods

Numerical methods for solving the world formula originate from Gauss but have been deeply enriched in the past few decencies.

The Power Method

If $\boldsymbol{\tau}$ is an eigenvector of a square matrix $\boldsymbol{A}$, then its corresponding eigenvalue is given by

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$$\lambda = \frac{\boldsymbol{\tau}^{\top} \boldsymbol{A} \boldsymbol{\tau}}{\boldsymbol{\tau}^{\top} \boldsymbol{\tau}} \tag{8}$$

This quotient is called the *Rayleigh Quotient*. For the proof, see for instance the authors' book about transfer functions (Fehlmann, 2016, p. 358).

The power iteration algorithm starts with a random vector $\boldsymbol{\tau}_0$, if possible, near to the principal eigenvector. The method is described by the recurrence relation

$$\boldsymbol{\tau}_{i+1} = \frac{\boldsymbol{A}\boldsymbol{\tau}_i}{\|\boldsymbol{A}\boldsymbol{\tau}_i\|} \tag{9}$$

At every iteration, the vector $\boldsymbol{\tau}_i$ is multiplied by the matrix $\boldsymbol{A}$ and normalized.

Hence, in cases for which the power method (9) generates a good approximation of a dominant eigenvector, the Rayleigh Quotient (8) delivers a good approximation of a dominant eigenvalue. Thus, (8) indicates whether the power iteration found the principal eigenvector, or some other.

The power iteration algorithm is robust but slow.

The Jacobi Iteration

In numerical linear algebra, the Jacobi eigenvalue algorithm is an iterative method for the calculation of the eigenvalues and eigenvectors of a real symmetric matrix. This process is known as diagonalization. The original algorithm was published by Rutishauser (Rutishauser, 1966).

The Jacobi eigenvalue method repeatedly performs rotations around the off-diagonal element with the largest absolute value, called the *Pivot*, until the matrix becomes almost diagonal. Then the elements in the diagonal are approximations of the (real) eigenvalues. For details, consult a textbook, e.g., *Numerical Recipes* (Press, et al., 2007).

The Jacobi iteration is popular because of its speed and intuitiveness. The figures below show the steps needed according to this method to calculate the eigenvalues and the eigenvectors of a typical QFD matrix.

Figure 2 is a matrix originating from a real QFD that does not provide a response profile $\boldsymbol{\tau}$ near to the goal profile $\boldsymbol{y}$. In this case, the domain consists of investments into product characteristics $tc_1, \ldots tc_6$, providing value for the user $cr_1, \ldots, cr_8$. Note that the total investment needed is represented by the solution profile below the matrix $\boldsymbol{A}$. The same investment has impact on various user values.

The corresponding world formula has a model – the cause-effect relations that written as arrow terms, were used to expert estimate the QFD matrix.

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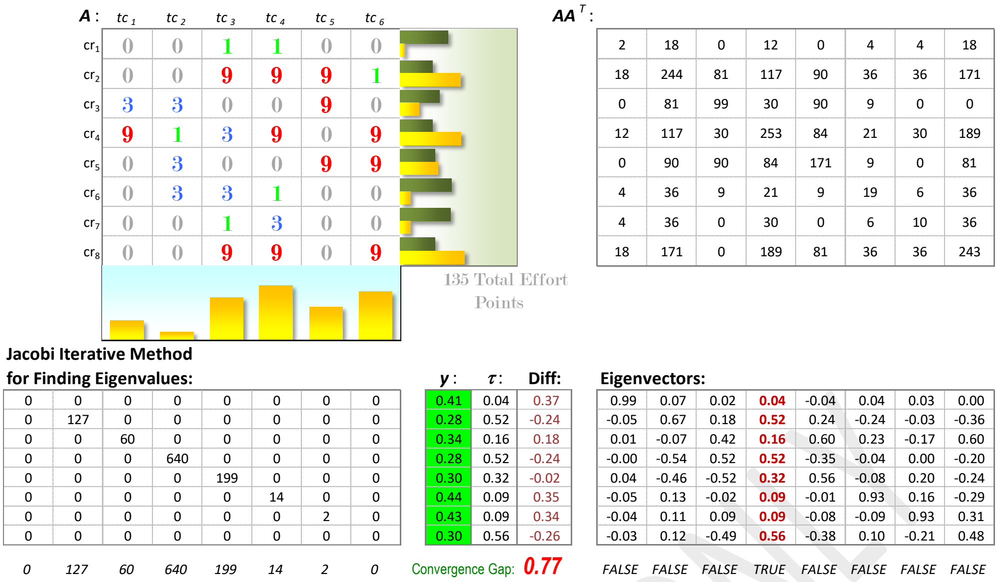
Figure 2. *Insolvable QFD Matrix*



A:

	tc_1	tc_2	tc_3	tc_4	tc_5	tc_6
cr_1	0	0	1	1	0	0
cr_2	0	0	9	9	9	1
cr_3	3	3	0	0	9	0
cr_4	9	1	3	9	0	9
cr_5	0	3	0	0	9	9
cr_6	0	3	3	1	0	0
cr_7	0	0	1	3	0	0
cr_8	0	0	9	9	0	9

135 Total Effort Points

AA^T:

2	18	0	12	0	4	4	18
18	244	81	117	90	36	36	171
0	81	99	30	90	9	0	0
12	117	30	253	84	21	30	189
0	90	90	84	171	9	0	81
4	36	9	21	9	19	6	36
4	36	0	30	0	6	10	36
18	171	0	189	81	36	36	243

Jacobi Iterative Method for Finding Eigenvalues:

0	0	0	0	0	0	0	0
0	127	0	0	0	0	0	0
0	0	60	0	0	0	0	0
0	0	0	640	0	0	0	0
0	0	0	0	199	0	0	0
0	0	0	0	0	14	0	0
0	0	0	0	0	0	2	0
0	0	0	0	0	0	0	0
0	*127*	*60*	*640*	*199*	*14*	*2*	*0*

y:	τ:	Diff:
0.41	0.04	0.37
0.28	0.52	-0.24
0.34	0.16	0.18
0.28	0.52	-0.24
0.30	0.32	-0.02
0.44	0.09	0.35
0.43	0.09	0.34
0.30	0.56	-0.26

Convergence Gap: **0.77**

Eigenvectors:

0.99	0.07	0.02	**0.04**	-0.04	0.04	0.03	0.00
-0.05	0.67	0.18	**0.52**	0.24	-0.24	-0.03	-0.36
0.01	-0.07	0.42	**0.16**	0.60	0.23	-0.17	0.60
-0.00	-0.54	0.52	**0.52**	-0.35	-0.04	0.00	-0.20
0.04	-0.46	-0.52	**0.32**	0.56	-0.08	0.20	-0.24
-0.05	0.13	-0.02	**0.09**	-0.01	0.93	0.16	-0.29
-0.04	0.11	0.09	**0.09**	-0.08	-0.09	0.93	0.31
-0.03	0.12	-0.49	**0.56**	-0.38	0.10	-0.21	0.48
FALSE	*FALSE*	*FALSE*	*TRUE*	*FALSE*	*FALSE*	*FALSE*	*FALSE*

Source: Own work, using a QFD matrix from Wu (Wu, et al., 2005).

However, at least for the domain under scrutiny, $\boldsymbol{A}$ is not a solution. This is made visible by the comparison between $\boldsymbol{y}$ and $\boldsymbol{Ax}$ shown right from the first upper matrix, the QFD matrix in Figure 2. Below this matrix is the solution profile $\boldsymbol{x}$. The authors of the original QFD matrix realized this and tried to use "Grey Theory" (Wu, et al., 2005) to better analyze dynamic customer requirements. Their SSTF $\boldsymbol{A}$ needs improvements, possibly additional or less columns for the product characteristics, or correct cause-effect relationships. Adapting the implementation $\boldsymbol{A}$ changes the solution profile $\boldsymbol{x}$ and thus $\boldsymbol{Ax}$.

Since cell values in QFD matrices dynamically reflect relative importance, not static, immutable physical measurements, improving these values is an excellent way for finding better solution for the world formula. For instance, this might be used to create better products at less cost. It is therefore highly desirable to find a way how to improve an "insolvable" QFD matrix. For more details, see the series of standards ISO 16355 (ISO 16355-1, 2015).

The Winding Stairs Method

Is it possible to improve the SSTF $\boldsymbol{A}$ with the existing product characteristics such that the investments are better focused on customer's needs? Applying the global sensitivity analysis *Winding Stairs Method*, see for instance the authors' respective paper (Fehlmann & Kranich, 2022), it is possible to mathematically improve the SSTF. However, such an improvement only considers the information that had been supplied to the original SSTF and therefore just optimizes distribution of effort, or budget, ignoring other possibilities such as adding another technical product characteristics. Figure 3 shows the automatically improved QFD matrix.

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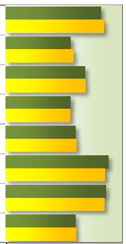
The convergence gap closes. Thus, this QFD reflects not just measurements of some cause-effect relationships but can be used as a planning matrix, indicating how much effort, or coupling, you need in each cell to achieve a response near to the goal profile.

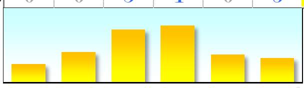
The *Total Effort Points* – the total sum of all the cause-effect relationships, an indicator for total cost of all the product improvement measures – even had slightly decreased compared to Figure 2. On the other hand, the need to invest in tc_2 has significantly increased and decreased somewhat for tc_6. This is a finding by machine intelligence, whilst there might be other constraints not reflected in the optimization method and thus not known to the "intelligent" machine that should be considered before deciding about investments.

Figure 3. *Optimized QFD Matrix*

A:

	tc_1	tc_2	tc_3	tc_4	tc_5	tc_6
cr_1	0	0	7	6	0	0
cr_2	0	0	3	4	3	1
cr_3	8	7	0	0	8	0
cr_4	3	1	2	4	0	2
cr_5	0	5	0	0	6	8
cr_6	0	4	6	5	0	0
cr_7	0	0	6	7	0	0
cr_8	0	0	3	4	0	5





118 Total Effort Points

AA^T:

85	45	0	38	0	72	84	45
45	35	24	24	26	38	46	30
0	24	177	31	83	28	0	0
38	24	31	34	21	36	40	32
0	26	83	21	125	20	0	40
72	38	28	36	20	77	71	38
84	46	0	40	0	71	85	46
45	30	0	32	40	38	46	50

Jacobi Iterative Method for Finding Eigenvalues:

1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	226	0	0	0	0	0
0	0	0	18	0	0	0	0
0	0	0	0	80	0	0	0
0	0	0	0	0	12	0	0
0	0	0	0	0	0	331	0
0	0	0	0	0	0	0	0
1	*0*	*226*	*18*	*80*	*12*	*331*	*0*

y:	*τ*:	Diff:
0.41	0.42	-0.01
0.28	0.29	-0.01
0.34	0.34	-0.00
0.28	0.28	0.00
0.30	0.30	-0.00
0.44	0.43	0.01
0.43	0.42	0.01
0.30	0.30	-0.00

Convergence Gap: **0.02**

Eigenvectors:

0.73	0.02	-0.33	-0.08	-0.11	-0.17	**0.42**	-0.37
-0.33	0.55	-0.05	0.04	0.04	-0.70	**0.29**	0.05
0.14	-0.07	0.71	0.12	-0.55	-0.04	**0.34**	0.20
-0.28	0.19	-0.02	0.61	-0.03	0.40	**0.28**	-0.52
-0.03	-0.18	0.49	-0.26	0.69	-0.04	**0.30**	-0.29
-0.22	0.31	-0.16	-0.60	-0.14	0.50	**0.43**	0.15
-0.37	-0.72	-0.33	0.06	-0.10	-0.19	**0.42**	0.11
0.27	0.11	-0.10	0.41	0.42	0.19	**0.30**	0.66
FALSE	*FALSE*	*FALSE*	*FALSE*	*FALSE*	*FALSE*	*TRUE*	*FALSE*

Source: Own work, based on an improved QFD matrix from Figure 2.

Various other only marginally less optimal solutions for the SSTF exist that also provide value for the user. A small convergence gap is also possible when improving different product characteristics. Solutions to the world formula are something that make fun to play with and are often quite insightful.

An Afterthought on the Windings Stairs Method

Without going into the details of the *Windings Stairs* (WS) method, shown elsewhere (Fehlmann & Kranich, 2022), the elegance of this numerical method is worth an afterthought. This method uses a principle often used in Artificial Intelligence (AI) and Big Data. Thus, it is capable of effectively surprising human users because it can do this faster than humans. However, it is not using any kind of creativity that otherwise is known for its uttermost importance in product design and improvement.

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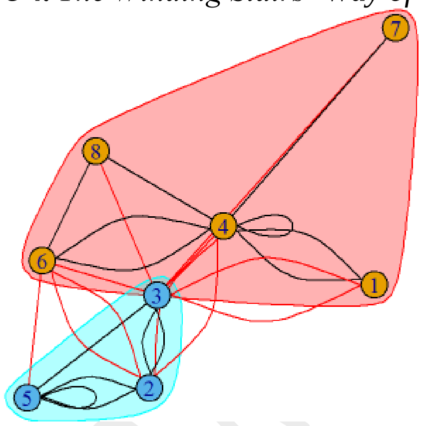
For a QFD matrix, there exists an undirected graph connecting the technical solution constraints with the goal topics, the edges represents the weighting of the connection. Note that the number of nodes is the maximum of both dimensions of the matrix. Cluster algorithms, well-known from AI, simulate the flow through the graph with the help of so-called random walks. A cluster algorithm over the permuted matrix yields at least one cluster – that would be the trivial case.

For minimizing the convergence gap, one strategy is to use the differences $\boldsymbol{y} - \boldsymbol{\tau_y}$ and reorder the absolute differences in descending order.

These are the nodes that may have a major impact on the reduction of the convergence gap. The original QFD matrix must of course be permuted, row by row.

The cluster algorithm provides an ordering of the nodes based on impact, as shown in Figure 4. This order can be used as strategy for WS, because at each iteration the nodes must be traversed once. Processing is cyclic.

Figure 4. *The Winding Stairs' Way of Solving the World Formula*



Source: Own work, based on R (The R Foundation, 1993).

The order of the promising nodes, i.e., the rows and associated entries of the QFD matrix, visited by the Winding Stairs' vertex access sequence in Figure 4 is 4-6-1-7-8-3-5-2; the first five being in the upper (red) area where the impact on the convergence gap supposedly is highest. Since the values of the elements of the QFD matrix are limited downward and upward, the convergence gap can be made smaller than any predefined limit.

Conclusions

Knowing how to solve the world formula is both important and useful. It involves not only linear algebra but numerical methods as well. Thanks to numeric, the world formula has become more democratic in the sense that today almost everyone has access to the necessary computing power. Everybody can solve problems and find solutions once they have access to the relevant facts.

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Indeed, even pandemics would be easier to defeat when providing information based on facts instead of imposing “rules” with questionable effects. People who understand how to solve the world formula are likely to less believe in allegations of the mighty or the majority. For education to democracy and self-determination, addressing the world formula and its solutions is paramount.

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Appendix

Using Matrix.xla from Volpi

A function that returns multiple values is called "array function". The *Matrix.xla* tool contains a wide range of array functions, described in the reference (Volpi & Team, 2006). The Tutorial (Volpi & Team, 2004) helps users learning how to use them.

The main principle is that calculations with vectors and matrices can be made by specifying using them as variables and arguments. Since Excel does not allow to have compound data stored in a single cell, but represent matrices by rectangular tables, a trick is used that was introduced to Excel in its very early versions. In order to calculate a matrix or vector at once, one can select the whole area where the result shall be placed and enter the formula into the selection, using the "magic" key sequence *Ctrl+Shift+Enter*.

The formula is then shown within curly brackets. Entering curly brackets is to no avail; just use the magic key sequence. For instance, in Figure 2 and Figure 3, to calculate $\boldsymbol{A}\boldsymbol{A}^{\intercal}$**,** enter the Excel formula

$$\{= \mathrm{MMULT}\big(TransferFunction; \mathrm{TRANSPOSE}(TransferFunction)\big)\} \qquad (10)$$

where $TransferFunction$ is an Excel name referring to the Excel range containing $\boldsymbol{A}$. The eigenvalues are the calculated by

$$\{= \mathrm{MatEigenvalue\_Jacobi}(AAT)\} \qquad (11)$$

where AAT is the Excel name for the range containing $\boldsymbol{A}\boldsymbol{A}^{\intercal}$. By virtu of the Jacobi method, the eigenvalues are in the diagonals of the resulting square matrix.

Finally, the eigenvectors are written in another square range by

$$\{= \mathrm{MatEigenvector\_Jacobi}(AAT)\} \qquad (12)$$

and he principal eigenvector is easily detectable by searching for the largest eigenvalue. MatEigenvector_pow() and MatEigenvalues_pow(), two array functions applying the Power Method, are also available.

Using the R Package

R has no table interface such as Excel. Similar to other programming languages, one has first to install the respective library before using it. In R,

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it is call *Package*. However, the eigenvector calculation does not require any additional package.

In the R GUI, an $n \times m$ matrix is entered by

$$\begin{aligned} \mathrm{A} \leftarrow \mathrm{matrix}(\mathrm{data} = \mathrm{c}(\mathrm{a}_{1,1}, \mathrm{a}_{1,2}, \dots, \mathrm{a}_{n,m}), \\ \mathrm{nrow} = n, \mathrm{ncol} = m, \mathrm{byrow} = \mathrm{TRUE}) \end{aligned} \tag{13}$$

without line breaks. Then,

$$\mathrm{eigenPairs} \leftarrow \mathrm{eigen}(\mathrm{tcrossprod}(A), \mathrm{symmetric} = TRUE) \tag{14}$$

simply fills into eigenPairs the eigen() decomposition $values and $vectors. Note that eigenPairs can become all negative and might need a sign change.

The Windings Stairs' Way is explained in (Fehlmann & Kranich, 2022).

The graphics in Figure 4 is constructed using the package igraph.

The following steps are needed:

- Create an igraph object, say actual_graph
- For this object create an adjacency table
- Apply the function cluster_walktrap originating from igraph
- For vector graphic output, use svg(cluster_walktrap(actual_graph)) or use any other appropriate R output function, see (Kabacoff, 2015).

That's all!

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