Finite Element and Experimental Investigation of Plate Vibrations Using Chladni Techniques

Teaching the fundamentals of vibration and modal behavior can be challenging because students often struggle to relate theoretical concepts to physical phenomena. This study presents an educational approach that integrates Chladni plate experiments, modal analysis theory, and finite element simulations to provide a visual and interactive framework for teaching structural dynamics. By exciting a square plate at its natural frequencies, students directly observed Chladni patterns and nodal lines, offering a clear demonstration of resonance and vibration mode shapes. Experimental results were compared with finite element modal analysis performed in ANSYS Workbench. Five experimentally observed mode shapes were matched with their numerical counterparts, showing excellent qualitative agreement in the predicted nodal patterns. A quantitative comparison of natural frequencies yielded an average error of approximately 2.8%, indicating good agreement between the numerical model and the physical system. This approach provides undergraduate students with early exposure to vibration theory, modal analysis, and computational modelling. The integration of experimental and numerical methods offers an accessible framework for connecting theoretical concepts with observable physical behavior.

Keywords: *Vibrations, Chladni Patterns, Modal Analysis, Finite Element Analysis, Engineering Education.*

Introduction and Literature Review

Vibration is a fundamental topic in mechanical, aerospace, civil, and structural engineering, playing a critical role in the design, analysis, and operation of engineering systems. Understanding the dynamic behavior of structures, including natural frequencies, mode shapes, and resonance phenomena, is essential for predicting performance, preventing failures, and ensuring structural reliability [1,2]. Despite its importance, vibration theory is often perceived by students as abstract and mathematically demanding, making it difficult to establish a clear connection between theoretical concepts and real physical behavior.

Traditional instruction in vibration courses frequently relies on analytical derivations and mathematical models. While these approaches provide a rigorous theoretical foundation, students may struggle to visualize how vibrating systems behave in practice. Educational studies have shown that active learning and visual demonstrations can significantly improve conceptual understanding of complex engineering concepts [3-4-5-6]. Consequently, the integration of experimental demonstrations with theoretical and computational methods has become increasingly important in engineering education.

One of the most effective experimental demonstrations of vibration phenomena is the Chladni plate. First introduced by Ernst Chladni in the eighteenth century, the technique involves exciting a thin plate at its natural frequencies and observing the formation of nodal patterns through the accumulation of fine particles along regions of zero displacement [7]. These patterns provide a direct visualization of vibration mode shapes and standing wave behavior, transforming abstract mathematical concepts into observable physical phenomena.

The scientific significance of Chladni patterns continues to attract research interest. Baker et al. (2024) investigated resonant modes in circular and polygonal plates and demonstrated the influence of plate geometry and boundary conditions on the resulting vibration patterns [8]. Their findings confirmed that experimentally observed nodal patterns correspond closely to theoretical mode shapes predicted by vibration theory. More recently, Abramian et al. (2025) examined the physical mechanisms responsible for particle accumulation in Chladni experiments and proposed a diffusion-based explanation for the formation of nodal patterns [9]. These studies highlight both the scientific relevance and educational value of Chladni figures as a means of understanding vibrational phenomena.

The interpretation of Chladni patterns is closely related to modal analysis, a widely used technique for characterizing the dynamic behavior of structures. Modal analysis determines the natural frequencies and corresponding mode shapes of a system, providing essential information for vibration control, structural design, and dynamic performance evaluation [1,10]. By decomposing complex structural responses into individual vibration modes, modal analysis offers a systematic framework for understanding resonance behavior. The visual patterns produced by Chladni plates serve as physical manifestations of these theoretical mode shapes, creating a natural bridge between experimental observation and analytical concepts.

In parallel with experimental developments, numerical simulation has become an indispensable tool for vibration analysis. Finite Element Analysis (FEA) software enables engineers to predict natural frequencies, mode shapes, and dynamic responses for structures with complex geometries and boundary conditions. Commercial software packages such as ANSYS are widely used in both industry and academia for modal and harmonic response analysis [11-12]. The integration of computational simulation into engineering education allows students to gain practical experience with industry-standard tools while reinforcing theoretical concepts through numerical experimentation.

In fact, when students compare experimentally observed mode shapes with finite element predictions, they gain insight into the assumptions underlying numerical models, the importance of boundary conditions, and the relationship between theoretical predictions and real-world behavior. Furthermore, simulation-based learning helps students develop professional skills that are increasingly expected in modern engineering practice [13-14].

Although previous studies have explored Chladni patterns, modal analysis, and finite element simulations individually, relatively few investigations have integrated these elements into a unified educational framework. Existing research has primarily focused on understanding the physics of pattern formation or improving modal characterization techniques, while less attention has been given to their combined use as pedagogical tools for teaching vibration concepts.

Therefore, this study presents an educational approach that integrates experimental visualization using Chladni plates, theoretical interpretation through modal analysis, and computational validation using ANSYS simulations. By comparing experimentally observed Chladni patterns with numerically predicted mode shapes, students are able to connect physical observations with analytical and computational methods. This integrated methodology aims to enhance student engagement, improve conceptual understanding of vibration behavior, and provide initial exposure to engineering simulation tools.

Project Description

Project Engineering Success at San José State University (SJSU) provides students with opportunities to participate in a variety of academic and professional development activities. During the Fall 2025 semester, selected students were invited to participate in faculty-mentored research projects across different engineering disciplines. Over a period of three months, students worked closely with faculty mentors, gaining hands-on experience in the research process and contributing to ongoing projects.

The program concluded with a poster presentation event during which students presented their research projects, preliminary results, and initial conclusions. This experience provided students with an opportunity to develop technical communication skills while sharing their work with faculty and peers, and the broader university community.

Participation in undergraduate research allows students to gain a deeper understanding of the systematic process of conducting engineering research, including problem identification, literature review, experimental design, data analysis, and interpretation of results. Furthermore, the experience helps students recognize how the concepts learned in their coursework are applied to solve real-world engineering problems. Demonstrating these connections between theory and practice is essential for increasing student engagement, strengthening motivation, and enhancing the overall learning experience.

The present project was conducted within the framework of the Project Engineering Success program and focused on the integration of experimental, analytical, and computational methods for teaching vibration concepts through Chladni plate demonstrations and finite element analysis.

Two students from the Aerospace Engineering Department, one junior and one senior, participated in the project and met with the faculty mentor for one hour each week throughout the three-month program. Neither student had prior coursework or formal background in vibration analysis, as the vibrations course is offered later in the aerospace engineering curriculum. As a result, the project provided an opportunity for the students to gain early exposure to fundamental concepts in structural dynamics and vibration theory before encountering these topics in their regular coursework. The initial phase of the project focused on introducing the students to the Chladni plate experiment and the fundamentals of Finite Element Analysis (FEA).

To establish a foundation in numerical modeling, the students were first introduced to static structural analysis. The faculty mentor explained the governing analytical equations in matrix form and demonstrated how finite element simulations can be used to reproduce theoretical results. A simple cantilever beam was selected as the initial case study. The analytical solution for the beam-tip deflection was derived and compared with results obtained from ANSYS simulations. Through a step-by-step tutorial, the students learned how to create geometry, assign material properties, generate finite element meshes, apply boundary conditions and loads, run the analysis, and interpret the results.

This approach allowed the students to become familiar with the terminology, theoretical foundations, and workflow associated with finite element modeling. By comparing analytical and numerical solutions, they gained an understanding of the assumptions and limitations associated with simulation-based methods.

Following the introduction to static analysis, the students progressed to dynamic analysis. Similar to the approach adopted for static analysis, the students were first introduced to the governing equation of motion of an undamped single-degree-of-freedom (SDOF) system. The analytical formulation was presented in matrix form, and the physical significance of mass, stiffness, natural frequency, and free vibration response was discussed.

Subsequently, finite element dynamic analyses were performed using the same cantilever beam subjected to different types of excitation loads. These simulations enabled the students to observe how a structure responds when the excitation frequency approaches one of its natural frequencies. The comparison between theoretical predictions and numerical results helped reinforce the concepts of natural frequency and resonance while providing a deeper understanding of structural dynamic behavior.

The students were then introduced to the equations governing the vibration of multi-degree-of-freedom systems and learned how these equations can be decoupled into a set of independent single-degree-of-freedom systems through modal analysis. Both the theoretical background and the finite element implementation were discussed.

Building on the experience gained from the cantilever beam example, the students applied finite element techniques to model vibrating plates and reproduce Chladni patterns through modal analysis. The numerically predicted mode shapes were compared with experimentally observed Chladni patterns. This process enabled the students to establish a direct connection between vibration theory, finite element simulations, and physical observations, thereby reinforcing their understanding of structural dynamics, modal behavior, and the relationship between analytical, computational, and experimental approaches in engineering.

Mathematical Background

The modal decomposition approach presented herein follows the classical formulation of structural dynamics [2, 15].

The finite element equation of motion of a damped n degree of freedom (DOF) system is given by:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t) + \boldsymbol{C}\dot{\boldsymbol{X}}(t) + \boldsymbol{K}\boldsymbol{X}(t) = \boldsymbol{f}(t) \qquad (1)$$

where $\boldsymbol{M} \in R^{n\times n}$ is the symmetric positive definite mass matrix; $\boldsymbol{C} \in R^{n\times n}$ is the symmetric damping matrix; $\boldsymbol{K} \in R^{n\times n}$ is the symmetric stiffness matrix and $f(t) \in R^{n}$ is the external load vector applied to the structure. The vectors $\ddot{\boldsymbol{X}}(t)$, $\dot{\boldsymbol{X}}(t)$ and $\boldsymbol{X}(t) \in R^{n}$ represent the acceleration, velocity and displacement responses of the system, respectively.

The modal solution of Eq. (1) requires first solving the associate eigenvalue problem. In the absence of damping and external excitation (undamped free vibration) Eq. (1) reduces to:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t) + \boldsymbol{K}\boldsymbol{X}(t) = \boldsymbol{0} \qquad (2)$$

To formulate the vibration problem into a symmetric eigenvalue form, a coordinate transformation is introduced. Let

$$\boldsymbol{X}(t) = \boldsymbol{M}^{-1/2}\boldsymbol{q}(\mathrm{t})$$

and $\qquad (3)$

$$\ddot{\boldsymbol{X}}(t) = \boldsymbol{M}^{-1/2}\ddot{q}(\mathrm{t})$$

Substituting Eq (3) into Eq. (2) and left multiply by $\boldsymbol{M}^{-1/2}$yields:

$$\boldsymbol{M}^{-1/2}\boldsymbol{M}\boldsymbol{M}^{-1/2}\ddot{q}(\mathrm{t}) + \boldsymbol{M}^{-1/2}\boldsymbol{K}\boldsymbol{M}^{-1/2}\boldsymbol{q}(\mathrm{t}) = \boldsymbol{0}$$

Which simplifies to:

$$\boldsymbol{I}\ddot{q}(\mathrm{t}) + \widetilde{\boldsymbol{K}}\boldsymbol{q}(\mathrm{t}) = \boldsymbol{0} \qquad (4)$$

Where $\boldsymbol{I}$ is the identity matrix and $\widetilde{\boldsymbol{K}}$ is the mass normalized stiffness matrix. Equation (4) is solved by assuming a harmonic solution of the form:

$$\boldsymbol{q}(\mathrm{t}) = \boldsymbol{v}e^{\boldsymbol{j\omega t}} \qquad (5)$$

Differentiating Eq. (5) twice with respect to time gives:

$$\ddot{\boldsymbol{q}}(t) = -\omega^2 \boldsymbol{v} e^{\boldsymbol{j\omega t}} \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) and rearranging yields:

$$(\widetilde{\boldsymbol{K}} - \lambda \boldsymbol{I})\boldsymbol{v} = \boldsymbol{0} \qquad \text{with} \quad \lambda = \omega^2 \tag{7}$$

Equation (7) has the standard form of eigenvalue problem. For a nontrivial solution to exist, the determinant of the coefficient matrix must be zero:

$$\left|\widetilde{\boldsymbol{K}} - \lambda \boldsymbol{I}\right| = 0 \tag{8}$$

The characteristic equation has n roots denoted by (λ_1, λ_2, …, λ_n), called eigenvalues. The square roots of these eigenvalues correspond to the natural frequencies of the system. For each eigenvalue λ_i, an associated n-dimensional eigenvector $\boldsymbol{v_i}$ is from Eq. (7):

$$\boldsymbol{v_i} = \{v_{1i}, v_{2i}, \dots, v_{ni}\}^T \tag{9}$$

Because the eigenvector describes the relative deformation of the structure during vibration, it is commonly referred to as mode shape or natural mode. Furthermore, the modal vectors satisfy orthogonality conditions with respect to both the identity matrix and the mass-normalized stiffness matrix, implying that all eigenvectors are linearly independent. Since modal vectors represent relative displacements rather than absolute magnitudes, they must be scaled to be compared. This scaling process is known as normalization.

It is possible to define an $n \times n$ normal modal matrix whose columns are the normalized mode shapes:

$$\boldsymbol{P} = [\boldsymbol{v}_1 \; \boldsymbol{v}_2 \; \dots . \boldsymbol{v}_n] \tag{10}$$

The orthogonality condition of the normal modal matrix with respect to the mass normalized stiffness matrix is given by:

$$\boldsymbol{P^T} \widetilde{\boldsymbol{K}} \boldsymbol{P} = \boldsymbol{\Lambda} \tag{11}$$

Where $\boldsymbol{\Lambda}$ is a diagonal matrix, referred to as the spectra matrix of $\widetilde{\boldsymbol{K}}$, defined as:

$$\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1 \;\; \lambda_2 \dots \lambda_n \;) \tag{12}$$

The diagonal entries are the eigenvalues of $\widetilde{\boldsymbol{K}}$, which correspond to the squares of the system's natural frequencies**.**

Next, as second coordinate system, $\boldsymbol{r}(t) = \{r_1, r_2\,, \dots, \mathrm{r_n}\}^T$, is introduced such that:

$$\boldsymbol{q}(t) = \boldsymbol{P}\,\boldsymbol{r}(t) \tag{13}$$

Differentiating Eq. (13) twice with respect to time yields:

$$\ddot{\boldsymbol{q}}(t) = \boldsymbol{P}\,\ddot{\boldsymbol{r}}(t) \tag{14}$$

Substituting Eq. (13) and Eq. (14) into Eq. (4), multiplying from the left by the matrix $\boldsymbol{P^T}$, and rearranging, gives:

$$\boldsymbol{P^T I P}\,\ddot{\boldsymbol{r}}(t) + \boldsymbol{P^T}\widetilde{\boldsymbol{K}}\boldsymbol{P}\,\boldsymbol{r}(t) = \boldsymbol{0}$$

This reduces to:

$$\boldsymbol{I}\ddot{\boldsymbol{r}}(t) \;\; + \boldsymbol{\Lambda}\boldsymbol{r}(t) = \boldsymbol{0} \tag{15}$$

Since the spectra matrix (**Λ**) is diagonal, the dynamic equilibrium equation given by Eq.(15) is uncoupled in the modal space. Consequently, the n differentials equation can be solved independently.

Material and Methods

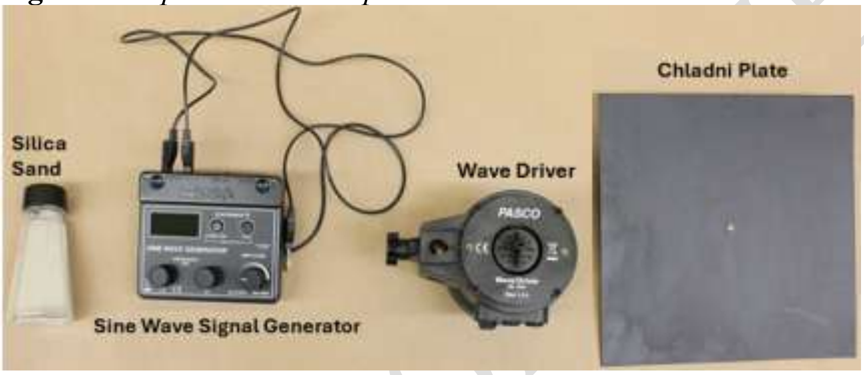
The plate used in this experiment is a Chladni plate consisting of a square stainless-steel plate measuring 24 cm × 24 cm, with a thickness of 1.25 mm and a 3.7 mm diameter hole at its center. The material properties are shown in Table 1.

Table 1. *Material properties*

Stainless Steel	
Density	7850 kg/m^3
Poisson’s ratio	0.3
Young’s Modulus	200 GPa

The experimental setup is shown in Figure 1. It consists of the Chladni plate, a wave driver, a sine wave signal generator, and silica sand. The signal generator is used to control the excitation frequency and amplitude. Its operating frequency range spans from 1 Hz to 800 Hz. The wave driver receives the electrical signal generated by the signal generator and converts it into mechanical vibrations at the specified frequency.

Figure 1. *Experimental Setup*



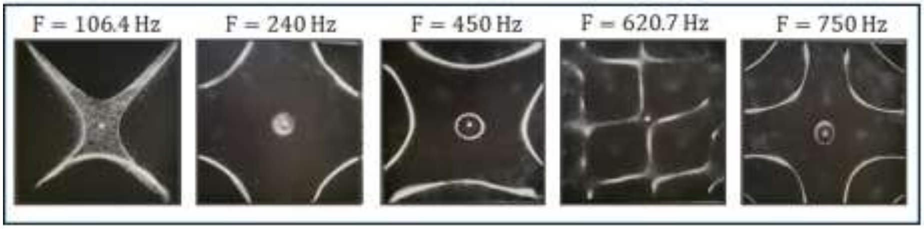
Initially, a thin layer of silica sand is uniformly distributed over the plate surface as illustrated in Figure 2. The plate is attached to the wave driver through the central hole using a screw connection.

Figure 2. *Initial Step of the Experiment*

The excitation frequency is then gradually increased until one of the plate’s natural frequencies is reached. The vibration amplitude is adjusted as needed to improve the visibility of the Chladni patterns. At resonance, the sand particles migrate and accumulate along the nodal lines, where the vibration amplitude is zero, forming distinct patterns that correspond to the vibration mode shapes of the plate.

The resulting Chladni pattern is photographed, and the corresponding resonant frequency is recorded. Five representative Chladni patterns obtained during the experiment, with their corresponding natural frequencies, are shown in Figure 3.

Figure 3. *Modal shapes obtained during the experiment.*

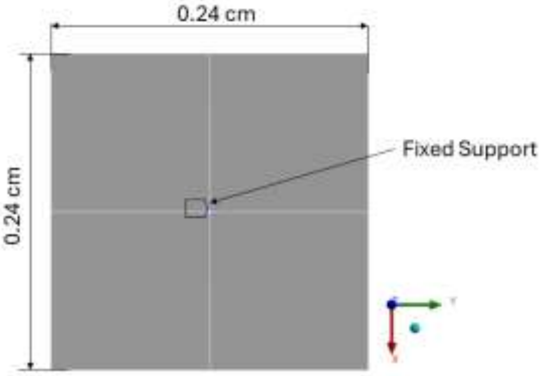


After each measurement, the sand is redistributed over the plate surface, and the excitation frequency is further increased to identify the subsequent natural frequencies and their associated mode shapes. The experimentally observed resonant frequencies and Chladni patterns are then compared with the mode shapes and natural frequencies predicted by finite element simulations performed using ANSYS Workbench.

Finite Element Analysis (FEA)

The Chladni plate was modelled in ANSYS Workbench as a surface body. A fixed-support boundary condition was applied along the edge of the central hole to represent the connection between the plate and the wave driver, as illustrated in Figure 4.

Figure 4. *Plate model with boundary conditions*

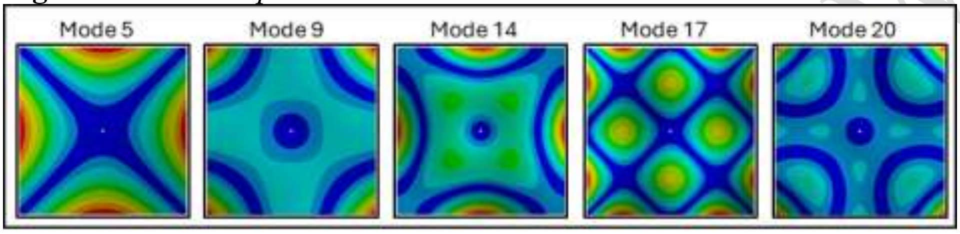


The finite element model employed a mesh consisting of 58,070 nodes and 57,586 shell elements (SHELL181). The selected mesh provided an accurate representation of the plate geometry, including the central hole, and ensured sufficient resolution for accurately predicting the vibration mode shapes and natural frequencies.

A modal analysis was then performed, and the first 20 mode shapes and their corresponding natural frequencies were extracted. Among these, five mode shapes were selected for comparison with the experimentally observed Chladni patterns, as shown in Figure 5.

The selected numerical mode shapes were qualitatively compared with the experimental Chladni patterns based on their geometric characteristics. In addition, a quantitative comparison was performed by evaluating the percentage error between the natural frequencies obtained experimentally and those predicted by the finite element model as plotted in Figure 6.

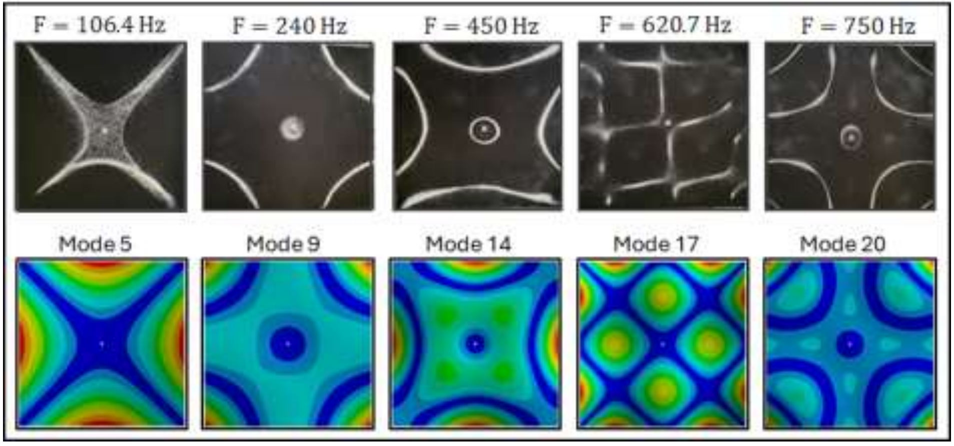
Figure 5. *Mode shapes obtained in simulation*



Results

The selected mode shapes predicted by the finite element model showed good agreement with the experimentally obtained Chladni patterns. A qualitative comparison between the experimental and numerical results indicates that the geometrical characteristics of the nodal lines were accurately captured by the finite element analysis, as can be observed in Figure 6. For all five selected modes, the predicted mode shapes closely matched the corresponding experimental patterns.

Figure 6. *Qualitative comparison between experimental and predicted mode shapes*



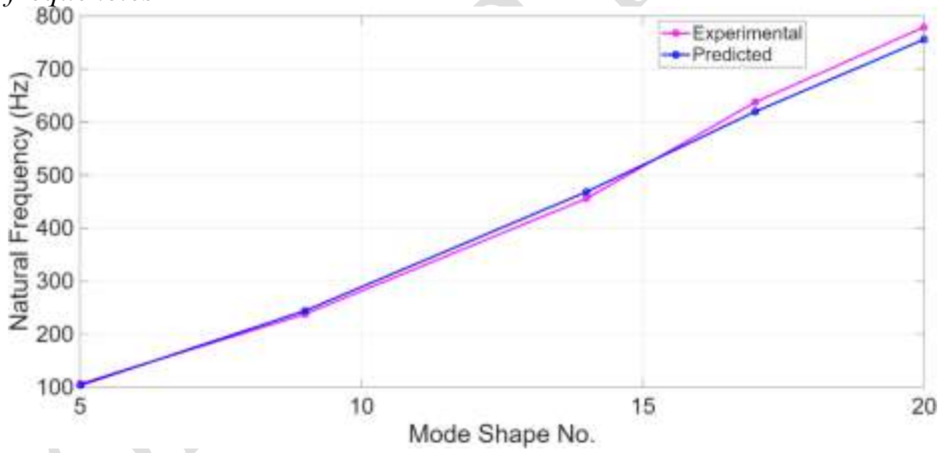
A quantitative comparison was performed by evaluating the percentage error between the experimentally measured natural frequencies and those predicted by the finite element model as reported in Table 2 and illustrated in Figure 7. The average percentage error was approximately 2.8%, indicating excellent agreement between the numerical and experimental results.

The small discrepancy between the experimental and numerical frequencies demonstrates that the modelling approach adopted in this study, including the material properties, boundary conditions, mesh density and element type, provided an accurate representation of the physical system.

Table 2. *Comparison between experimental and predicted natural frequencies and Error %*

Mode	Experimental (Hz)	Predicted (Hz)	Error %
5	106.4	103.5	2.73
9	238	244.21	2.61
14	456	468.48	2.74
17	637.7	619.8	2.81
20	779.3	756	2.99

Figure 7. *Quantitative comparison between experimental and predicted natural frequencies*



Conclusions

This study presented an educational approach for teaching vibration concepts through the integration of experimental Chladni plate demonstrations, modal analysis theory, and finite element simulations. The methodology enabled students to visualize vibration mode shapes and establish connections between theoretical concepts, numerical predictions, and physical observations.

The experimental investigation successfully identified several resonant frequencies and corresponding Chladni patterns of a square stainless-steel plate. These experimentally observed mode shapes were compared with the results obtained from finite element modal analysis. The qualitative comparison demonstrated excellent agreement between the experimental and numerical mode shapes, with the finite element model accurately reproducing the geometrical characteristics of the nodal patterns.

A quantitative comparison of the natural frequencies showed an average error of approximately 2.8% between the experimental measurements and the numerical predictions. This small discrepancy indicates that the finite element model, including the selected material properties, boundary conditions, mesh density, and element type, provided an accurate representation of the physical system.

From an educational perspective, the integration of experimental demonstrations with computational modelling provided students with an accessible way to connect theoretical vibration concepts with physical behavior. The approach supports early exposure to modal analysis and finite element modelling in undergraduate engineering education.

Future work could extend the educational scope through parametric finite element studies, where students investigate the influence of geometry, material properties, thickness, and boundary conditions on natural frequencies and Chladni patterns. Such investigations would deepen understanding of the relationship between structural parameters and dynamic behavior while strengthening the link between vibration theory, computational modelling, and experimental observation. They would also introduce students to sensitivity analysis and design-oriented thinking commonly used in engineering practice.

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