

Unified Substrate Closure under Admissible Regime Determination

This work completes a constructive determination of physical mechanics under an explicit admissibility hypothesis: a regime qualifies as physical only if it admits a governing equation, a numerical calibration, and a directly testable observable. Regimes that fail this equation–number–observable criterion do not define predictive physics within the present framework. Under this hypothesis, the admissible expressions of a single physical substrate are sharply constrained. Exactly five regimes satisfy the criterion: Propagation (light), Ordering (time and causality), Localization (matter), Coupling (gravity), and Global Drift (cosmology). Each regime is derived independently, calibrated arithmetically, and associated with an observable channel. No regime introduces additional primitive structures or unfixed constants; all numerical content is fixed internally by calibration. The central result is that, under the same admissibility hypothesis, these five regimes are not independent constructions. They arise as controlled limits of a single mechanical law. The unifying object is the principal phase of a substrate excitation, evolved with respect to an induced ordering parameter rather than a primitive time. From this phase, canonical energy–momentum are defined, and admissible dynamics reduce to a single ordering-covariant Hamilton–Jacobi master constraint involving three induced scalars only: an ordering-distortion potential, a localization gap, and a global ordering drift. Within this admissible class, the master constraint constitutes the minimal closure compatible with all five calibrated regimes. In the gapless limit it enforces null propagation with a single invariant speed. With nonzero gap it yields gapped dispersion and the inertial mass relation. With ordering distortion it reproduces gravitational time dilation and weak-field coupling. With drift of the ordering background it yields redshift as an ordering ratio and enforces the observed one-mode cosmological structure. Any extension of the constraint by additional terms or degrees of freedom violates at least one regime calibration and is therefore excluded under the stated hypothesis. The resulting framework is a closed mechanics conditional on the admissibility criterion: one substrate, one ordering observable, one invariant propagation speed, one localization gap–mass identity, one infrared coupling closure, and one global drift law, unified by a single master constraint whose form and coefficients are fixed by calibration rather than postulated. Under these conditions, the five regimes exhaust the admissible ways in which the substrate can manifest in physical observables.

1 Jurisdiction, Aim, and Risk Posture

2
3 The aim of this work is not to introduce an additional interpretive layer atop
4 existing theories, but to determine—by elimination—the invariant mechanical
5 content common to all admissible predictive descriptions. The framework is
6 deliberately austere and deliberately fragile: where the structural conditions
7 stated herein cannot be satisfied, the framework is not patched or extended. It
8 becomes undefined. This absence of rescue mechanisms is not an aesthetic
9 preference; it is the only way necessity arguments retain scientific authority
10 under hostile inspection.

11 The manuscript imposes a non-negotiable constructive discipline, later
12 applied regime-by-regime and used again to certify the master closure:

$$13 \text{equation} \rightarrow \text{number} \rightarrow \text{observable}$$

14
15
16 Any structure that cannot be commissioned in this way is excluded as non-
17 physical within the present method.

18 The pre-regime portion of the paper has a single task: to demonstrate that
19 the methodological ledger—admissible re-description, invariant elimination,
20 finite diagnostic closure, logarithmic ordering, scalar collapse to phase, and first-
21 order phase closure—is *forced*, not chosen. Only after this demonstration may
22 regime constructions be admitted as legitimate expressions of a single substrate
23 mechanics.

24 25 26 Prediction, Admissibility, and Invariants

27 28 *Prediction as a Minimal Formal Object*

29
30 Let \mathcal{C} denote the space of admissible experimental conditions (preparations,
31 boundary settings, interventions), and let \mathcal{M} denote the space of admissible
32 measurement outcomes. A predictive description is a mapping

$$33 \text{D} : \mathcal{C} \rightarrow \mathcal{P}(\mathcal{M}),$$

34
35
36 where $\mathcal{P}(\mathcal{M})$ denotes probability measures on outcomes. Deterministic
37 descriptions appear as delta measures.

38 At this stage, no assumption is made regarding spacetime, fields, probability
39 interpretation, or dynamics. This is intentional: the admissible structure is
40 derived from the weakest possible starting point.

41 42 *Admissible Re-description*

43
44 Prediction is meaningful only if it is comparable across descriptions. If two
45 descriptions yield different conditional predictions for the same physical

1 situation while remaining observationally indistinguishable, then prediction
2 collapses into bookkeeping.

3
4 Define an equivalence relation:

$$5 \quad D_1 \sim D_2 \Leftrightarrow \forall c \in \mathcal{C}, D_1(c) = D_2(c).$$

6
7
8 A transformation R is *admissible* if it preserves predictive content:

$$9 \quad D \sim R(D).$$

10
11
12 This condition is not philosophical; it is the minimal requirement for the
13 phrase “same physical situation” to retain meaning across representational
14 encodings.

15 *Invariants and Elimination*

16
17 An invariant is any functional I on descriptions such that

$$18 \quad I(D) = I(R(D)) \text{ for all admissible } R.$$

19
20
21
22 Invariants are fixed by elimination. Any relation removable without altering
23 conditional predictions is representational; any relation whose modification
24 changes predictions is invariant. The objective is not to decorate descriptions
25 with symmetry but to isolate what cannot be removed without destroying
26 predictivity.

27 28 29 **The Invariant Core and Operational Substrate**

30
31 Define the invariant algebra of a description D as the set of all invariants
32 preserved under admissible re-description. The operational “substrate” is
33 nothing beyond the equivalence class of D under predictive equivalence together
34 with its invariant algebra.

35 This designation carries no ontic commitment. It is a bookkeeping label for
36 the invariant core forced by predictivity and admissibility. A reader who dislikes
37 the word “substrate” may delete it and retain only the equivalence class and
38 invariant content. The results that follow are unchanged.

39 40 41 **Why Constructivity Is Forced: Derivation of the CRC**

42
43 The equation–number–observable criterion is binding only if it is necessary.
44 This section establishes that necessity.

45
46

1 *No Equation \Rightarrow No Transport*

2

3 If a proposed regime supplies no governing relation linking predictions
4 across conditions, the description reduces to a lookup table. No structural
5 transport exists between distinct experimental conditions, and the description
6 does not constitute mechanics.

7

8 *Equation Without Numbers \Rightarrow Underdetermined Continuations*

9

10 If a regime supplies an equation involving parameters that are not
11 numerically commissioned, then it admits a family of inequivalent predictive
12 continuations. Under admissible re-description, parameterizations may be
13 reshuffled without changing formal structure while predictions vary. Such a
14 regime is not physically fixed.

15

16 *No Observable \Rightarrow Unfalsifiability*

17

18 If equations and numerical values are declared but no observable channel is
19 specified, then no tribunal exists by which predictions may be compared. The
20 regime is scientifically empty.

21

22 **Constructive Regime Criterion (CRC).**

23

24 A regime qualifies as physical only if it supplies a governing equation, a
25 numerical commissioning, and an observable diagnostic channel.

26 This criterion is not stylistic. It is the minimal closure required for prediction to
27 remain stable under admissible re-description.

28

29

30 **Ordering Enters Only Through Ratios: Logarithmic Inevitability**

31

32 Ordering information enters physics only through comparisons of intervals.
33 No experiment measures “duration” or “scale” in isolation. All measurement is
34 comparative.

35 Let $r > 0$ denote an ordering interval. The only physically meaningful
36 comparison between r_1 and r_2 is the ratio r_2 / r_1 . Composition of comparisons is
37 multiplicative.

38 Define a logarithmic coordinate:

39

$$40 \quad x = \ln(r / r_0),$$

41

42 where r_0 is an arbitrary reference scale. In this coordinate, ratio composition
43 becomes additive translation. No alternative coordinate achieves additive
44 closure under multiplicative composition while remaining invariant under
45 admissible rescaling.

1 Logarithmic structure is therefore forced by composition, by metrological
2 invariance, and by finite diagnostic closure under translation covariance.

3 4 5 **Finite Diagnostic Closure and the Admissible Operator Class**

6
7 Once ordering lives on a translation-invariant logarithmic scale, admissible
8 substrate dynamics must satisfy locality, translation covariance, finite diagnostic
9 closure, and saturation. These constraints uniquely select a band-selecting,
10 saturating normal form.

11 Linear analysis reveals a unique dominant logarithmic mode. Finite closure
12 prohibits persistent plurality of independent stable modes, since such plurality
13 would generate independent diagnostics under regeneration. A single
14 logarithmic frequency is therefore forced.

15 16 17 **Scalar Collapse: Phase as the Unique Admissible Scalar**

18
19 Any scalar intended to encode ordering must be invariant under admissible
20 re-description, comparable across carriers, and monotonic under composition.
21 Amplitude fails due to normalization dependence. Correlation length fails due to
22 window dependence. Power-law scalars fail under scale changes. Phase alone is
23 dimensionless, additive under composition, and stable under regeneration.

24 25 **Scalar Collapse Theorem.**

26
27 Under admissibility, finite diagnostic closure, and logarithmic ordering,
28 phase is the unique scalar ordering diagnostic that survives regeneration.

29 This result is substrate-level and licenses the use of phase throughout all
30 subsequent regimes.

31 32 33 **Induced Ordering Observable**

34
35 From logarithmic samples x_i , define a complex order parameter as the mean
36 of $\exp(i\beta_0 x_i)$. The argument of this quantity defines an induced ordering
37 observable τ_{ord} , up to a conventional reference scale.

38 At this stage, τ_{ord} is an ordering observable only. It is not yet identified
39 with primitive time. All subsequent rates will be defined with respect to it once
40 the regimes commission it numerically.

41 42 43 **Transport, Principal Phase, and Closure Class**

44
45 Predictive transport requires stable propagation of distinctions.
46 Distinguishability boundaries define characteristic surfaces, which are level sets

1 of a scalar. By scalar collapse, that scalar must be a phase. Denote the principal
2 phase by S .

3 Second-order evolution introduces surplus diagnostics and violates finite
4 closure. Nonlocal kernels introduce memory and scale dependence. Higher-
5 derivative closures proliferate independent diagnostic scales. All are
6 inadmissible.

7 The only admissible closure class is therefore a first-order constraint on
8 phase gradients, of Hamilton–Jacobi type.

11 **Predictivity Exhaustion and Dependency Order**

12
13 Predictive mechanics must answer exactly five invariant questions:
14 propagation, ordering, localization, coupling, and global drift. Omission of any
15 one destroys prediction. No sixth independent invariant question survives
16 admissibility.

17 The dependency order is fixed by definability:

18
19 Propagation \rightarrow Ordering \rightarrow Localization \rightarrow Coupling \rightarrow Global Drift.
20
21

22 **Minimality and Redundancy**

23
24 Each independent invariant parameter corresponds to diagnostic freedom.
25 Surplus parameters undermine closure. Once an invariant is commissioned in
26 one regime, it may not re-enter later as an independent constant. Later
27 appearances must be redundancy constraints.

28 Unification therefore reduces to numerical identity tests, not narrative
29 coherence.
30
31

32 **Substrate Invariants vs. Effective Parameters**

33
34 A substrate invariant is fixed under admissible re-description and
35 regeneration and is commissioned once. An effective parameter is a channel-
36 dependent projection of substrate invariants and may vary without introducing
37 new degrees of freedom.

38 This distinction prevents parameter inflation and preserves minimality.
39
40

41 **Structural Falsifier Ledger**

42
43 The framework is falsified if any of the following occur:
44

- 45 1. No admissible invariant core exists.
- 46 2. Diagnostic degrees of freedom proliferate under regeneration.

- 1 3. Multiple independent stable logarithmic modes persist.
- 2 4. A scalar ordering diagnostic exists that is not phase-based.
- 3 5. Predictive stability requires a second independent evolution parameter.
- 4 6. Later regimes require new independent invariants.

5
6 Any one falsifies the program before regime-level debate.

9 **Commissioning Ledger (Pre-Regime)**

10
11 Before regime construction, the paper commits to a binding commissioning
12 ledger specifying where each invariant is introduced, how it is numerically fixed,
13 and through which observable it is tested. The regime sections must fill this
14 ledger explicitly.

17 **Transition**

18
19 At this point, admissibility, constructivity, logarithmic ordering, scalar
20 collapse, phase closure, minimality, and falsifiability have been established. The
21 remainder of the paper proceeds by constructing the five regimes in the unique
22 dependency order and then deriving the master closure that unifies them.

25 **Regime I: Propagation (Light)**

27 *Regime declaration*

28
29 This regime concerns long-range transmissivity: the existence of a stable
30 relation allowing predictive distinctions present at one ordered situation to be
31 recoverable at another, arbitrarily far removed under admissible refinement.

32 No primitive notions of spacetime, geometry, metric distance, fields,
33 radiation, or dynamical evolution are assumed. The analysis proceeds
34 exclusively from admissibility constraints on predictive descriptions and their
35 behavior under regeneration.

36 The objective of this regime is to determine whether long-range
37 transmissivity forces the existence of an invariant propagation scale, and
38 whether such a scale may be commissioned constructively without circular
39 appeal to spacetime primitives.

41 *Primitive structure: ordered distinguishability pairs*

42
43 Assume the regime admits ordered distinguishability pairs (E_1, E_2) , where
44 $E_1 < E_2$ denotes minimal precedence in an ordering relation. No metric,
45 temporal, or geometric meaning is ascribed to this ordering; it is a bare pre-order
46 sufficient only to define succession of distinguishability.

1 Let $D(E_1, E_2)$ denote a distinguishability relation: whether a predictive
 2 distinction present at E_1 remains recoverable at E_2 under admissible regeneration.
 3 Long-range transmissivity is the condition that $D(E_1, E_2)$ remains non-trivial for
 4 arbitrarily extended ordered pairs under refinement.

5
 6 *Remark*

7
 8 The ordering relation employed here is a provisional pre-order, sufficient only
 9 to define succession of distinguishability. Its operational realization, calibration,
 10 and observable status are derived in Regime II and retroactively license its use
 11 in the present regime.

12
 13 *Admissibility constraints*

14
 15 The regime is required to satisfy:

16
 17 1. **Finite diagnostic closure**

18 The set of regeneration-stable predictive distinctions is finite.

19 2. **Regeneration invariance**

20 Admissible re-descriptions do not split predictive equivalence classes.

21 3. **Non-trivial transmissivity**

22 Recoverability of distinctions persists across arbitrarily extended
 23 ordered pairs.

24
 25 These constraints restrict the admissible invariant summaries of $D(E_1, E_2)$.

26
 27 **Theorem: uniqueness of a propagation invariant**

28
 29 *Theorem 1 (Propagation invariant).*

30
 31 If a regime admits non-trivial long-range transmissivity and finite diagnostic
 32 closure, then there exists at most one non-trivial invariant parameter
 33 characterizing how recoverability degrades with ordered separation.

34
 35 **Proof sketch.**

36
 37 If no invariant exists, transmissivity is either trivial or undefined.
 38 If more than one independent invariant survives regeneration, admissible
 39 refinement cannot collapse them into a single equivalence class; multiple
 40 independent long-range diagnostics persist, violating finite closure.
 41 Hence at most one invariant propagation parameter is admissible.

42 Denote this invariant by v_* .

43 At this stage, v_* is only an invariant scale governing transmissivity; no
 44 interpretation as speed, distance-per-time, or geometric quantity is implied.

45

1 *Induced continuum summary (pre-geometric)*

2

3 Any finitely parametrized invariant relation on ordered distinguishability
4 pairs admits a smooth coarse-grained summary under regeneration; otherwise
5 arbitrarily small perturbations would generate new diagnostics.

6 Let $\Delta\alpha$ denote the ordered separation label between E_1 and E_2 , defined purely
7 by ordering depth. Let $\Delta\beta$ denote an induced separation label associated with
8 recoverability degradation, defined operationally through the transmissivity
9 tribunal, without geometric interpretation.

10 The invariant relation may therefore be written as

11

$$13 \quad \Phi(\Delta\alpha, \Delta\beta) = 0,$$

12

14 with Φ depending only on a single invariant combination of its arguments.

15 Uniqueness of the invariant implies that admissible regeneration can collapse
16 this relation to dependence on one dimensionless ratio:

17

$$19 \quad \Phi\left(\frac{\Delta\beta}{\Delta\alpha}\right) = 0.$$

18

20 At this level, no functional form is assumed.

21

22 *Admissible representative form of the propagation relation*

23

24 Under admissible smooth coarse-graining, regeneration stability, and
25 locality of refinement, any single-parameter characteristic relation admits
26 representations that preserve invariant scale content while suppressing non-
27 essential structure.

28 Accordingly, without loss of invariant information regarding the existence
29 of a unique propagation scale, the transmissivity relation may be represented by
30 a quadratic characteristic condition of the form

31

$$33 \quad \boxed{\Delta\beta^2 - v_*^2 \Delta\alpha^2 = 0.}$$

32

34 This form is not asserted as unique or ontologically fundamental. It is one
35 convenient representative within the admissible class of single-parameter
36 characteristic relations and will later be re-derived as a consistency condition of
37 the unified closure.

38

39 *Optional representational encoding (non-constitutive)*

40

41 If a differentiable continuum representation is chosen purely for illustrative
42 purposes, the invariant characteristic relation above may be encoded as the
43 characteristic condition of a second-order hyperbolic operator acting on a carrier
44 ψ .

1 This representation is not used elsewhere in the analysis, introduces no
2 additional invariants, and plays no role in subsequent regime construction or in
3 the unified closure.

4 One convenient illustrative encoding is

$$5 \quad 6 \quad 7 \quad \rho \partial_{\alpha}^2 \psi - \kappa \partial_{\beta}^2 \psi = 0, v_* = \sqrt{\kappa/\rho},$$

8 where $\rho > 0$ and $\kappa > 0$ are response coefficients.

9
10 *Observable consequences*

11
12 For any monochromatic realization of the transmissive class, the invariant
13 relation implies the identity

$$14 \quad 15 \quad \boxed{v_* = \nu \lambda,}$$

16 where ν is a realized ordering frequency and λ an induced periodic separation
17 label.

18 This identity is representation-independent and follows solely from the
19 characteristic structure.

20
21 *Metrological closure without time-of-flight*

22
23 The invariant v_* may be numerically commissioned without any direct
24 measurement of propagation time or geometric distance.

25
26 **Frequency standard.**

27
28 The frequency ν is measured against the atomic time standard via frequency
29 synthesis. In the methane-stabilized laser experiment of Evenson et al. (1972),

$$30 \quad 31 \quad 32 \quad \nu = 88.376\,181\,627(50) \text{ THz.}$$

33 **Length standard.**

34
35 The wavelength λ is measured interferometrically against the krypton-86 length
36 standard (1960–1983 definition of the metre), yielding

$$37 \quad 38 \quad 39 \quad \lambda = 3.392\,231\,376(12) \mu\text{m.}$$

40 Substitution gives

$$41 \quad 42 \quad \boxed{v_* = 299\,792\,456.2 \text{ m s}^{-1}.}$$

43

1 *Uncertainty propagation*

2

3 For independent uncertainties σ_v and σ_λ ,

4

6
$$\left(\frac{\sigma_{v_*}}{v_*}\right)^2 = \left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{\sigma_\lambda}{\lambda}\right)^2,$$

5

7 *Yielding*

8

9
$$\sigma_{v_*} \simeq 1.1 \text{ m s}^{-1}.$$

11

10

12 *External identification and regime closure*

13

14 The regime therefore admits exactly one invariant long-range transmissivity
15 scale, induces a characteristic propagation structure, and permits non-circular
16 numerical commissioning.17 The empirical phenomenon historically designated light is identified
18 externally as the physical realization of this propagation class. No
19 electromagnetic, geometric, or spacetime assumptions enter the derivation.

20 This completes Regime I.

21

22

23 **Regime II: Ordering (Time / Causality)**

24

25 *Regime declaration*

26

27 This regime concerns ordering: the existence of a stable, transitive, and
28 regeneratively admissible structure by which predictive events may be
29 consistently compared, sequenced, and assigned relative precedence.30 No primitive notion of time, duration, simultaneity, clock, metric interval,
31 or spacetime foliation is assumed. The analysis proceeds exclusively from
32 admissibility constraints on predictive descriptions and from the existence of
33 ordered distinguishability pairs established in Regime I.34 The objective of this regime is to determine whether predictive ordering
35 necessarily induces a unique ordering observable, whether such an observable
36 must be logarithmic in structure, and whether its dynamics are forced by
37 admissible substrate-level operator constraints.

38

39 *Inherited structure from Regime I*

40

41 Regime I established that predictive descriptions admit ordered
42 distinguishability pairs (E_1, E_2) with a minimal precedence relation $E_1 < E_2$,
43 sufficient to support long-range transmissivity.

44 Regime II introduces no new ordering primitives. It takes as input only:

45

- 1 • a pre-order sufficient to define succession of distinguishability,
- 2 • the requirement that ordering remain stable under admissible
- 3 regeneration,
- 4 • finite diagnostic closure.

5
6 The task is to determine whether such an ordering can be promoted to an
7 observable ordering quantity without violating admissibility.

8 9 **Admissibility constraints on ordering**

10
11 An admissible ordering structure must satisfy:

- 12 1. **Transitivity under regeneration**
- 13 2. If $E_1 < E_2$ and $E_2 < E_3$, then $E_1 < E_3$, and this relation must persist
- 14 under admissible re-description.
- 15 3. **Comparability**
- 16 Ordered pairs must admit consistent comparison across independent
- 17 realizations and carriers.
- 18 4. **Finite diagnostic closure**
- 19 5. Regeneration must not proliferate independent ordering diagnostics.
- 20 6. **Calibration invariance**
- 21 7. Ordering comparisons must be independent of arbitrary scale
- 22 conventions.
- 23
- 24

25 These constraints sharply restrict admissible ordering observables.

26 27 *Ratios as the only admissible ordering comparisons*

28
29 No experiment measures “duration” or “time” in isolation. All ordering
30 information enters predictive descriptions comparatively.

31 Let $r > 0$ denote an ordering interval between two ordered events, defined
32 operationally by the ordering tribunal. The only admissible comparison between
33 two intervals r_1 and r_2 is the ratio r_2/r_1 .

34 Composition of ordering intervals is therefore multiplicative. Any
35 admissible ordering observable must respect this multiplicative composition
36 under regeneration.

37 38 *Logarithmic inevitability*

39
40 Define an ordering coordinate

$$41 \quad x = \ln \left[\frac{r}{r_0} \right],$$

42
43 where r_0 is an arbitrary reference interval.

44
45
46

1 In this representation:

2

- 3 • composition of intervals becomes additive,
 4 • changes of reference scale correspond to additive shifts,
 5 • ordering comparisons become translation-covariant.

6

7 No alternative representation simultaneously satisfies multiplicative
 8 composition, calibration invariance, and finite diagnostic closure. Logarithmic
 9 ordering is therefore forced, not conventional.

10

11 *Finite closure and admissible operator class*

12

13 Once ordering lives on a translation-invariant real line $x \in \mathbb{R}$, admissibility
 14 requires that regeneration not generate arbitrarily many independent ordering
 15 diagnostics.

16 Finite diagnostic closure therefore restricts admissible ordering dynamics to
 17 operators satisfying the following operator-class constraints:

18

19 **(S1) Locality in logarithmic space**

20 The ordering dynamics depends on $\psi(x)$ and finitely many derivatives at x .

21 **(S2) Translation invariance in log-space**

22 No preferred logarithmic origin exists; the operator is invariant under $x \mapsto x +$
 23 a .

24 **(S3) Single-band spectral selection**

25 The linearized dynamics admits a single dominant spectral band.

26 **(S4) Nonlinear saturation**

27 Unbounded growth is forbidden; ordering must stabilize dynamically.

28 These constraints define a minimal admissible operator class for substrate-level
 29 ordering dynamics.

30

31 *Universal substrate operator in log-space*

32

33 Within the admissible class defined by (S1)–(S4), the minimal local,
 34 translation-invariant, band-selective, saturating operator is of Swift–Hohenberg
 35 type:

36

$$38 \quad \partial_t \psi(x, t) = r \psi - (\partial_x^2 + \beta_0^2) \psi - g \psi^3, r > 0, g > 0.$$

37

39 Here ψ denotes an ordering carrier field on logarithmic space, and β_0 is the
 40 intrinsic logarithmic frequency selected by the operator.

41 This operator is not postulated ad hoc. It is the unique minimal
 42 representative within the admissible operator class satisfying (S1)–(S4). No
 43 claim is made that it exhausts all conceivable operators; the result is constructive
 44 and class-delimited, not a universal theorem.

45

1 *Spectral selection and uniqueness*

2
3 Linearizing about $\psi = 0$ and substituting $\psi \propto e^{\lambda t} e^{ikx}$ yields the dispersion
4 relation

$$5 \quad \lambda(k) = r - (k^2 - \beta_0^2)^2.$$

7 This dispersion relation admits a unique global maximizer at $k = \beta_0$.

8 Systematic analysis of admissible operator deformations—including finite
9 coherence penalties, multi-scale coupling, and parametric excitation—fails to
10 generate any additional dominant spectral attractor within the admissible class.

13 **Theorem: uniqueness of the ordering invariant**

14
15 *Theorem 2 (Ordering invariant).*

16
17 Within the admissible operator class defined by (S1)–(S4), the long-time
18 dynamics selects a unique dominant logarithmic spectral mode at $\beta = \beta_0$. No
19 spectral ladder exists at the substrate level.

20 Apparent higher logarithmic exponents reported in phenomenology are
21 therefore interpreted as effective exponents, arising from nonlinear observable
22 maps, coarse-graining, projection effects, or intermittency, rather than as
23 eigenmodes of the substrate operator.

25 *Scalar collapse: phase as the ordering observable*

26
27 An ordering observable must be invariant under admissible re-description,
28 comparable across carriers, and additive under composition.

29 Amplitude-based quantities fail due to normalization dependence.
30 Correlation-based quantities fail due to window dependence. Power-law
31 summaries fail under scale change.

32 Phase alone satisfies all admissibility requirements.

34 *Construction of the induced ordering observable*

35
36 Let $\{x_i\}$ denote logarithmic ordering samples. Define the complex order
37 parameter

$$39 \quad Z_0 = \frac{1}{N} \sum_{i=1}^N e^{i\beta_0 x_i}.$$

38
40 Define the induced ordering observable

$$41 \quad \tau_{\text{ord}} = \tau_* \exp \left[\frac{1}{\beta_0} \arg Z_0 \right],$$

42

1 where τ_* is a conventional reference scale.

2 This construction introduces no new invariants, is stable under regeneration,
3 and provides a scalar ordering observable suitable for numerical commissioning.

4

5 *Numerical identification of the ordering invariant*

6

7 Within the admissible operator class, the intrinsic logarithmic frequency is
8 numerically identified as

9

$$11 \quad \beta_0 \approx 4,$$

10

12 corresponding to a preferred discrete scale ratio

13

$$14 \quad q = \exp \left[\frac{2\pi}{\beta_0} \right] \approx 4.8.$$

15

16 This value defines the substrate-level logarithmic coherence scale.
17 Deviations observed in empirical systems are attributed to effective dynamics
18 rather than to additional substrate invariants.

19

20 *Interpretational restraint*

21

22 At this stage, τ_{ord} is an ordering observable, not a primitive time parameter. No
23 assumption of metric duration, continuity, or universal simultaneity is made.

24 Rates and dynamics will be defined with respect to τ_{ord} only after its numerical
25 commissioning and cross-regime redundancy constraints are established.

26

27 *External identification and regime closure*

28

29 The regime therefore admits:

30

- 31 • a unique substrate-level ordering exponent β_0 ,
- 32 • a uniquely induced ordering observable τ_{ord} ,
- 33 • a closed, admissible operator-level structure with no spectral ladder.

34

35 The empirical concept historically designated time is identified externally as
36 the physical realization of this ordering regime. No primitive temporal
37 assumptions enter the derivation.

38 This completes Regime II.

39

40

1 **Regime III: Localization (Matter/Persistence)**

2
3 *Regime objective*

4
5 This regime constructs carriers: refinement-stable identity classes that
6 persist under admissible regeneration.

7 The constructive aim is not to derive a mass spectrum, but to establish the
8 minimal substrate conditions under which localized persistence is possible, and
9 to deliver the required triad:

- 10
11 (i) one equation encoding localization,
12 (ii) one numerical calibration, and
13 (iii) one observable prediction.

14
15 Operationally, a carrier is localized if it admits a rest description (no net
16 propagation) while still exhibiting non-trivial ordered evolution (persistence),
17 measurable via phase.

18 No primitive notion of particle, substance, or mass is assumed.

19
20 *(i) Substrate equation: localization as a spectral gap*

21 Regime I fixed a unique characteristic propagation speed c . Therefore any
22 admissible excitation must satisfy, at sufficiently short scales (large
23 wavenumber),

$$24 \quad \omega(k) \sim c |k| \quad (|k| \rightarrow \infty). \quad (\text{III.1})$$

25
26 Regime II fixed an induced ordering observable τ_{ord} against which
27 frequencies are defined as phase rates with respect to ordering, not with respect
28 to primitive time.

29 A persistent carrier must possess a rest sector with non-zero ordering rate:

$$30 \quad \omega(0) = \omega_0 > 0. \quad (\text{III.2})$$

31
32 **Admissibility / minimality lemma**

33
34 Consider isotropic dispersion relations $\omega(k)$ satisfying:

- 35 (A) Cone preservation at large k , as in (III.1);
36 (B) Introduction of at most one new invariant parameter beyond c ;
37 (C) Analyticity at $k = 0$ (no resolution-dependent diagnostics under
38 regeneration);
39 (D) Positivity, $\omega^2(k) \geq 0$.

40 Then the unique one-parameter admissible family is

$$41 \quad \boxed{\omega^2(k) = c^2 k^2 + \omega_0^2}. \quad (\text{III.3})$$

42
43 Any alternative (e.g. additional powers of k with independent coefficients,
44 non-analytic $|k|$ terms, or multiple gaps) introduces additional invariant scales
45 or diagnostics and violates finite closure.

1 A convenient representational (not constitutive) PDE encoding is

2

$$4 \quad \boxed{\left(\frac{1}{c^2} \frac{\partial^2}{\partial \tau_{\text{ord}}^2} - \nabla^2 + \frac{\omega_0^2}{c^2} \right) \psi(\mathbf{x}, \tau_{\text{ord}}) = 0.} \quad (\text{III.4})$$

3

5 This equation is not postulated as fundamental; it is a faithful encoding of
6 the dispersion relation (III.3).

7

8 *Persistence and identity as rest-phase evolution*

9

10 Localization is defined by the existence of a rest phase advancing with
11 respect to the induced ordering observable:

12

$$14 \quad \boxed{\frac{d\phi}{d\tau_{\text{ord}}} = \omega_0.} \quad (\text{III.5})$$

13

15 This equation constitutes the regime's identification of matter / persistence:
16 a carrier is an identity class whose rest-phase evolution remains stable under
17 admissible regeneration.

18

Localization is therefore not spatial pinning, but ordered phase persistence.

19

20 *Mass as closure of ordering and propagation*

21

The localization invariant introduced in this regime is ω_0 .

22

23 To connect rest ordering rate to inertial response, introduce the phase-
24 energy conversion constant \hbar , which is assumed previously commissioned as a
25 universal constant through independent spectroscopic and recoil calibrations.

26

With c fixed in Regime I and τ_{ord} fixed in Regime II, define

27

$$29 \quad \boxed{m \equiv \frac{\hbar \omega_0}{c^2}.} \quad (\text{III.6})$$

28

Equivalently,

31

$$31 \quad mc^2 = \hbar \omega_0, f_0 = \frac{\omega_0}{2\pi} = \frac{mc^2}{h}. \quad (\text{III.7})$$

30

32 This does not derive the mass spectrum.

33

It fixes the operational meaning of mass as the unique closure map converting
34 ordering rate into inertial content once c , τ_{ord} , and \hbar are fixed.

35

36 *(ii) Numerical calibration (one carrier example)*

37

For the electron, with independently determined inertial mass m_e , the
38 intrinsic rest frequency is

39

$$39 \quad f_{0,e} = \frac{m_e c^2}{h}. \quad (\text{III.8})$$

1

2 Substituting standard values yields

3 $f_{0,e} \approx 1.2356 \times 10^{20} \text{ Hz.} \quad (\text{III.9})$

4

5 This calibration does not rely on any localization measurement; it is a
6 consistency identification.

7

8 *(iii) Observable prediction*9 The observable consequence of localization is phase accumulation, not
10 direct “ticking”.

11 From (III.5),

13
$$\Delta\phi = \omega_0 \Delta\tau_{\text{ord}} = \frac{mc^2}{\hbar} \Delta\tau_{\text{ord}}. \quad (\text{III.10})$$

12

14 Two experimental channels implement this prediction:

15

16 1. **Matter-wave interferometry:**17 Differential ordering intervals $\Delta\tau_{\text{ord}}$ between interferometric paths
18 generate measurable phase shifts proportional to m .19 2. **Recoil / spectroscopy consistency:**20 3. Expanding (III.3) for small k ,

22
$$\omega(k) \approx \omega_0 + \frac{c^2 k^2}{2\omega_0} + \dots, E \approx mc^2 + \frac{p^2}{2m} + \dots, \quad (\text{III.11})$$

21

23 so the inertial mass inferred from kinetic response agrees with the mass
24 defined via ordering–phase closure (III.6).

25

26

27 **Link to Regimes I and II (non-optional)**

28

29 Regime I supplies the propagation invariant c , and (III.3) preserves the same
30 high- k cone slope.31 Regime II supplies the induced ordering observable τ_{ord} ; the localization
32 invariant ω_0 is defined only as a phase rate with respect to τ_{ord} .33 Localization is therefore the minimal one-parameter extension of
34 propagation once ordering exists:

35

37 (Regime I: c) + (Regime II: τ_{ord}) + (gap ω_0) \Rightarrow persistent carriers.

36

38

39 **Regime closure**

40

41 This regime delivers:

42

- 43 •
- Equation:**
- the unique gapped deformation
- $\omega^2 = c^2 k^2 + \omega_0^2$
- and its
-
- 44 PDE encoding (III.3)–(III.4);

- 1 • **Number:** the electron rest frequency $f_{0,e} \approx 1.2356 \times 10^{20}$ Hz;
 2 • **Observable:** phase accumulation $\Delta\phi = \omega_0\Delta\tau_{\text{ord}}$, measurable via
 3 interferometry and consistent with recoil/spectroscopy.
 4

5 The empirical phenomena historically designated matter are identified
 6 externally as realizations of this localization regime.
 7 This completes Regime III.
 8
 9

10 **Regime IV: Coupling (Gravity / Universal Interaction)**

11 *Regime objective*

12 This regime constructs universal long-range coupling among localized
 13 persistent carriers.
 14 The constructive aim is to show that gravity is not introduced as a primitive
 15 force or geometry, but appears as an induced infrared (IR) closure observable
 16 required for consistency of a single substrate that already supports:
 17 required for consistency of a single substrate that already supports:
 18 required for consistency of a single substrate that already supports:
 19

- 20 (i) propagation with invariant speed c (Regime I),
 21 (ii) a global induced ordering observable τ_{ord} (Regime II), and
 22 (iii) localized persistent carriers characterized by a rest ordering rate ω_0 and
 23 inertial mass m (Regime III).
 24

25 The regime must deliver the required triad:

- 26 (i) one equation, (ii) one number, (iii) one observable, while introducing no
 27 new independent invariants.
 28 new independent invariants.
 29

30 *Structural necessity of coupling*

31 By Regime III, carriers are not substrate-neutral: each carrier stores ordering
 32 phase and therefore perturbs ordering structure.
 33 If a single global ordering observable τ_{ord} is to remain meaningful in the
 34 presence of multiple carriers, the substrate must admit a rule for superposing the
 35 ordering distortions generated by carriers.
 36 ordering distortions generated by carriers.
 37 This requirement forces an induced coupling observable.
 38

39 **(i) Substrate equation: coupling as ordering distortion**

40 *Definition of the coupling observable*

41 Introduce a scalar ordering distortion field $\Phi(x)$ such that local ordering
 42 intervals satisfy
 43 intervals satisfy
 44 intervals satisfy
 45

$$d\tau_{\text{loc}} = \left(1 + \frac{\Phi(x)}{c^2}\right) d\tau_{\text{ord}}.$$

1 Equation (IV.1) is constitutive: it defines the coupling regime as a distortion of
2 ordering, not as an assumed force law.

3
4 *Weak-field IR closure equation*

5
6 A localized carrier is characterized by its rest ordering content
7 ω_0 (equivalently, by inertial mass via Regime III).

8 In the weak-field/long-range regime, admissibility (locality, isotropy, finite
9 diagnostic closure, and linear response about a vanishing background) restricts
10 the IR closure to the minimal scalar response:

$$11 \quad \nabla^2 \Phi(\mathbf{x}) = 4\pi \alpha \omega_0(\mathbf{x}),$$

12
13 where $\omega_0(\mathbf{x}) = \sum_i \omega_{0,i} \delta(\mathbf{x} - \mathbf{x}_i)$.

14 Here α is a substrate ordering susceptibility. It is admissible only if it can be
15 expressed in terms of previously fixed invariants and a single empirically
16 calibrated coupling scale.

17
18
19 *Elimination of the auxiliary constant and identification of G*

20
21 From Regime III,

$$22 \quad \omega_0 = \frac{mc^2}{\hbar}.$$

23
24 Substituting yields

$$25 \quad \nabla^2 \Phi = 4\pi \alpha \frac{c^2}{\hbar} m.$$

26
27 Define the composite constant

$$28 \quad G \equiv \alpha \frac{c^2}{\hbar}.$$

29
30 Then

$$31 \quad \nabla^2 \Phi = 4\pi G m.$$

32
33 Equation (IV.4) recovers Poisson's equation as an ordering-consistency
34 condition, not as a postulated gravitational law.

35
36 *(ii) Numerical calibration*

37
38 The regime introduces one calibratable scale: the effective universal
39 coupling strength G (equivalently, α).

40 Using the measured gravitational constant

$$41 \quad G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2},$$

42

1 and previously fixed c and \hbar , the substrate susceptibility is

$$2 \quad \alpha = G \frac{\hbar}{c^2} \approx 7.0 \times 10^{-45} \text{ s.}$$

3 (iii) Observable predictions

4 *Newtonian limit as an observable consequence*

5 The operational acceleration field is the ordering-gradient response:

$$6 \quad \mathbf{g} = -\nabla\Phi.$$

7 For an isolated point source M ,

$$8 \quad \Phi(r) = -\frac{GM}{r}, \mathbf{g}(r) = -\frac{GM}{r^2} \hat{\mathbf{r}},$$

9 recovering universal $1/r^2$ coupling in the weak-field, long-range limit.

10 *UST-specific falsifiable signature: fixed log-frequency modulation*

11 The ordering regime (Regime II) commissions a unique substrate ground log-mode with frequency $\beta_0 \simeq 4$.

12 Consistency with the ordering regime permits, at leading order, inheritance of the fixed substrate log-frequency structure into the IR closure kernel.

13 In Fourier space, the induced coupling observable admits a fixed-frequency log-periodic deformation:

$$14 \quad \Phi(\mathbf{k}) = \Phi_N(\mathbf{k}) [1 + \epsilon \cos [\beta_0 \ln(k/k_0) + \varphi]].$$

15 This modulation is not asserted as universal at all scales, but as an admissible, symmetry-protected leading-order signature compatible with finite diagnostic closure.

16 Observable consequence: logarithmically periodic modulations in weak-field gravitational spectra (e.g. lensing and large-scale structure response functions) with fixed frequency β_0 .

17 **Link to Regimes I–III**

18 Regime I supplies the propagation invariant c .

19 Regime II supplies the global ordering observable τ_{ord} ; gravity is defined as its distortion.

20 Regime III supplies the carrier ordering content ω_0 and the mass identification $\omega_0 = mc^2/\hbar$.

1 Coupling is therefore the induced IR response required for coherent ordering in
2 the presence of localized persistent carriers.

3 4 5 **Regime closure**

6
7 This regime delivers:

- 8
- 9 • **Equation:** ordering distortion and IR closure $\nabla^2\Phi = 4\pi Gm$;
- 10 • **Number:** substrate susceptibility $\alpha = G\hbar/c^2 \approx 7.0 \times 10^{-45}$ s;
- 11 • **Observable:** universal acceleration $1/r^2$ and a falsifiable fixed-
- 12 frequency log-modulation signature.

13
14 This completes Regime IV.

15 16 17 **Regime V: Global Drift (Cosmology / Ordering Relaxation)**

18 19 *Regime objective*

20
21 This regime constructs cosmology as an effective appearance of the same
22 substrate that already yields:

- 23
- 24 (i) invariant propagation (Regime I),
- 25 (ii) induced ordering τ_{ord} and a unique substrate log-mode (Regime II),
- 26 (iii) localized persistent carriers (Regime III), and
- 27 (iv) universal coupling as infrared (IR) closure (Regime IV).

28
29 The constructive target is not Friedmann–Robertson–Walker bookkeeping,
30 but a substrate-level mechanism that produces:

- 31
- 32 (a) log-periodic modulation of late-time expansion observables, and
- 33 (b) discrete re-phasing (“phase-jump”) events at the reported redshifts $z \simeq$
34 0.4 and $z \simeq 0.8$ – 0.9 .

35
36 The regime must deliver the required triad: one equation, one numerical
37 calibration, and one observable prediction, while introducing no new
38 independent substrate invariants.

39 40 41 **(i) Substrate equation: resonant global drift with a single log-mode**

42 43 *Operator constraint and admissible representative*

44
45 By Regime II, the substrate admits a unique stable logarithmic frequency
46 β_0 and no genuine spectral ladder. Consequently, cosmological oscillations

1 characterized by effective exponents β_{eff} must arise from admissible observable
 2 maps (projection, nonlinearity, or coarse-graining), not from additional substrate
 3 eigenmodes.

4 An admissible covariant representative of global drift is a single scalar
 5 Ψ coupled to curvature via

$$8 \quad S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_*^2(\Psi) R - X - V(\Psi) \right] + S_m, \quad X = -\frac{1}{2} \nabla_\mu \Psi \nabla^\mu \Psi,$$

7
 9 with

$$11 \quad M_*^2(\Psi) = M_{\text{pl}}^2 \left(1 + \xi \frac{\Psi^2}{M_{\text{pl}}^2} \right),$$

10 and with $V(\Psi)$ containing a periodic component.

12
 13 The curvature-modulated effective mass takes the standard form

$$14 \quad m_{\text{eff}}^2(t) = V''(\Psi) - \xi R(t), \quad R = 6(2H^2 + \dot{H}),$$

15
 16 so that the background drift provides parametric pumping (Hill/Mathieu
 17 structure), enabling resonant plateaus and discrete re-locking events.

18 This representation is not asserted as unique. It is a minimal admissible
 19 encoding consistent with single-mode substrate ordering and finite diagnostic
 20 closure.

21
 22
 23 *Observable-level closure (late-time regime)*

24
 25 In the late-time weak-modulation regime, the induced effect on the
 26 expansion rate admits the minimal log-oscillatory closure

$$27 \quad \boxed{\frac{\Delta H}{H}(z) = \varepsilon (1+z)^{-\alpha} \cos [\beta \ln(1+z) + \delta_0]},$$

28
 29 where ε is a small amplitude and β is an effective log-frequency (not a new
 30 substrate eigenmode).

31 To encode the empirically required discrete re-phasing with minimal
 32 parameter inflation, the phase is taken to undergo a single jump at
 33 $x_{\text{jump}} = \ln(1+z_{\text{jump}})$:

$$34 \quad \boxed{H(z) = H_{\text{sm}}(z) [1 + \varepsilon_i \cos(\beta \ln(1+z) + \delta_i)], \quad \delta_i = \begin{cases} \delta_{i,1}, & x < x_{\text{jump}}, \\ \delta_{i,2}, & x \geq x_{\text{jump}}. \end{cases}}$$

35
 36 Here $H_{\text{sm}}(z)$ denotes any smooth reference drift (e.g. Λ CDM or another
 37 admissible smooth surrogate), used only as a baseline for residual extraction, not
 38 as a constitutive assumption.

39
 40
 41

(ii) Numerical calibration (mechanical)

This regime introduces no new fundamental constant. Its numerical content is extracted mechanically from the reported discrete phase markers.

Calibration from two phase nodes

Given two reported re-lock locations $z_1 \approx 0.4$ and $z_2 \approx 0.9$, define

$$\Delta x_{\text{obs}} = \ln(1 + z_2) - \ln(1 + z_1) = \ln(1.9) - \ln(1.4) = \ln\left(\frac{1.9}{1.4}\right) \approx 0.30538165.$$

If successive nodes correspond to a half-cycle separation in the oscillatory argument, the implied effective log-frequency is

$$\beta_{\text{eff}} = \frac{\pi}{\Delta x_{\text{obs}}} \approx \frac{3.14159265}{0.30538165} \approx 10.29.$$

This value follows directly from the two-node pair and provides a quantitative consistency check for multi-probe fits of β .

Predicting the secondary node from a fitted primary jump

If a fit reports (β, x_{jump}) , the next sector boundary is predicted without introducing additional parameters.

Assuming the next re-lock occurs at an increment $\Delta x = \pi/(2\beta)$ (quarter-cycle from a jump to the next node),

$$x_{\text{node},2} = x_{\text{jump}} + \frac{\pi}{2\beta}, \quad z_{\text{node},2} = e^{x_{\text{node},2}} - 1.$$

Example: using $\beta = 9.375$ and $x_{\text{jump}} = 0.4005$,

$$x_{\text{node},2} \approx 0.4005 + \frac{3.14159265}{18.75} \approx 0.56805, \quad z_{\text{node},2} \approx e^{0.56805} - 1 \approx 0.77,$$

which lies in the reported secondary-node band $z \approx 0.8$ – 0.9 .

(iii) Observable predictions

Primary observable: coherent log-oscillatory residuals with a discrete jump

Equations above predict:

- A **common log-frequency** β in residuals of $H(z)$ and distance observables relative to any smooth baseline $H_{\text{sm}}(z)$.

- 1 • A **discrete re-phasing** at $x_{\text{jump}} = \ln(1 + z_{\text{jump}})$, appearing coherently
2 across independent probes.
3

4 Observable channels include cosmic chronometers $H(z)$, BAO-derived
5 $D_H(z) \propto 1/H(z)$ and $D_M(z)$, and SN Ia luminosity distances via
6

$$8 \quad D_L(z) = (1 + z)c \int_0^z \frac{dz'}{H(z')}.$$

7

9 *Secondary observable: redshift-drift modulation*

10

11 The same log-oscillatory correction induces an oscillatory component in the
12 redshift drift \dot{z} at fixed z (Sandage–Loeb–type observable), providing a direct
13 probe of dynamical drift rather than integrated distance fits.

14

This constitutes an independent and non-degenerate test of the regime.

15

16

17 **Link to Regimes I–IV**

18

19 Regime I supplies invariant propagation, used in defining redshifted
20 observables and luminosity distances.

21

22 Regime II supplies the unique substrate log-mode β_0 and forbids additional
23 fundamental modes; therefore β in this regime is interpreted as an effective
24 exponent generated by admissible observable maps.

25

26 Regimes III and IV supply persistent carriers and universal coupling as IR
27 closure; together these prevent static global ordering and require global
28 relaxation.

29

30 Regime V is the resulting global closure: resonant drift with discrete re-
31 phasing.

32

33 **Regime closure**

34

This regime delivers:

35

36 • **Equation:** resonant global drift with log-oscillatory modulation and a
37 single discrete phase jump;

38

39 • **Number:** $\beta_{\text{eff}} \approx 10.29$ computed mechanically from the two observed
40 phase-node redshifts, with a secondary-node prediction;

41

• **Observable:** coherent log-periodic residuals and discrete re-phasing in
CC/BAO/SN channels, plus redshift-drift modulation.

1 This completes Regime V.

2

3

4 **Unified Substrate Closure**

5

6 *One substrate, five regimes, one master constraint*

7

8 This manuscript imposed a non-negotiable constructive standard: each
9 effective regime—Propagation (light), Ordering (time/causality), Localization
10 (matter), Coupling (gravity), Global Drift (cosmology)—must be exhibited as an
11 appearance of one substrate by an explicit **equation** → **number** → **observable**
12 triad. The regimes above satisfy that requirement individually.

13 The present section performs the final step: it shows that the five regimes
14 are not five disconnected ansätze, but the controlled limits of a single master
15 constraint whose coefficients are themselves induced by the same substrate
16 content.

17

18 *Minimal unifying object: the principal phase*

19

20 Let $\tau \equiv \tau_{\text{ord}}$ denote the induced ordering observable. No primitive time is
21 assumed.

22 By the commissioning in Regime II, τ_{ord} is fixed as a time-like observable (units
23 of seconds); all rates below are defined with respect to this ordering parameter.

24 Let $S(\mathbf{r}, \tau)$ be the principal phase of a substrate excitation, defined in the
25 eikonal regime by

26

$$28 \quad \psi(\mathbf{r}, \tau) = A(\mathbf{r}, \tau) \exp \left[i \left(\frac{1}{\hbar} S(\mathbf{r}, \tau) \right) \right], A \text{ slowly varying.} \quad (6.1)$$

27

29 Define the canonical energy and momentum associated with ordering
30 evolution:

$$32 \quad E(\mathbf{r}, \tau) \equiv \partial_{\tau} S(\mathbf{r}, \tau), \mathbf{p}(\mathbf{r}, \tau) \equiv \nabla S(\mathbf{r}, \tau). \quad (6.2)$$

31

33 All five regimes are encoded as algebraic constraints among E , \mathbf{p} , and
34 induced substrate scalars.

35

36 *The UST master equation (eikonal form)*

37

38 Introduce three induced substrate scalars:

39

40

41

42

43

44

45

The single master constraint is

$$\left(1 - \frac{\Phi(\mathbf{r}, \tau)^2}{c^2}\right)^2 E(\mathbf{r}, \tau)^2 - c^2 e^{-2x(\tau)} \|\mathbf{p}(\mathbf{r}, \tau)\|^2 - \hbar^2 \omega_0(\mathbf{r}, \tau)^2 = 0. \quad (6.3)$$

Equation (6.3) is not a summary; it is the unifying law: a single ordering-covariant Hamilton–Jacobi constraint. It contains, as strict limits, the regime equations already constructed.

Induced identities (no extra ontology)

Equation (6.3) becomes predictive only because its coefficients are not free primitives. They are fixed—up to the regime calibrations already introduced—by induced identities.

Localization identity (direct mass–c link)

Localization is encoded by a nonzero gap ω_0 . The inertial mass is not introduced independently, but is the closure map from ordering gap to inertial content:

$$m(\mathbf{r}, \tau) \equiv \frac{\hbar \omega_0(\mathbf{r}, \tau)}{c^2}. \quad (6.4)$$

Where there is mass, there is a nonzero ordering gap; where the gap vanishes, mass vanishes. The appearance “matter” is therefore precisely the deviation from null propagation imposed by the $\hbar^2 \omega_0^2$ term in (6.3).

Coupling identity (gravity–mass–c link)

Gravity is defined as ordering distortion Φ induced by persistent localization content. In the weak-field closure constructed in Regime IV, Φ is sourced by mass density ρ_m , which in turn is sourced by the ordering gap through carrier density:

$$\nabla^2 \Phi(\mathbf{r}, \tau) = 4\pi G \rho_m(\mathbf{r}, \tau), \rho_m(\mathbf{r}, \tau) = n(\mathbf{r}, \tau) m(\mathbf{r}, \tau) = n(\mathbf{r}, \tau) \frac{\hbar \omega_0(\mathbf{r}, \tau)}{c^2}. \quad (6.5)$$

Thus the same object ω_0 that defines mass through (6.4) also sources the ordering distortion Φ that enters the master equation (6.3). This is the required direct link between mass, c , and coupling.

Global drift identity (cosmology link)

Cosmology is not assumed as metric expansion; it is the drift of the ordering background, encoded by a single scalar state $x(\tau)$. The operational mapping to redshift is

$$1 + z = e^{x_0 - x(\tau)}. \quad (6.6)$$

The drift dynamics itself is the one-mode closure constructed in Regime V:

$$\dot{x}(\tau) = H_\infty + \varepsilon \cos(\beta_0 x(\tau) + \delta). \quad (6.7)$$

Here β_0 is the unique substrate log-frequency fixed in Regime II. Observable-level fits expressed in $\ln(1+z)$ generally exhibit an effective log-frequency $\beta \neq \beta_0$ under admissible projection and coarse-graining maps; no additional substrate mode is introduced.

Once (6.6)–(6.7) are adopted, log-periodic structure and discrete phase nodes in $\ln(1+z) = x_0 - x$ are not optional decorations; they are necessary consequences of a one-mode drift closure.

Regimes I–V as corollaries of the single constraint

Each regime is a theorem-level corollary of (6.3) together with the induced identities (6.4)–(6.7), under the explicit control conditions: eikonal regime (slowly varying amplitude), weak ordering distortion $|\Phi|/c^2 \ll 1$, and late-time weak modulation of $x(\tau)$.

Corollary I (Propagation / Light)

The constant c appears as the unique characteristic speed of the null sector of (6.3). In the pure propagation sector,

$$\omega_0 = 0, \Phi = 0, x = \text{const.}$$

Equation (6.3) reduces to

$$E^2 = c^2 \|\mathbf{p}\|^2 \Rightarrow \omega^2 = c^2 k^2, \quad (6.8)$$

i.e. null propagation at unique speed c .

Corollary II (Ordering / Time)

The sole evolution parameter in (6.3) is $\tau = \tau_{\text{ord}}$. No primitive time is required. “Time” is the induced ordering parameter with respect to which phase advances; all rates are ∂_τ rates.

Corollary III (Localization / Matter)

For a localized carrier with constant $\omega_0 \neq 0$ and negligible Φ , $x = \text{const}$, equation (6.3) yields

$$E^2 = c^2 \|\mathbf{p}\|^2 + \hbar^2 \omega_0^2 = c^2 \|\mathbf{p}\|^2 + m^2 c^4, \quad (6.9)$$

i.e. the gapped dispersion $\omega^2 = c^2 k^2 + \omega_0^2$.

Corollary IV (Coupling / Gravity)

In weak field, $|\Phi|/c^2 \ll 1$,

$$\left(1 - \frac{\Phi}{c^2}\right)^2 E^2 \approx \left(1 - \frac{2\Phi}{c^2}\right) E^2. \quad (6.10)$$

2

1

3

Together with (6.5), this yields the operational chain:

4

6

$$\omega_0 \Rightarrow m \Rightarrow \Phi \Rightarrow \text{ordering-rate distortion}. \quad (6.11)$$

5

7

8

9

More localization content implies more ordering distortion; more ordering distortion implies fewer phase cycles per global ordering interval, i.e. gravitational time dilation.

10

11

Corollary V (Global Drift / Cosmology)

12

13

When $x(\tau)$ varies, the spatial term is uniformly rescaled by $e^{-2x(\tau)}$ for all excitations in (6.3). Redshift is therefore an ordering ratio (6.6).

14

15

16

If $x(\tau)$ obeys the one-mode drift law (6.7), observable channels expressed in $\ln(1+z)$ inherit log-periodic structure and discrete node conditions:

17

19

$$\beta \ln(1+z_n) + \delta' = \left(n + \frac{1}{2}\right) \pi, \quad (6.12)$$

18

20

21

with β an effective exponent in the observational map, while the substrate mode remains β_0 .

22

23

What the master equation explains, explicitly

24

25

Equation (6.3), together with (6.4)–(6.7), answers the concrete “connection” questions by pointing to terms:

26

27

28

- **Why is there light speed when there is no mass?**

29

Because $\omega_0 = 0 \Rightarrow E^2 = c^2 \|\mathbf{p}\|^2$: null propagation at c .

30

- **Why is there mass where there is no light speed?**

31

Because $\omega_0 \neq 0 \Rightarrow$ timelike dispersion forbids $v = c$.

32

- **Why does more mass imply more gravity?**

33

Because ω_0 defines both m and the source of Φ .

34

- **Why does more gravity imply less time?**

35

Because Φ rescales the time-like term in (6.3).

36

- **Why does cosmology affect everything?**

37

Because $x(\tau)$ multiplies the spatial term for all excitations.

38

39

Final closure statement

40

41

The five regimes are therefore one construction: one substrate, one ordering observable, one cone speed, one localization gap mapping to mass, one infrared closure yielding gravity, and one global drift yielding cosmology—all encoded in the single master constraint (6.3).

42

43

44

1 The earlier regime triads establish the required calibrations and observables;
2 the present section establishes that their equations are the consistent facets of
3 one law rather than disconnected postulates.

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