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Dark Energy and Matter as Essential Components of the Expanding Universe

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The present work reviews the facts of some models related to the dark matter and energy with variation of gravitational and cosmological constants with the cosmic time by giving the evidence in support of expanding universe due to Big Bang. Three important time epochs of BBN like Inflationary era, Radiation dominated era and Matter dominated era have also been discussed. The paper discusses also about the age of the universe using Hubble law in terms of Hubble constant and density parameter including the concept of space-time, geography of the universe, new cosmology, dark energy and some challenges and says that the correct theory of gravity is massive scalar-tensor gravity drawn from the detection of the gravitational wave from the collision of binary black holes as predicted by Albert Einstein's general theory of relativity, 100 years ago.

Keywords: *Dark energy, dark matter, gravitational & cosmological Constant*

Introduction

Edwin Hubble demonstrated that the spiral nebulae observed in the telescope are in the fact galaxies like the Milky Way (Hubble 1926). He showed that the spectral lines of most galaxies are shifted towards the red, which suggests that they are moving away to each other in all possible directions and the universe is expanding (Hubble 1929, 1982). On the basis of the Hubble diagram, it is clear that the red shift of the galaxy is directly proportional to the distance indicating that the higher the red shift more distant the galaxy (Kirshner 2004). The recent observations show that there are about10^{22} to 10^{24} galaxies and they have wide range in size. Each galaxy contains approximate 10^{11}-10^{12} stars (Staff 2019). The theoretical calculations based on the general theory of relativity led the big-bang theory which suggested that the universe has expanded from a very hot, very dense state existing at some finite time about 14 billion years ago in the past (A142 2012). Zwicky (1933) showed that the visible matter accounts for only a tiny fraction of all of the mass in the universe during 20[th] century (Zwicky 1993). Kahn and Woltjer (1959) pointed out that M 31 and the Galaxy were moving towards each other so that they must have completed most of an orbit around each other

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during a Hubble time (Kahn and Woltjer 1959). Wang (1991) studied the astrophysical bounds on the change of the gravitational constant with time and found that $\dot{G}/G = 10^{-12} yr^{-1}$ is the condition that has to be satisfied in order not to cause a conflict with the observations (Wang 1991). Singh and Agrawal (1993) studied the Einstein field equations with perfect fluid source and variation of cosmological and gravitational constants with cosmic time for Bianchi-type universe (Singh and Agrawal 1993). Melek (2000) studied the limits on cosmic time-scale variations of gravitational and cosmological constants regarding the spatially perturbed FRW metric (Melek 2000). Boyle et al. (2002) investigated a class of models for dark matter and negative pressure, dynamical dark energy consisting of "spintessence" a complex scalar field spinning in a U(1) symmetrical potential (Boyle et al. 2002). Singh (2006) considered the Einstein field equitation in zero-curvature R-W Cosmology with perfect fluid source and time-dependent gravitational and cosmological constants (Singh 2006). Buchert (2008) described the effective evolution of an inhomogeneous universe models in any theory of gravitation in terms of spatially averaged variables (Buchert 2008). Aguirre (2008) reviewed the initial condition for the standard cosmology model and gave confrontation of this theory with observations of the Cosmic Microwave Background (CMB) (Aguirre 2008). Bilic (2010) discussed the thermodynamic properties of dark energy in terms of a self-interacting complex scalar (Bilic 2010). Silva et al. (2011) discussed thermodynamic properties of dark energy components assuming a general time-dependent equation of state parameters (Silva et al. 2011). Dallal and Azzam (2012) discussed that the discovery of the Higgs boson had provided new momentum for the Standard Model as a cornerstone in understanding the universe (Dallal and Azzam 2012). Bilic (2011) studied a non-interacting supersymmetric model in de Sitter space-time providing matching between the vacuum energy density and the cosmological constant to the de Sitter expansion parameter (Bilic 2011). Shaikh and Wankhade (2015) discussed a dark energy model with EoS parameter for hypersurface-homogeneous space-time filled with perfect fluid source in the frame work of f(R,T) gravity (Shaikh and Wankhade 2015).

In the present work, we have reviewed the fact regarding some models related to the dark matter and energy with the variation of Gravitational and cosmological constants with cosmic time giving the evidences in support of expanding universe due to Big Bang. Three important time epochs of BBN like Inflationary era, Radiation dominated era and Matter dominated era have been discussed. We also have discussed about the age of the universe using Hubble law in terms of Hubble constant and density parameter with the concept of space-time, geography of the universe, New Cosmology, Dark energy and some challenging issues that we had to face in predicting the secret of expanding universe.

Concepts of Space and Time

The Newtonian mechanics defines the space and time as rigid and immutable. This means that the space and time is absolute. The concept of space and time in general theory of relativity differs from the Newtonian space time. According to general theory of relativity, the structure of the space time is affected by the presence of matter and hence it becomes soft and malleable. In the String theory, the coupling constants are determined by the vacuum expectation values of some scalar fields and hence they are no longer constant at all. Regarding this theory, the space and time can be viewed as a frame work for softening the laws of physics. This can be summarized as follows (Chiba 2011).

Table 1. *Concept of Space & Time*

S. No.	Theory as proposed	Space	Time
1.	Newtonian Theory	Rigid	Rigid
2.	Einstein Theory	Soft	Rigid
3.	String Theory	Soft	Soft

We know that the universe is expanding just like balloon expands in all possible directions. Hence the parameters like the gravitational constant, cosmological constant. Hubble constant and energy density are varying with cosmic time from the evolution of the Universe. Any detection/non-detection for the variations of all above parameters could provide useful information about the nature of dilation/moduli fixing and the coupling of the quintessence field. The quintessence is meant by the cosmic acceleration induced by a slowly rolling light scalar field and this can couple the electromagnetic field and gravitational field. Due to this reason, we discuss about the parameters discussed above.

Geography of the Universe

The entire universe consists of 10^{11} galaxies. Each galaxy has 10^{11} stars, few black holes, neutron stars, white dwarfs and other asteroids. The entire universe is just like balloon and expanding so that stars of the galaxy are receding to each other. In comparison to the stars in any galaxies, the black holes, neutron stars, white dwarfs, etc., are few in numbers. The star is the basic building blocks of the universe. A galaxy has luminous centre containing nearly all stars and a dark halo of unknown compositions. In the most of cases, there is a nucleus consisting of black holes which is gobbling up stars and gas and emitting radiation. When the nucleus is the dominant feature, then it is known as an AGN. Seyfert galaxies and quasars belong to this category. Generally, large galaxies have a size of .1 Mpc and are of order 1Mpc apart. Many galaxies belong to gravitationally bound clusters containing from 2 to1000 galaxies. Small clusters are usually called groups. Big clusters are the biggest gravitationally bound objects in the universe. The biggest distance can be observed in the order of 10^4 Mpc. The observable universe is extremely homogenous and isotropic and closed analogous to the surface of an extremely deformed sphere. The universe is expanding isotropically. In the astronomy, the astronomers use the unit Mega parsec to measure the astronomical distance as follows (David 1993):

$1pc = 3.26\ \ light\ \ years = 3.09 \times 10^{16} meters.\ \ 1Mpc = 10^{6}\ pc$,
The mass of stars are ranging roughly from $1M_{\oplus}$ to $10M_{\oplus}$. This range may be $10^{6} M_{\oplus}$ to $10^{12} M_{\oplus}$ for dwarf and large galaxies respectively.

Big Bang Nucleosynthesis

Big Bang nucleosynthesis (BBN) is known as primordial nucleosynthesis. This is one of the most important pillars of the modern cosmology and testing ground upon which many cosmological models must ultimately rest (Patrignani et al. 2016). Primordial nucleosynthesis process has taken place after the Big Bang in the interval from roughly 10 seconds to 20 minutes and the most part of the helium of Universe are formed as the isotope helium ($^{4}He_{2}$) along with small amounts of the hydrogen isotope deuterium ($^{2}H_{1}$ or D), the helium isotope helium ($^{3}He_{2}$), and a very small amount of the lithium isotope lithium-($^{7}Li_{3}$). In this process, two unstable or radioactive isotopes named the heavy hydrogen isotope tritium ($^{3}H_{1}$ or T); and beryllium isotope beryllium-7 (^{7}Be) were also produced. These two unstable isotopes later decayed into $^{3}He_{2}$ and $^{7}Li_{1}$ (Coc and Vangioni 2017).

There are three important time epochs of BBN as follows (Mathews et al. 2017):

1. Inflationary era: In this stage, the energy density is dominated by the potential of a scalar field. In this era. In this stage, the time is about 10^{-2} second and temperature is approximately equal to 10MeV and the energy density of the universe is dominated by relativistic particles like neutrinos, antineutrinos, positive electrons and negative electrons in thermal equilibrium. In this era, the weak interaction rates are faster than the universal expansion rate.
2. Radiation dominated era: In this stage, the energy density is dominated by relativistic particles called radiation. In this stage, the time is about 1 second and temperature is approximately equal to 1MeV and the radiation density of the universe is dominated by relativistic particles like proton, neutron, neutrinos, antineutrinos, positive electrons and negative electrons in thermal equilibrium. In this era, the weak interaction rates can no longer maintain the thermal equilibrium. The reactions are as follows:

$$n + \nu_{e} \Leftrightarrow p + e^{-}$$

$$n + e^{+} \Leftrightarrow p + \nu_{e}^{-}$$

$$n \Leftrightarrow p + \nu_{e}^{-} + e^{-}$$

3. Matter dominated era: In this stage, the energy density is dominated by the mass of non-relativistic particles called matter. At the temperature from T=0.5 to 0.1MeV, positive and negative electron pairs begin to annihilate the photon gas. The neutrino gas is however, unaffected by this time. At t=100s, the n/p ratio has diminished from 1/6 to 1/7 by neutron decay, then the helium mass fraction becomes approximately equal to 0.25 which is very close to observed abundance.

The best current understanding of the composition of the universe comes from the analysis of the Planck Surveyor. When combined analysis of the Planck temperature plus polarisation and gravitational lensing data with Baryan Acoustic Oscillation in the matter power spectrum and supernovae data from the Joint Light-Curve analysis of type Ia supernovae is taken into consideration, then, we obtain

$$\Omega_b h^2 = 0.02230 \pm 0.00014$$

where $h = 0.6774 \pm 0.0066$. The other key quantity to study for the formation of the universe starting from BBN is a term called η representing the ratio of the baryon number density to the photon number density, which relates the quantity $\Omega_b h^2$. The calculated value of η concludes that there are about 400 photons per cm^3 in the universe and roughly 2 billions photons per baryon. This number was fixed in the epoch of baryon-genesis. From the Planck analysis, the baryonic is $\Omega_b = 0.0486 \pm 0.011$, total matter content is $\Omega_m = 0.3089 \pm 0.062$ and completely unknown component of cold dark matter is $\Omega_c = 0.260 \pm 0.006$ are calculated. In addition to above analysis, the universe is predominantly made completely exotic form of mass energy called dark energy denoted by $\Omega_\Lambda = 0.691 \pm 0.006$. There are some other contribution of presence of matter due to relativistic photons and neutrinos given by $\Omega_\gamma = 5.46 \times 10^{-5}$ (Planck Collaboration 2016). Thus, we see that the composition of present universe is a great cosmic mystery and about 95% form of matter of the universe called Dark matter & Energy are present about which we have little Knowledge. Now a day, this subject is a challenge for the Astrophysicists and cosmologists to solve this puzzling matter.

The Age of the Universe

The universe is expanding isotropically so that the distance between any pair of the galaxies separated by more than 100Mpc is proportional to a universal scale factor a(t). The Hubble parameter is defined as

$$H = \dot{\alpha} / \alpha, \qquad (1)$$

where dot is the time derivatives. The present value of H is H_0 known as Hubble constant. The galaxies are receding from us with velocity $v << 1$ can be measured by the red shift $z \equiv \Delta\lambda / \lambda$ and velocity of such galaxies is given by

$$v = Hr. \qquad (2)$$

The Hubble constant H_0 is in the range from 40 to 100km/sMpc^{-1} and defining a quantity h so that as H_0==100hkm/sMpc^{-1}. The red shift determination gives 0.4< h <1. Since, $H = \dot{\alpha} / \alpha$, the time taken for the universe to expand by an appreciable amount is of order the Hubble time given by following

$$H_0^{-1} = 9.7 h^{-1} \times 10^9 Mpc \qquad (3)$$

During the Hubble time, the light travels a distance of order the distance given by the following equation

$$H_0^{-1} = 2998 h^{-1} Mpc \qquad (4)$$

Hubble law holds good only for those galaxies whose distance is much less than the Hubble distance. This principle is based on the non-relativistic Doppler shift and it requires $v << 1$. At the present the epoch, the Hubble time is of order 10^{10} years. When we go back to an era, when the universe was very hot and dense, the estimation and theoretical calculation shows that the Hubble time was only a tiny fraction of second and popularly known as the Hot Big Bang. It is presumably assumed that when the Hubble time is of order the Planck time given by the following equation

$$t_{Pl} = G^{1/2} = 5.39 \times 10^{-44} \sec. \qquad (5)$$

then it is known as the Planck epoch.

From the study of the nucleo-synthesis, baryonic matter contributes the density parameter $\Omega_o = 0.01 - 0.09$. The theoretically favoured value of density parameter is $\Omega_o = 1$. This fact indicates the presence of non-baryonic dark matter in addition to baryonic dark matter. The luminous matter accounts for only $\Omega_o = 0.01$. Even if Ω is not equal to1 at the present epoch, it quickly approaches 1 as we go back in time. If the density parameter $\Omega_o << 1$, then we are able to estimate the epoch before which Ω is practically equal to 1 as follows:

As long as $\Omega_o << 1$, the gravity is negligible and therefore $\dot{\alpha}$ is constant, leading that

$$H \propto \alpha \qquad (6)$$

and

$$\Omega \propto \rho / H^2 \propto \alpha^{-1} \quad (7)$$

If Ω is approximately equal to 1 before the epoch, then

$$\alpha \equiv \Omega_0 \quad (8)$$

Since $\Omega_o \geq 1$, we see that Ω is certainly close to 1 before $\alpha \equiv 0.1$.

The Friedmann equation allows to calculate the present age of the universe in terms of Ω_o and for $\alpha = \Omega_o$, we have

$$\alpha \propto t^{2/3} \text{ at H=2/3t} \quad (9)$$

Therefore

$$t_0 = \frac{2}{3} H_0^{-1} = 6.5 \times 10^9 h^{-1} year \quad (10)$$

If $\Omega_o = .1$, then we have

$$t_0 = .9 H_0^{-1} = 8.8 \times 10^9 h^{-1} year \quad (11)$$

where

h<0.65 for $\Omega_o = 1$ and h<0.88 for $\Omega_o = .1$ (12)

Using the Hubble law, different astronomers estimated the value of h as .4<h<1, but the true value of h is as:

0.40<h<0.65 (13)

Theoretical Discussion

The evolution of the universe is described by Einstein field equations together with perfect fluid and equation of state in the relativistic and observational cosmology. Einstein's theory of gravitation involves two fundamental constants called Newtonian gravitational constant and cosmological constant (Singh 2006). These two fundamental constants have vital role to explain about the evolution of the universe. The variation of these two constants with cosmic time gives us the idea about the expanding universe initially from Big-Bang. The Gravitational constant (G) and Cosmological constant (Λ) may be defined as: When two bodies of unit mass are placed at unit distance, the force of attraction acting between them is called the gravitational constant (Mahto et al. 2013), while the homogeneous energy density of the vacuum that causes the acceleration of expansion of the universe is called the Cosmological constant (Davis and Griffen 2010). This cosmological constant (Λ) was assumed to zero by the most cosmology researchers from 1929 to 1990 $(\Lambda = 0)$, but the supernovae explosion in 1998 has shown that about 68% of the mass-energy density of the universe can be attributed to dark energy the so called cosmological constant. It may not be less than zero or equal to zero $(\Lambda \leq 0)$, but in practice always greater than zero $(\Lambda > 0)$. This cosmological constant is understood as the fundamental force of nature in the particle physics (Felix 2012), while the modern field theory defines the cosmological constant as energy density of the vacuum. The universe contains a bizarre form of matter/energy, which is gravitationally repulsive and hence, it is an anti-gravity effect. Due to this effect, the galaxy is moving from us and the universe is expanding as shown by Hubble (Turner 2001).

There are so many experimental evidences and data available from different research papers and literatures that the gravitational constant (G) vary with time and we obtain that $\dot{G}/G = 10^{-12}$ per year is the condition that has to be satisfied in order not to cause a conflict with the observation. As per the model for variation limit of $\dot{G}/G = 10^{-12}$ per year, the radius changes by about 1% and the luminosity by 3% at the solar stage (Wang 1991). The Newtonian universal constant (G) has vital role of coupling constant between geometry of space and matter content in Einstein's field equation (Singh 2006). Dirac (1937) first proposed the idea regarding the variation of gravitational constant with time which gives a good understanding of cosmology and particle physics (Dirac 1937). Dicke (1961) gave a gravitational theory with a cosmic time increasing scalar field $\phi(t)$ which is the inverse of decreasing gravitational constant (G) i.e. $\phi(t) \propto 1/G(t)$ (Dicke 1961). Shapiro et al. (1971) discussed about the problem of gravitational constant and its variation (σ) with cosmic time and possible experimental bound (Shapiro et al. 1971). Canuto et al. (1977) gave many suggestions based on different arguments that G is indeed time dependent (Canuto et al. 1977). Dyson (1978) proposed that G does change with a rate of variation that should be of order of the expansion rate of the universe $G/\dot{G} = \sigma H_0$, where H_0 is the Hubble parameter ($H_0 = 2 \times 10^{-10}$) and σ is a dimensionless parameter (Dyson 1978). Beesham (1986) studied the creation with variable G and pointed out that the variation of G with cosmic time as: $G \propto 1/t$ originally proposed by Dirac in 1937 (Beesham 1986). Halpern (1987) collected several works on the measurement of cosmological variations of the gravitational constant (Halpern 1987). Berman (1992) generalised the large number of hypothesis regarding the cosmological constant and concluded that the cosmological constant is the time dependent scalar field and inversely proportional to the square of cosmic time ($\Lambda = c/t^2$), where c is the proportionality constant (Berman 1992).

The Robertson-Walker Line Element for a Spatially Homogeneous and Isotropic Medium

The Robertson-Walker line element for a spatially homogeneous and isotropic medium is given by the following equation (Singh 2006).

$$ds^2 = dr^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\right)d\phi^2\right] \quad (14)$$

where R(t) is the scale factor.

When k=-1 represents the curvature parameter for open universe and the above equation takes as

$$ds^2 = dr^2 - R^2(t)\left[\frac{dr^2}{1+r^2} + r^2\left(d\theta^2 + \sin^2\theta\right)d\phi^2\right] \quad (15)$$

When k=1 represents the curvature parameter for closed universe and the above equation takes as

$$ds^2 = dr^2 - R^2(t)\left[\frac{dr^2}{1-r^2} + r^2\left(d\theta^2 + \sin^2\theta\right)d\phi^2\right] \quad (16)$$

When k=0 represents the curvature parameter for flat universe and the above equation takes as

$$ds^2 = dr^2 - R^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\right)d\phi^2\right] \quad (17)$$

The Einstein field equation with variable cosmological and gravitational constants are given by the following equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu} \quad (18)$$

The term $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ be the metric tensor and R be the tensor of the scalar curvature in the left hand side of above equation. These all terms describe the properties of space time. The term $T_{\mu\nu}$ in the right hand side of the above equation represents energy momentum tensor of a perfect fluid given by the following equation

$$T_{\mu\nu} = -pg_{\mu\nu} + (p+\rho)u_\mu u_\nu \quad (19)$$

where ρ is the energy density of the cosmic matter, p is the pressure and u is the four-velocity vector such that $u_\mu u_\nu = 1$

The co-variant divergence of the equation (5), taking account the Bianchi identity gives

$$8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu} = 0 \tag{20}$$

This equation also may be considered as the fundamental equation of gravity with gravitational constant (G) and cosmological constant (Λ) in addition to the equation (5).

The Einstein field equation containing the speed of light (c) can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G(t)}{c^4}T_{\mu\nu} + \Lambda(t)g_{\mu\nu} \tag{21}$$

The general relativity does not account G and gives no theoretical explanation like the fundamental quantity speed of light c, although G is treated as fundamental constant in nature.

According to the energy momentum conservation equation (Singh and Agrawal 1993), we have

$$T_{\mu\nu} = 0 \tag{22}$$

Putting the above value in equation (8), we have

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda(t)g_{\mu\nu} \tag{23}$$

or

$$R_{\mu\nu} = \frac{1}{2}g_{\mu\nu}R + \Lambda(t)g_{\mu\nu} \tag{24}$$

or

$$R_{\nu\nu} = \left(\Lambda + \frac{R}{2}\right)g_{\mu\nu} \tag{25}$$

When the Hubble parameter ($H = \dot{R}/R$) is used to solve the Einstein Field equations(1) using co-moving coordinates given by the following equation

$$u_k = (1,0,0,0) \tag{26}$$

Then we obtain the following results as follows:

$$H^2 + \dot{H} = -\frac{4\pi}{3}G(t)(3p + p) + \frac{1}{3}\Lambda(t) \quad (27)$$

$$H^2 = \frac{8\pi}{3}G(t)\rho + \frac{1}{3}\Lambda(t) - \frac{k}{R^2} \quad (28)$$

Let us assume that

$$G(t) = \alpha H \quad (29)$$

and

$$\Lambda(t) = \beta H^2 \quad (30)$$

where α and β are dimensionless positive constants.

Then the solution of Einstein field gives the following physical parameters

$$H = \frac{3}{\left[(1-2a)\beta + 3\right]}\left(\frac{1}{t}\right) \quad (31)$$

$$G = \frac{3\alpha}{\left[(1-2a)\beta + 3\right]}\left(\frac{1}{t}\right) \quad (32)$$

$$\Lambda = \frac{9\beta}{\left[(1-2a)\beta + 3\right]^2}\left(\frac{1}{t^2}\right) \quad (33)$$

$$\rho = \frac{3(3-\beta)}{8\pi\alpha\left[(1-2a)\beta + 3\right]}\left(\frac{1}{t}\right) \quad (34)$$

and

$$R = R_*\left[(1-\beta)\left(\frac{A}{1-A}\right)^{2\beta/3} H_* t\right]^{1/(1-\beta)} \quad (35)$$

where a is the free parameter related to the power of cosmic time and lies $0 \le a < 1$. R_* is the reference value such that if $R \le R_*$

From the observation of above equations, we see that for the positive values of Hubble parameter, gravitational constant, cosmological constant and energy density, both the dimensionless parameters α and β are should be as follows:

$(\alpha \le 1/2)$ & $\beta < 3$ (36)

When $\beta = 0$, then the equation (35) can be written as follows:

$$R = R_0 H_0 t \quad (37)$$

or

$$R \propto t \quad (38)$$

The above equation shows the radius of curvature increases linearly with the age of time at $\beta = 0$.

The variation of gravitational constant and energy density with the cosmic time using equations (32) & (34) clearly shows that the gravitational constant as well as energy density will be collapsed, when cosmic time tends to infinity.

But when each Bianchi type model (I to IX) is applied to the Einstein field equation, then we obtain the following solutions for radius of curvature, Hubble parameter, gravitational constant, cosmological constant and energy density respectively:

$$R \propto t^3 \quad (39)$$

$$H \propto \frac{1}{t} \quad (40)$$

$$\Lambda \propto \frac{1}{t^2} \quad (41)$$

$$\rho \propto \frac{1}{t} \quad (42)$$

$$G \propto \frac{1}{t} \quad (43)$$

Chen and Wu (1990) suggested that the cosmological constant (Λ) vary as the square of the scale factor R in the Robertson-Walker model as per the following relation (Chen and Wu 1990).

$$\Lambda \propto \frac{1}{R^2} \quad (44)$$

The above relation shows that the cosmological constant decreases with increase of the radius of curvature.

The equation (39) shows that the curvature of the space increases with the cosmic time, while the equations (40), (41), (42) & (43) show that the Hubble parameter, gravitational constant and energy density decrease with the increase of cosmic time and for infinite time, the gravitational constant will be collapse and hence no gravity will act at that time. But when we go in the reverse direction, then these models $G \propto 1/t$, $\Lambda \propto 1/t^2$ and $\Lambda \propto 1/R^2$ become singular at t=0. (Singh and Agrawal 1993, Davis and Griffen 2010, Felix 2012). This fact gives the evidence in support to the Big-Bang theory for the evolution of universe, because, the universe was a singular point at t=0.

Hypersurface-Homogenous Dark Energy Model with Variable E$_o$S Parameter in f(R,T) Gravity

The line element for the hypersurface homogenous space time is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)[dy^2 + \Sigma^2(y,K)dz^2] \qquad (45)$$

where K=1, 0, -1 as discussed in equations (15), (16) and (17) in the subsection 6.1.

When the above equation is solved with proper mathematical operation for suitable choice of co-ordinates and constants, we have

$$ds^2 = dt^2 - (\alpha_1 t + \alpha_2)^{6n/m(n+2)} dx^2 - (\alpha_1 t + \alpha_2)^{6/m(n+2)}[dy^2 + \Sigma^2(y,K)dz^2] \qquad (46)$$

This model has no initial singularity and represents hypersurface-homogenous Dark energy model in f(R,T) gravity.

This model gives the parameters like the spatial volume (v), the generalised Hubble parameter (*H*), scalar expansion (θ), mean isotropic parameter (Λ), shear scalar (σ), energy density (ρ) and equation of state & Skewness parameter (ω) respectively as follows:

$$V = (\alpha_1 t + \alpha_2)^{3/n} \qquad (47)$$

$$H = \alpha_1 / m(\alpha_1 t + \alpha_2) \qquad (48)$$

$$\theta = 3\alpha_1 / m(\alpha_1 t + \alpha_2) \qquad (49)$$

$$\Lambda = 3(n^2 + 2)/(n+2)^2 \qquad (50)$$

$$\rho = \frac{1}{(8\pi + 2\mu)}\left[\frac{3\alpha_1^2(n+1)(3 - mn - 2m) + 9\alpha_1^2(n^2 - 2n - 1)}{m^2(n+2)^2(\alpha_1 t + \alpha_2)^2} - \frac{K}{\{(\alpha_1 t + \alpha_2)\}^{6/m(n+2)}}\right] = -p \qquad (51)$$

$$w = \frac{1}{\rho(8\pi + 2\mu)}\left[\frac{3\alpha_1^2(3 - mn - 2m) - 3n\alpha_1^2(3n - mn - 2m) + 9\alpha_1^2}{m^2(n+2)^2(\alpha_1 t + \alpha_2)^2} - \frac{K}{\{(\alpha_1 t + \alpha_2)\}^{6/m(n+2)}}\right] = -\delta \qquad (52)$$

The equation (47) shows that the spatial volume of the universe increases with increasing cosmic time and gives the accelerated expansion of the universe. The equation (48) shows that the Hubble parameter decreases with increasing cosmic time and gives the null value of H for infinite cosmic time. The equation (49) shows that the expansion scalar decreases with increase of the cosmic time. The equation (50) shows that the mean anisotropic parameter is uniform throughout the evolution of the universe, because it is independent of cosmic time. The equation (51) shows that in the beginning of the universe the shear scalar was infinitely large and with increasing the cosmic time, it initially decreases rapidly to reach null value as the cosmic time tends to infinity. The equation (52) represents the equation of state and Skewness parameter depends on the cosmic time. The parameter E_OS remains the same from beginning to till time, but it acquires different positions for K= -1,0 , 1. These all variations of the parameters like *V*, *H*, *θ*, *ρ* and *ω* with cosmic time are shown graphically in the reference (Shaikh and Wankhade 2015).

Line Element from a GW from Standard General Relativity

The line element for a gravitational arising from standard general relativity and propagating in the z direction using natural units equal to unity is given by following equation

$$ds^2 = dt^2 - dz^2 - (1+h_+)dx^2 - (1-h_+)dy^2 - 2h_\times dxdy \qquad (53)$$

where h_+ and h_x are the weak perturbations due to the (+) and (x) polarisation.

The total frequency and angle dependent response function to the (+) and (x) polarisation of an interferometer with arms in the u and v directions w.r.t. the propagating gravitational waves are given as follows:

$$H_+(w) = \frac{\cos^2\theta\cos^2\phi - \sin^2\phi}{2L} H_u(w,\theta,\phi) - \frac{\cos^2\theta\sin^2\phi - \cos^2\phi}{2L} H_v(w,\theta,\phi) \qquad (54)$$

and

$$H^\times(w) = \frac{-\cos\theta\cos\phi\sin\phi - \sin^2\phi}{L}[H_u(w,\theta,\phi) - H_v(w,\theta,\phi)] \qquad (55)$$

In the case of low frequency limit ($w \to 0$), the low frequency response functions tend to the low values for both the equations (54) & (55).

The total frequency and angle dependent response function of an interferometer to this scalar polarisation is given by the following equation.

$$H^\phi(w) = \frac{\sin\theta}{2i\omega L}\{\cos\phi[1+\exp(2i\omega L) - 2\exp i\omega L(1+\sin\theta\cos\phi)] - \sin\phi[1+\exp(2i\omega L) - 2\exp i\omega L(1+\sin\theta\sin\phi)]\} \qquad (56)$$

If the interferometer arm is parallel to the propagating gravitational wave, the longitudinal response function with the bouncing photon analysis associated with such a massive mode is given by the following equation.

$$\begin{aligned}\mathrm{T}_l(\omega) = \frac{1}{m^4\omega^2 L}[1/2\{1+\exp(2i\omega L)m^2\omega^2 L(m^2-2\omega^2) - i\exp(2i\omega L)\omega^2\sqrt{-m^2+\omega^2}(4\omega^2+m^2(-1-i\omega L))\} \\ +\omega^2\sqrt{-m^2+\omega^2}(-4i\omega^2+m^2(i+\omega L)+\exp iL(\omega+\sqrt{-m^2+\omega^2})(m^6 L+m^4\omega^3 L \\ +8i\omega^4\sqrt{-m^2+\omega^2}+m^2(-2\omega^4 L)-2i\omega^2\sqrt{-m^2+\omega^2})\} \\ +2\exp(i\omega L)\omega^3(-3m^2+4\omega^2)\sin(\omega L)]\end{aligned} \quad (57)$$

The theory of gravitation as per general relativity admits only two response functions (54) and (55) corresponding to the two canonical polarisations h_+ and h_x respectively and concludes that the general relativity is definitive. If the third polarisation is present, the third response function (57) will be detected by the gravitational wave interferometers.

Astronomers have made a new detection of gravitational waves and for the first time have been able to trace the shape of ripples sent through spacetime when black holes collide. Three scientists named: Rainer Weiss, Kip Thorne and Barry Barish jointly awarded by Nobel prize (2017) for the detection of gravitational waves coming from the collision of two massive black holes situated a billion light years away from us as predicted by Albert Einstein 100 years ago (Delvin 2017). The detection of the gravitational wave rules out the general relativity as definitive theory of gravity. Hence the presence of a longitudinal response function (equation 57) for a wave propagating parallel to one interferometer arm gives that the correct theory of gravity is massive scalar-tensor gravity (Corda 2009).

Dark Matter and Dark Energy

In physical cosmology, dark energy is a hypothetical form of energy that permeates all of space and tends to increase the rate of expansion of the universe. The modern concept regarding the history of observable universe is broadly divided into three stages (David 1993). Inflationary era, Radiation dominated era and Matter dominated era.

Naturally, the dark energy is diffuse and a low-energy phenomenon. It is probably, neither found in galaxies nor in the clusters of galaxies and can't be produced at accelerators. The Universe itself is perhaps only the natural laboratory in which we can study it (Turner 2001). The observation shows that the universe is expanding and contains about 75% dark energy, 20% dark matter and 5% normal (atomic) matter, with smaller contributions from photons and neutrinos (Caldwell and Kamionkowski 2009). There is some compelling evidence that about 90% of the mass of the Universe is invisible. This fact was explained by Oort during the analysis of Doppler shift in the spectra of stars in the galactic disk. He also concluded that the amount of gravitating matter implied by the measured velocities could not be explained by the mass of visible stars. This gives the evidence the presence of invisible matter. He also studied that a large spherical collection of stars called globular clusters move within the galaxy too fast to be bound to the galaxy by the gravitational influence of the luminous matter alone and half of the Galaxy disk was made of dark matter before astronomers decided to investigate the problem further (Freeman and McNamara 2006). Zwicky (1933) concluded that the velocity dispersion in rich clusters of galaxies require a huge amount of mass to keep them bound could be accounted by the luminous galaxies themselves, when he was observing the Coma Cluster of galaxies (Zwicky 1933). The matter and energy used to keep bound the luminous galaxies are known as dark matter and energy. The most new burning and revolutionary topics for research in the cosmology and astronomy is dark matter and dark energy. Ram and Chandel (2014) have shown that the universe exhibits transition from deceleration to acceleration (Ram and Chandel 2014). Reddy and Kumar obtained Bianchi type II cosmological model and an exact cosmological model with an appropriate choice of the function f(RT) (Reddy and Shanthi 2013). Rao and Davuluri (2014) gave the solution of the field equations to use the anisotropy feature of the universe in the Bianchi type-VI 0 space time (Rao and Davuluri 2014). Singh & Singh obtained a model of the universe containing some part of quintessence form of dark energy and some part of cosmological constant form of dark energy and found that this new model is viable form of model of the universe consisting of dark energy (Singh and Singh 2014).

New Cosmology and Dark Energy

The New Standard Cosmology is characterised by the following features (Mathews et al. 2017). It is flat and accelerating Universe with early period of expansion (inflation).

1. It has inhomogeneous density produced from quantum fluctuations during inflation.
2. It consists of 2/3rds dark energy; 1/3rd dark matter; 1/120th bright stars.
3. Matter content: $(29 \pm 4)\%$ cold dark matter; $(4 \pm 1)\%$ baryons and 0.3% neutrinos.

The above model for New Cosmology is certainly not as well established as the standard hot big bang. However, the evidence is mounting.

Dark energy is a cosmological term for the causative agents of the current epoch of accelerated expansion. The dark energy has following defining properties:

i. It emits no light.
ii. It has large negative pressure.
iii. It is approximately homogeneous
iv. It is more “energy-like” than “matter-like”. Dark energy is qualitatively very different from dark matter.

v. Dark energy by its nature is diffuse and a low-energy phenomenon.
vi. It probably cannot be produced at accelerators.
vii. It is a mysterious substance with negative pressure and accounts for nearly 70% of the total matter-energy of the universe, but has no clear explanations.
viii. It is not very dense.
ix. There is a possibility that dark energy may become dark matter when buffeted by baryonic particles.

Some Challenges to the Dark Energy

The New Standard Cosmology leaves some burning questions as follows:

1. What is physics underlying inflation?
2. What is the dark-matter particle?
3. How was the baryon asymmetry produced?
4. What is the nature of dark energy?

Regarding this context, the physicists and researchers associated with astrophysics and astronomy should take it as a challenge to solve all these questions so that an exact solution about the dark energy problems is solved.

Results and Discussion

In the present work, we have studied different existing models of dark matter and energy for the universe evolution as proposed by many cosmology and astronomy researchers. Out of these different models, each Bianchi type models (I to IX) is applied to the Einstein field equation with the principle of conservation of energy, gives that the universal constant or gravitational constant is inversely proportional to the cosmic time for which the model finally becomes singular at t=0.

We know that at the time of Big Bang, the universe was also singular at zero time. From that time, the universe is expanding like balloon in all possible outward directions from the beginning point of the Big Bang/explosion of the primordial black hole. The universe contains 10^{11} galaxies and each galaxy has 10^{11} stars. Due to the expansion of universe, all the galaxies together with their stars are moving or receding from us. This will give rise the Doppler shift/red shift due to the spectra of stars in the galactic disk which can determine the distance of the galaxy/star from us. As per the relation $\Lambda \propto t^{-2}$ and $G \propto t^{-1}$, on increasing the value of the cosmic time(t), the cosmological constant as well as gravitational constant will decrease and finally, when this cosmic time will approach to the infinity, both the constants will vanish.

At this stage, the spatial volume of the universe becomes enough large acquiring its extreme values as is clear from equation (47). The expansion scalar function, shear scalar function and energy density all tend to zero and no change takes place in the mean isotropic parameter. The energy density gives the negative pressure as predicted by equation (52) justifying the property of dark energy.

This will lead the stoppage of the expansion of the universe and shear will die out. Hence, an unbalanced position will be reached and anti-gravity will fail to act and hence the contraction of the universe may start and after billions and billions years, the shape of the universe will become an egg like. After acquiring the certain limit of the shape of the universe like an egg, another Big Bang will happen and hence in his way, the new universe will form.

The theory of gravitation as per general relativity indicated by equations (54) and (55) for two canonical polarisations h_+ and h_x respectively concludes that the general relativity is definitive. But when the third polarisation is present indicating the third response function (57) will rule out the general relativity as definitive theory of gravity and gives that the correct theory of gravity is massive scalar-tensor gravity.

From whole study of this work, we conclude that the variation of Gravitational constant, cosmological constant, spatial volume, expansion shear, expansion scalar, energy density with the cosmic time have vital responsibility to hold in balancing the position of the universe together with their galaxies and stars by invisible matter and energy so called dark matter and dark energy.

Conclusions

The fact of dark matter and dark energy explains the evolution of the universe and gives some evidences in the support of expanding universe and is also concluded that the variation of the both gravitational and cosmological constants with the cosmic time have vital responsibility to hold in balancing position of the universe together with their galaxies and stars. The detection of the gravitational waves from collision of binary black holes shows that the massive scalar-tensor gravity is definitive theory of gravity drawn from the detection of gravitational wave coming from the collision of binary black holes.

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