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A Modified System of Nonlinear Fractional-Order Differential Equations in the Study of the Dynamics of Marital Relationships and their Behavioural Features

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Urbanization and modernization have effects on marital relationships in Nigeria which led to high divorce rate among legitimate couples prompting unstable environment. This situation design and uses scientific means to study the dynamics of marital relationships and their behavioural features to check excesses in marriage and to promote stability. A modified system of nonlinear fractional-order differential equations was used to categorize people of different personalities and different Impact Factors of Memory, using different sets of parameters. The equations predict and interpret the features of the union of different individuals with external circumstance(s). Equations were adapted to a local environment where data collections were carried out to investigate factors affecting marriages. Data collected by the use of questionnaire validate the model. An Iterative Decomposition Method was adopted to solve the fractional system in which fractional derivatives were given in the Caputo sense; the obtained results were interpreted appropriately. The modified model shows the trajectory of the couple from the state of indifference and as the impact factor memory increases it affects their togetherness making the love between them to decay easily. Numerical simulation results were presented to show the effectiveness of the model and the accuracy of the statements established.

Keywords: *Differential model, dynamical system, impact factors of memory, iterative decomposition method, marital relationship*

Introduction

Marital relationship can be referred to as a relationship governed by feedback, or circular causality, in which each person continually responds to the other in predictable ways that sustain patterns of interaction in the system. However, the presence of chaos in marriage is undesirable. Wauer et al. (2007) reported that romantic relationships in human can be studied via system dynamics methodology. It was reported by Weigel and Murray (2000) that to apply chaos theory to marital relationship, there should be thorough and rigorous work so that its value to marital relationships studies will come from its ability to solve problems and explain behaviours. The mathematics to the study of marital relationship was presaged by Von Bertalanffy (1968) and since then the study began to hold both the artistic imaginations and interest of various scholars in the fields of

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Mathematics, Physics, Sociology, Biology, Neuroscience, Psychology and Anthropology (Barley and Cherif 2011). Strogatz (1988) first analyzed love affairs in his 1-page paper and later contributed to a book. Gottman et al. (2005) had applied Mathematical power to the dynamics of marriage. Experiment in studying the application of Mathematics to marital relationships has been difficult to design because marriage is difficult to quantify. However, earthly observations are the only sources of description of such relationship expressed within individuals, groups or set of people. It could be inferred that mathematical models may play a major role in studying dynamics of marital relationships by using a set of nonlinear and linear differential equations. Control of chaos in marriage promotes a peaceful co-existence and it could show the behavioural time series and phase plots (Paul et al. 2012). Modelling dynamical system or phenomena in life sciences, economics, engineering, particularly in marriage dynamics using classical integer order differential equations has attracted researchers several years ago, but there is a limit to integer order models because love is a state that is always influenced by historical information (Liu and Chen 2015). There are many excellent introductions to this general approach to qualitative nonlinear dynamic modelling and its subtopics of chaos and catastrophe theory (Morrison 1991, Beltrami 1993, Lorenz 1993, Vallacher and Nowack 1994, Baker and Gollub 1996).

Nonlinear Dynamical Models in the Study of Complex Phenomena

Vallacher et al. (2010) in a recent issue along with an American Psychologist supported the importance of dynamical systems for the modelling and study of complex phenomena using simple linear formulae. They also discussed the feature of social life that degenerate into pattern of behaviour which brings out the worst in human nature and the attractor in their work but no model and phase portraits were used to back up the work. Strogatz's aim was to teach harmonic oscillation phenomena in the classroom (Sprott 2004, Barley and Cherif 2011), but ended up by suggesting an unusual approach to the teaching of coupled integer order differential equations (Sprott 2005). His approach related to the model of marriage relationship that is time-evolution of a romantic affair between two individuals. His model was referred to as minimal model on love which explains love affairs between Romeo and Juliet. Strogatz linear model was modified by Rinaldi (1998) using two linear differential equations to describe romantic life between two individuals and considered in his work the mechanisms of love, growth and decay which he referred to as return (pleasure of being loved), instinct (reaction to the appeal of the partner) and oblivion (the forgetting process). Rinaldi's model can only be used for a short period of time (months/years) to predict if marriage will be ideal or fragile. The model is a dynamic system with two state variables, also called a typical minimal model but love is a complex mixture of different feelings (esteem, friendship, sexual satisfaction, emotional attachment, qualifications, career work, finance and so on and on) and can barely be captured by a single variable. In this model, the tensions and emotions involved in the social life of human being cannot be considered in a simple model, therefore only the interactions between the two individuals are modelled, while the rest of the world is kept frozen and does not participate explicitly in the structure of love dynamics (Rinaldi 1996). Dharna and Arun (2012), discovered Rinaldi's model does not consider some parameters like the effect of synergism, adaptation and learning after living together (in the case of arranged marriages in India, which have succeeded over centuries). They suggested that, the emotional interaction of two individuals must be considered in the process of modelling, that is, emotion of a particular individual with respect to another cannot increase infinitely and they assumed that emotion is proportional to $x_1x_{2.}$ Dharna and Arun (2012) in their work reported that to the psychologists it is not difficult to study human behaviour but it is not easy studying it to accuracy.

Sprott (2001) disagreed with Strogatz work because the researcher did not consider appeal of the couple in his differential equation. Sprott (2001) therefore proposed a model of love and happiness which he referred to as Love model for Romeo and Juliet. Orsucci (2001) reported that, it is essential to define happiness as many emotions which includes excitement, pleasure, joy, contentment, serenity, fulfillment, and satisfaction among others. In the world today, people get married for several reasons which may include legal, social, libidinal, emotional, financial, spiritual, and religion purposes (Cynthia 2015). Kefalas et al. (2011) identified two groups of marriage namely marriage naturalists and marriage planners. In their work, Kefalas et al. (2011) defined marriage naturalists to be largely from the rural area and are eager to get married while marriage planners are based in metropolitan areas. Chaos is surely in marriages and has been defined from different perspectives namely ancient time, ancient classics, general science and religious books. It is a term used in Physics to refer to the state of extreme disorder which results due to unpredictable behaviour that depends so sensitively on the system's precise initial conditions. Nature has given us amazing different complex systems that show chaotic behaviour. For example, animal populations, the red spot of Jupiter, electrical signal in the human heart, the motion of the planets round the sun, pulsations of the brain, the stock market, the population, marriage instability and so on. All these are different complex systems with different reasons but with the help of chaos theory, the complexity of these systems can be explained to some extent. Many publications and books have been written on the subject ‘chaos’ as it relates to various behaviours in nonlinear dynamics, mathematics, mechanics, computer, religion, humanities, social life and other branches of science. Gleick (1987) stated that classical science stops where chaos begins.

Adenugba (2015) reported that sex is important in marriage. Sex between male and female can be described as an excited electron after the bombardment by an atom. As the excited atom is ground seeking for stability, so also a couple that is sexually aroused is yearning for sex. If the husband or the wife has the urge to meet with his or her partner and the partner is not in the mood or not aroused then such a person will not be balanced. According to Adenugba (2015) sexual tension could drive any person to misbehave, commit sexual immorality with any person including the under aged and/or go into prostitution. By the application of external force in terms of flow of electron that is, conductors allows electric current to flow through them while insulators do not allow the passage of electric current and

Adenugba (2015) related it to impotency in human whereby the body is not charged and excited. Also, there are virtually no free electrons hence there is no electron gas and therefore no electrical conductivity. Both the male and the female shall not be attracted to each other in this case hence when they see each other, their bodies are not excited. Applications of fractional order differential (FOD) are recent research focus of interest which can be used to describe life sciences and psychological processes (Gu and Xu 2011). Many dynamical phenomena in physics have been studied by the application of fractional-order differential equations. Liu and Chen (2015) reported in their work that the dynamics of love have not been studied properly. Due to the effectiveness of fractional order integrals and derivatives, several models in sciences and engineering have been formulated and analyzed successfully (Okyere et al. 2016). Fractional order differential operator is non local (Khan et al. 2020). This makes fractional differential equation to be veritable tools for modelling marital relations. The theory of FOD was first constrained to the field of pure theoretical mathematics three centuries ago which is useful to both the theoretical Physicists and Mathematicians only (Mohammadi 2014). Also, it has been successfully used in modelling many physical phenomena nowadays such as nonlinear oscillation of earthquake, acoustics, electromagnetism, electrochemistry, diffusion process, fluid flow, polymer physics, mathematical biology and several areas of science and technology (Alawad et al. 2013, Odetunde and Taiwo 2015, Ahmad et al. 2020). In solving fractional order differential equation efficiently, several methods have been applied such as Taylor Collocation Method (TCM) (Noeiaghdam et al. 2020), Variational Iteration Method (VIM) (Vilu et al. 2019), Iterative Decomposition Method (IDM), Adomian Decomposition Method (ADM) (Thabet and Kendre 2018), Homotopy Analysis Method (HAM) (Mohammadi 2014). However, most nonlinear fractional order differential equations cannot be solved analytically but numerical methods can be used in solving them.

Recently, decomposition methods receive more attention as efficient techniques for the solution of deterministic or stochastic, nonlinear, linear, partial differential equations (Osilagun and Taiwo 2011). Talabi et al. (2019) applied Adams Predictor-Corrector Method (APCM) to test the chaoticity of the system and was confirmed via numerical simulations. Many researchers put efforts into the studies of chaotic dynamics and control of fractional order differential system but no research has been done in applying iterative decomposition method in solving love model. Nonlinear coupled dynamical fractional model of romantic and interpersonal relationships using fractional variation iteration method (FVIM) and fractional homotopy perturbation transform method (FHPTM) was applied to inspect the dynamics of love affairs among couples (Goyal et al. 2019). Owolabi (2019) modelled a new fractional-order love dynamics equation using the Caputo–Fabrizio (exponential decay-law), Caputo (power-law) and Atangana–Baleanu (Mittag-Leffler-law) fractional operators and Adams-Bash forth new fractional schemes was utilize for the approximation of these derivatives. Koca and Yaprakdal (2019) reported that mathematical modeling is a very important work in applied sciences and considered mathematical modeling of nuclear family model with fractional order Caputo derivative in their work. Zhang (2018) uses the coincidence degree theory, to get result for a coupled system of nonlinear fractional differential equations with multi-point boundary conditions at resonance.

According to Ahmad and El-Khazali (2007) it was emphasized that modelling psychological phenomena is more complex than engineering systems due to many parameters involved in the phenomena. The parameters used in psychological phenomena is time varying. Recently, it has been noticed by some researchers that relationships do not always follow patterns of linear change. Some relationships end and are then reborn in a new form while others begin and end. Therefore this makes it difficult to identify any stable sequence.

This work compels evidence of the power of nonlinear dynamical models for understanding complex psychological phenomena which will give people different ideas of love in couples. Song and Yang (2009) described real world processes as the most likely fractional order system and numerical simulations are often used to investigate the behaviour of these systems.

Methodology and Mathematical Model

Survey (using questionnaire), was conducted among young and old married men/women and the questionnaire was designed to get views from different angles to help find answers to the research questions. Two different states were used as the sample studies which are Lagos and Ogun states of Nigeria because of the settlement pattern (urbanization and modernization), population and environmental dynamics. The focus of this study is based on micro-experiment, the statistics techniques used was Point Pattern Analysis (PPA) and the criteria chosen were random sampling. The research was not limited to controlled environment but respondent were picked from different locations in the study areas. The study areas had been under survey for two years to better understand the way of living in that environment and gain access to participants.

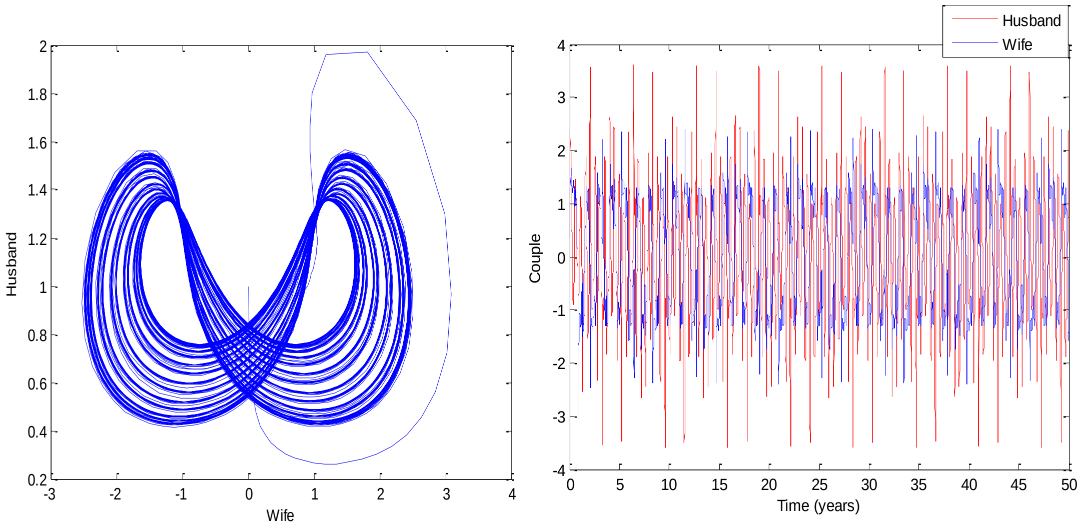
However, in this work; all items in the research data set were labeled for proper identification whereby data from the field were transformed into numbers so that the computer software can understand after listing all the variables to be measured. For the close ended questions all the categories had been decided and common responses were ascribed the same code.

Barley and Cherif (2011) proposed a general model on couples dynamics to show the behavioural features of romantic relationship but only showed the phase plot of ideal relationship in their work where they considered $\varepsilon = 0$. The model of consideration in this work is an improvement of the existing model of Barley and Cherif which was modified to fractional nonlinear differential equation that uses the result of the data analysis as its parameters. It describes the time-variation displayed by individuals in their marriages under certain external circumstance(s) which is given as:

$$\begin{cases} \dfrac{d^q x}{dt^q} = -ax + bx(1-ey^2) + \sin t \\ \dfrac{d^q y}{dt^q} = -cy + dy(1-ex^2) + \sin t. \end{cases} \tag{1}$$

For convenience of interpretation, some practical physical meanings were imposed on the parameters such as a, b, c, d, e, x, y, $\sin(t)$ and q as shown in eq. (1). The parameters used in this study are to categorize people with different personality in marriage, a; maltreatment, b; loneliness, c; wrong advice, d; phone calls, e; fission (external influence), measure of disorderliness in marriage of individual 1 and 2 (x(t) and y(t)), sin (t) is a time dependent forcing function representing ones external circumstances. The order q represents the Impact Factor of Memory (IFM) of the couple, that is, as the value of q increases from 0 to 1, the IFM of an individual increases correspondingly. The derivative is of the order $0 < q \le 1$. The system can produce chaos according to different competition coefficients (Figure 1).

Figure 1. *Strange Attractor and Time Series Plot from the Nonlinear Fractional-Order Love System in (1) with* α *= 0.75*



There are several definitions of fractional derivatives but this study uses the Caputo-type fractional derivative defined by Caputo (1967). Caputo's definition is a modification of the Riemann-Liouville definition and has the merit of dealing properly with initial value problems. It is more favoured because of its ease in adaptability to initial conditions for physical problems whereby the initial conditions are given in terms of the field variables (Odibat and Momani 2008). The fractional derivative in the Caputo sense is defined as

$$D^{\alpha} y(x) = J^{m-\alpha} y^{(m)}(x),\ \alpha > 0 \tag{2}$$

where $m = [\alpha]$ is the value α rounded up to the nearest integer, y^m is the ordinary *mth* derivative of y, and

$$J^{\beta} f(x) = \frac{1}{\Gamma(\beta)} \int_0^x (x-t)^{\beta-1} f(t)dt, \tag{3}$$

Is the Riemann-Liouville integral operator of order $\beta > 0$ where $\Gamma(\beta)$ is the gamma function.

To illustrate the suitability of the proposed method.

$$\begin{cases} D^{\alpha} x(t) = -ax + bx(1-ey^2) + \sin t \\ D^{\alpha} y(t) = -cy + dy(1-ex^2) + \sin t. \end{cases} \tag{4}$$

where $0 < \alpha < 1$, the fractional-order α will be determined later, to point out that the order α has its practical physical meanings which represents the impact factor of memory of an individual, with initial condition $y(0) = x(0) = 0.1$,
We consider the equation

$$D^{\alpha} x(t) = -ax + bx - ebxy^2 + \sin t. \tag{5}$$

sin *t* in equation (5) can be decomposed into infinite series as follows

$$D^{\alpha} x(t) = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + bx - ax - ebxy^2. \tag{6}$$

By applying the operator J^{α} to both sides of (6), we have

$$x(t) = \frac{\Gamma(2)t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{\Gamma(4)t^{\alpha+3}}{6\Gamma(\alpha+4)} + \frac{\Gamma(6)t^{\alpha+5}}{120\Gamma(\alpha+6)} - \frac{\Gamma(8)t^{\alpha+7}}{5040\Gamma(\alpha+8)} + J^{\alpha}\{bx_o - ax_o - ebx_o\}. \tag{7}$$

Take x_o to be

$$x_o = \frac{\Gamma(2)t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{\Gamma(4)t^{\alpha+3}}{6\Gamma(\alpha+4)} + \frac{\Gamma(6)t^{\alpha+5}}{120\Gamma(\alpha+6)} - \frac{\Gamma(8)t^{\alpha+7}}{5040\Gamma(\alpha+8)} \tag{8}$$

and

$$x_1 = J^{\alpha}\{bx_o - ax_o - ebx_o\}. \tag{9}$$

On substituting equation (8) into equation (9), we have

$$x_1 = J^{\alpha}\left\{\begin{array}{l}\frac{b\Gamma(2)\Gamma(\alpha+2)t^{2\alpha+1}}{\Gamma(\alpha+2)\Gamma(2\alpha+2)} - \frac{b\Gamma(4)\Gamma(\alpha+4)t^{2\alpha+3}}{6\Gamma(\alpha+4)\Gamma(2\alpha+4)} + \frac{b\Gamma(6)\Gamma(\alpha+6)t^{2\alpha+5}}{120\Gamma(\alpha+6)\Gamma(2\alpha+6)} + \frac{b\Gamma(8)\Gamma(\alpha+8)t^{2\alpha+7}}{5040\Gamma(\alpha+8)\Gamma(2\alpha+8)} - \\ \frac{a\Gamma(2)\Gamma(\alpha+2)t^{2\alpha+1}}{\Gamma(\alpha+2)\Gamma(2\alpha+2)} + \frac{a\Gamma(4)\Gamma(\alpha+4)t^{2\alpha+3}}{6\Gamma(\alpha+4)\Gamma(2\alpha+4)} - \frac{a\Gamma(6)\Gamma(\alpha+6)t^{2\alpha+5}}{120\Gamma(\alpha+6)\Gamma(2\alpha+6)} - \frac{a\Gamma(8)\Gamma(\alpha+8)t^{2\alpha+7}}{5040\Gamma(\alpha+8)\Gamma(2\alpha+8)} - \\ \frac{eb\Gamma(2)\Gamma(\alpha+2)t^{2\alpha+1}}{\Gamma(\alpha+2)\Gamma(2\alpha+2)} + \frac{eb\Gamma(4)\Gamma(\alpha+4)t^{2\alpha+3}}{6\Gamma(\alpha+4)\Gamma(2\alpha+4)} - \frac{eb\Gamma(6)\Gamma(\alpha+6)t^{2\alpha+5}}{120\Gamma(\alpha+6)\Gamma(2\alpha+6)} - \frac{eb\Gamma(8)\Gamma(\alpha+8)t^{2\alpha+7}}{5040\Gamma(\alpha+8)\Gamma(2\alpha+8)}\end{array}\right\}. \quad (10)$$

$$x_0 + x_1 = \left\{\begin{array}{l}\frac{\Gamma(2)t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{t^{\alpha+3}}{\Gamma(\alpha+4)} + \frac{t^{\alpha+5}}{\Gamma(\alpha+6)} + \frac{t^{\alpha+7}}{\Gamma(\alpha+8)} + \frac{b\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} - \frac{bt^{2\alpha+3}}{\Gamma(2\alpha+4)} + \frac{bt^{2\alpha+5}}{\Gamma(2\alpha+6)} + \frac{bt^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{a\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{at^{2\alpha+3}}{\Gamma(2\alpha+4)} - \\ \frac{at^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{at^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{eb\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{ebt^{2\alpha+3}}{\Gamma(2\alpha+4)} - \frac{ebt^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{ebt^{2\alpha+7}}{\Gamma(2\alpha+8)}\end{array}\right\}. \quad (11)$$

By applying the operator J^{α} on equation (11) we have

$$x_2 = J^{\alpha}\{b(x_o + x_1) - a(x_o + x_1) - eb(x_o + x_1)\} - J^{\alpha}\{bx_o - ax_o - ebx_o\}. \quad (12)$$

On integrating equation (12) we obtained.

$$x_2 = \left\{\begin{array}{l}\frac{b^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{b^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{b^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{b^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} - \frac{2ab\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2abt^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2abt^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2abt^{3\alpha+7}}{\Gamma(3\alpha+8)} - \\ \frac{2eb^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2eb^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2eb^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2eb^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \frac{a^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{a^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{a^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{a^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \\ \frac{2eab\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{2eabt^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2b^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{e^2b^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2b^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{e^2b^2t^{3\alpha+7}}{\Gamma(3\alpha+8)}\end{array}\right\}. \quad (13)$$

Then, $x(t)$ is approximated as

$$x(t) = x_0 + x_1 + x_2. \quad (14)$$

Therefore,

$$x(t) = \left\{\begin{array}{l}\frac{\Gamma(2)t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{t^{\alpha+3}}{\Gamma(\alpha+4)} + \frac{t^{\alpha+5}}{\Gamma(\alpha+6)} + \frac{t^{\alpha+7}}{\Gamma(\alpha+8)} + \frac{b\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} - \frac{bt^{2\alpha+3}}{\Gamma(2\alpha+4)} + \frac{bt^{2\alpha+5}}{\Gamma(2\alpha+6)} + \frac{bt^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{a\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{at^{2\alpha+3}}{\Gamma(2\alpha+4)} - \\ \frac{at^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{at^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{eb\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{ebt^{2\alpha+3}}{\Gamma(2\alpha+4)} - \frac{ebt^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{ebt^{2\alpha+7}}{\Gamma(2\alpha+8)} + \frac{b^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{b^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{b^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{b^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} - \\ \frac{2ab\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2abt^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2abt^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2abt^{3\alpha+7}}{\Gamma(3\alpha+8)} - \frac{2eb^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2eb^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2eb^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2eb^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \frac{a^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{a^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \\ \frac{a^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{a^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \frac{2eab\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{2eabt^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2b^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{e^2b^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2b^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{e^2b^2t^{3\alpha+7}}{\Gamma(3\alpha+8)}\end{array}\right\}. \quad (15)$$

We then consider x(t) for particular cases $\alpha = 0.25, 0.50, 0.75 and 1.00$, the numerical simulations are shown below.

From equation (4)

$$D^{\alpha} y(t) = -cy + dy - edyx^2 + \sin t. \tag{16}$$

To solve for (16), same procedure used in solving (6) was applied to establish the accuracy and efficiency of the method.

Therefore,

$$y(t) = \left\{ \begin{array}{l} \frac{\Gamma(2)t^{\alpha+1}}{\Gamma(\alpha+2)} - \frac{t^{\alpha+3}}{\Gamma(\alpha+4)} + \frac{t^{\alpha+5}}{\Gamma(\alpha+6)} + \frac{t^{\alpha+7}}{\Gamma(\alpha+8)} + \frac{d\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} - \frac{dt^{2\alpha+3}}{\Gamma(2\alpha+4)} + \frac{dt^{2\alpha+5}}{\Gamma(2\alpha+6)} + \frac{dt^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{c\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{ct^{2\alpha+3}}{\Gamma(2\alpha+4)} - \\ \frac{ct^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{ct^{2\alpha+7}}{\Gamma(2\alpha+8)} - \frac{ed\Gamma(2)t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{edt^{2\alpha+3}}{\Gamma(2\alpha+4)} - \frac{edt^{2\alpha+5}}{\Gamma(2\alpha+6)} - \frac{edt^{2\alpha+7}}{\Gamma(2\alpha+8)} + \frac{d^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{d^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{d^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{d^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} - \\ \frac{2cd\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2cdt^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2cdt^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2cdt^{3\alpha+7}}{\Gamma(3\alpha+8)} - \frac{2ed^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \frac{2ed^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} - \frac{2ed^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{2ed^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \frac{c^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{c^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \\ \frac{c^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} + \frac{c^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} + \frac{2ecd\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{2ecdt^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2d^2\Gamma(2)t^{3\alpha+1}}{\Gamma(3\alpha+2)} - \frac{e^2d^2t^{3\alpha+3}}{\Gamma(3\alpha+4)} + \frac{e^2d^2t^{3\alpha+5}}{\Gamma(3\alpha+6)} - \frac{e^2d^2t^{3\alpha+7}}{\Gamma(3\alpha+8)} \end{array} \right\}. \tag{17}$$

For the particular case $\alpha = 0.25$

The results for y(t) was then solved for different cases when $\alpha = 0.25, 0.50, 0.75 and 1.00$. The parameters used (c, d and e) were taken from the data analysis; the data of the results when $\alpha = 0.25, 0.50, 0.75 and 1.00$ were used to show the time series plots.

Results

Based on the Iterative Decomposition Method results above, the results of the fractional order love model can be proved by computer simulations. According to what have been discussed above, Numerical simulations were conducted on both the modified and the existing models using the fractional differential equation. The following are the plots of the results of our investigations for various cases studied

Table 1. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 1a*

(t)	x(t) for α=0.25 Modify	y(t) for α=0.25 Modify	x(t) for α=0.25 Existing	y(t) for α=0.25 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	7.304214356E-01	4.244793214E-01	-2.892194748E-01	2.751858042E+00
0.2	2.515290497E+00	1.463791253E+00	-2.219194574E+00	2.360731075E+00
0.3	5.163624373E+00	3.009833545E+00	-3.813756018E+00	1.969697227E+00
0.4	8.569007204E+00	5.001575057E+00	-5.212717650E+00	1.596293123E+00
0.5	1.264557709E+01	7.389417875E+00	-6.478279693E+00	1.240875821E+00
0.6	1.731469709E+01	1.012770811E+01	-7.644869554E+00	9.016065644E-01
0.7	2.250004977E+01	1.317196466E+01	-8.734018027E+00	5.765295815E-01
0.8	2.812551189E+01	1.647766682E+01	-9.760299237E+00	2.639340799E-01
0.9	3.411420844E+01	1.999969842E+01	-1.073414431E+01	-3.761458280E-02
1.0	4.038813691E+01	2.369210174E+01	-1.166333600E+01	-3.293112943E-01

Table 2. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 1b*

(t)	x(t) for α=0.5 Modify	y(t) for α=0.5 Modify	x(t) for α=0.5 Existing	y(t) for α=0.5 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	-1.033244507E-01	-1.237625056E-01	3.904169924E+00	3.569993597E+00
0.2	-5.695277085E-01	-7.014697501E-01	4.167967943E+00	4.232027564E+00
0.3	-1.532946223E+00	-1.915567443E+00	3.729735120E+00	4.465458881E+00
0.4	-3.094919015E+00	-3.901556392E+00	2.823869996E+00	4.432357257E+00
0.5	-5.344811772E+00	-6.775299939E+00	1.554224406E+00	4.201095971E+00
0.6	-8.366875547E+00	-1.064183329E+01	-1.953058719E-02	3.809487942E+00
0.7	-1.224377152E+01	-1.559977739E+01	-1.858204951E+00	3.281599784E+00
0.8	-1.705872195E+01	-2.174397089E+01	-3.933846720E+00	2.634148296E+00
0.9	-2.289694736E+01	-2.916720295E+01	-6.225369636E+00	1.879461044E+00
1.0	-2.984667663E+01	-3.796142903E+01	-8.716208328E+00	1.027033337E+00

Table 3. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 2a*

(t)	x(t) for α=0.25 Modify	y(t) for α=0.25 Modify	x(t) for α=0.5 Existing	y(t) for α=0.5 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	1.312525905E-01	3.046455524E-01	1.867862502E+00	6.109091020E-01
0.2	4.422230900E-01	1.049177774E+00	1.389765176E+00	-6.670279515E-01
0.3	9.034316491E-01	2.157795048E+00	9.547245161E-01	-1.738535113E+00
0.4	1.497833906E+00	3.587340842E+00	5.551577841E-01	-2.685604756E+00
0.5	2.211302685E+00	5.302565504E+00	1.833011644E-01	-3.546431545E+00
0.6	3.030650803E+00	7.270909162E+00	-1.663435073E-01	-4.342625903E+00
0.7	3.942899115E+00	9.460568568E+00	-4.976899938E-01	-5.087888882E+00
0.8	4.934957495E+00	1.183965175E+01	-8.136157012E-01	-5.791583973E+00
0.9	5.993479581E+00	1.437579136E+01	-1.116304024E+00	-6.460461401E+00
1.0	7.104798537E+00	1.703597419E+01	-1.407457430E+00	-7.099586297E+00

Table 4. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 2b*

(t)	x(t) for α=0.75 Modify	y(t) for α=0.75 Modify	x(t) for α=0.75 Existing	y(t) for α=0.75 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	6.155577598E-03	2.895632885E-03	1.970623848E+00	2.191082404E+00
0.2	5.219094311E-03	-1.845667942E-02	2.414176187E+00	2.594006269E+00
0.3	-9.639837374E-02	-1.007135085E-01	2.657487129E+00	2.674349279E+00
0.4	-2.342601224E-01	-2.804235074E-01	2.761154513E+00	2.523544203E+00
0.5	-2.342601224E-01	-5.955334252E-01	2.754112656E+00	2.186316953E+00
0.6	-4.554791476E-01	-1.085750764E+00	2.653686938E+00	1.689915427E+00
0.7	-7.798133170E-01	-1.792775141E+00	2.471556839E+00	1.052956970E+00
0.8	-1.227804269E+00	-2.760511951E+00	2.216200694E+00	2.891050823E-01
0.9	-1.820895223E+00	-4.035291009E+00	1.894093963E+00	-5.911083376E-01
1.0	-2.581543730E+00	-5.666093907E+00	1.510369287E+00	-1.579269656E+00

Table 5. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 3a*

(t)	x(t) for α=0.25 Modify	y(t) for α=0.25 Modify	x(t) for α=0.25 Existing	y(t) for α=0.25 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	7.769339754E-02	2.069033088E-01	2.411166256E+00	3.651986618E+00
0.2	2.498818864E-01	7.086550603E-01	2.437305294E+00	3.912897698E+00
0.3	5.011067765E-01	1.455661253E+00	2.411906698E+00	4.044918132E+00
0.4	8.225614617E-01	2.419428702E+00	2.368621031E+00	4.123406017E+00
0.5	1.206893805E+00	3.576535993E+00	2.317264752E+00	4.172723969E+00
0.6	1.647197834E+00	4.905259900E+00	2.261861399E+00	4.203850215E+00
0.7	2.136647029E+00	6.384333950E+00	2.204369407E+00	4.222658726E+00
0.8	2.668338185E+00	7.992421331E+00	2.145846553E+00	4.232649343E+00
0.9	3.235224058E+00	9.707897187E+00	2.086904413E+00	4.236068325E+00
1.0	3.830088010E+00	1.150878607E+01	2.027913104E+00	4.234438406E+00

Table 6. *Numerical Results x(t), y(t) in Equation 15 and 17 for Case 3b*

(t)	x(t) for α=0.5 Modify	y(t) for α=0.5 Modify	x(t) for α=0.5 Existing	y(t) for α=0.5 Existing
0	0.000000000E+00	0.000000000E+00	1.000000000E+00	1.000000000E+00
0.1	-1.462256518E-03	3.760936169E-03	2.332430963E+00	3.046080610E+00
0.2	-4.805752367E-02	-2.019925049E-03	2.750177860E+00	3.759430869E+00
0.3	-1.609068464E-01	-1.425974674E-02	2.979204591E+00	4.215282038E+00
0.4	-3.534475222E-01	-2.906783061E-02	3.095651948E+00	4.522951241E+00
0.5	-6.367567758E-01	-4.393004618E-02	3.130878072E+00	4.726647193E+00
0.6	-1.020812528E+00	-5.799756505E-02	3.101880974E+00	4.849958463E+00
0.7	-1.515062564E+00	-7.221543975E-02	3.019292920E+00	4.907432408E+00
0.8	-2.128746296E+00	-8.940831237E-02	2.890386777E+00	4.908892795E+00
0.9	-2.871100573E+00	-1.143429875E-01	2.720452974E+00	4.861401914E+00
1.0	-3.751501770E+00	-1.537720040E-01	2.513516669E+00	4.770275003E+00

Figure 2a. *Time Series Plot of Case 1a Depicts the Optimal Height where the Love Reached Before Depreciating After Three Years of Togetherness. The Plot Compares the Existing and Modified Model where α =0.25*

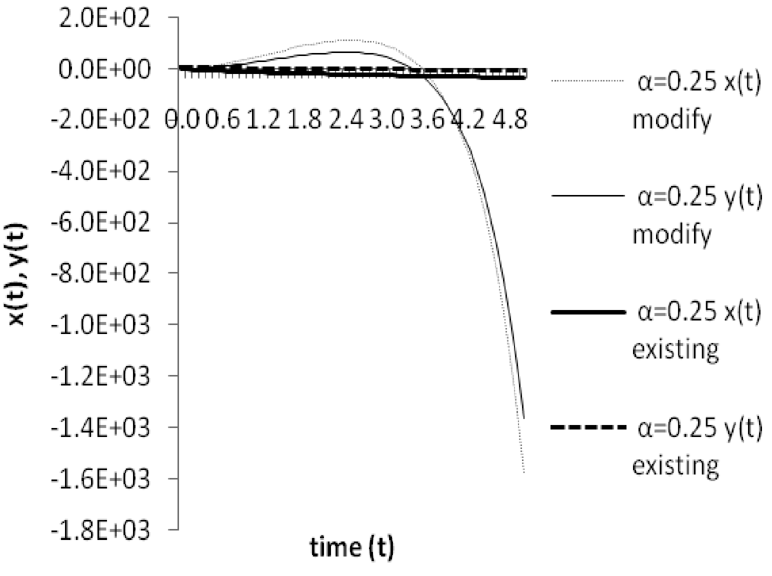
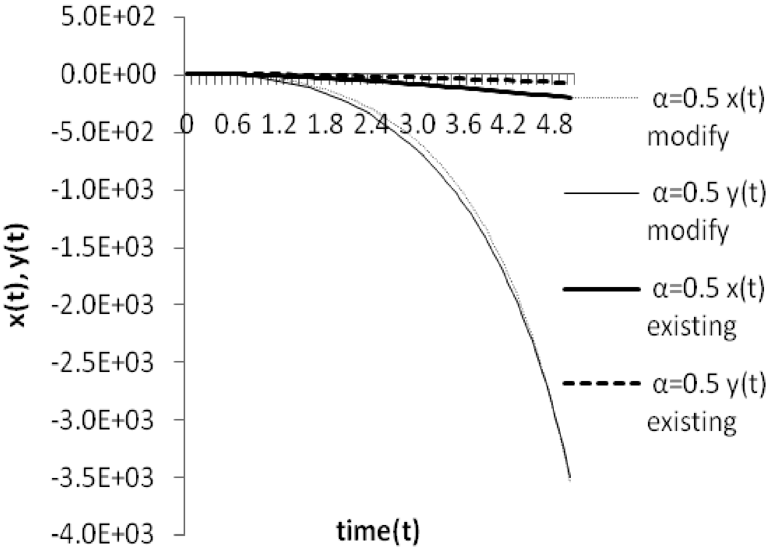
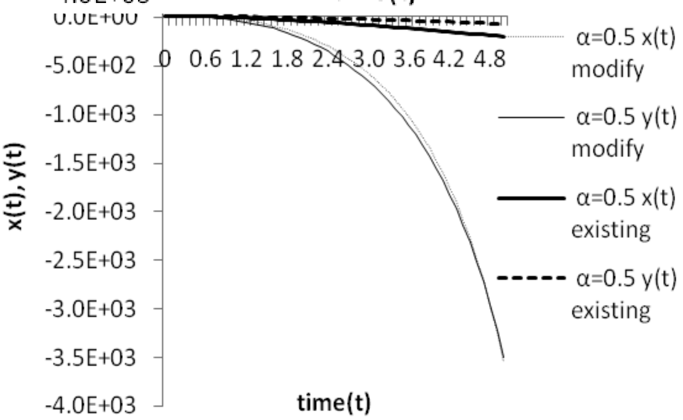


Figure 2b. *Time Series Plot of Case 1a Depicts Rapid Drop in the Love Life of the Couple due to Increase in the Impact Factor Memory where* α *=0.50*





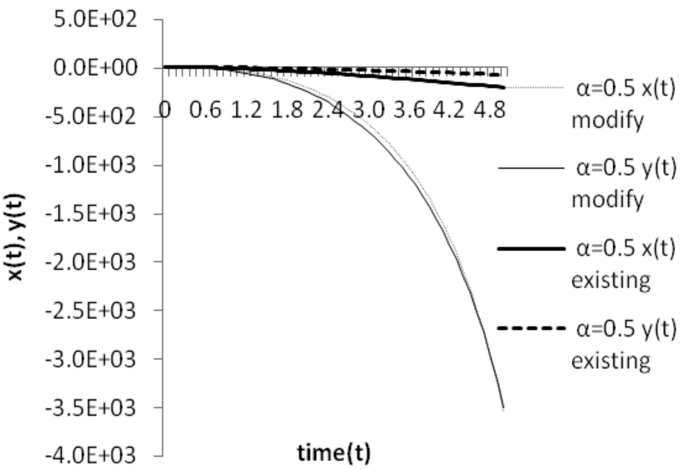


Figure 3a. *Time Series Plot of Case 3 Depicts the Optimal Height where the Love Reached Before Depreciating After Three Years of Togetherness where* α *=0.25*

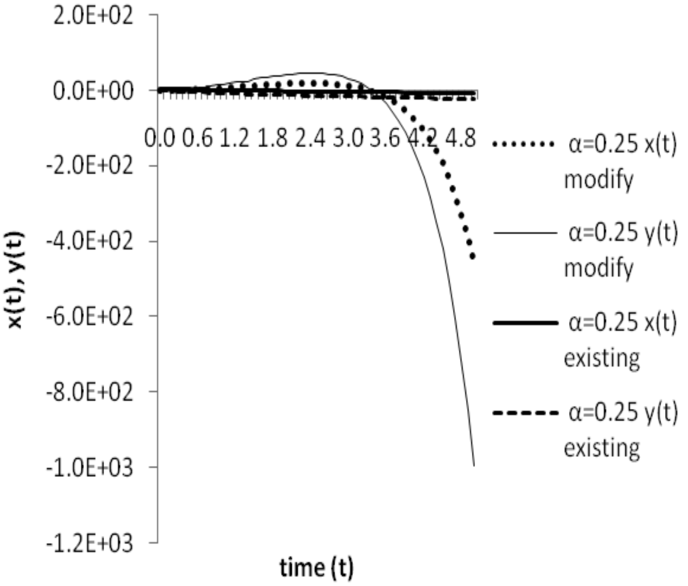


Figure 3b. *Time Series Plot of Case 3 Depicts Rapid Drop in the Love Life due to Increase in the Impact Factor Memory where* α *=0.75*

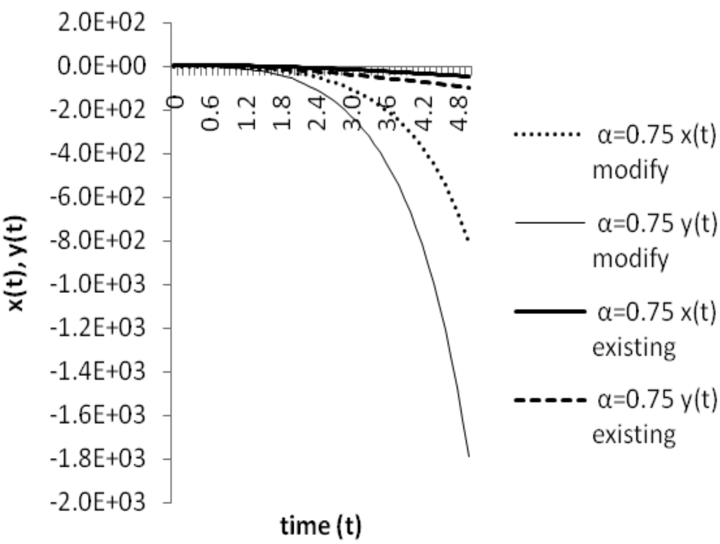


Figure 4a. *Time Series Plot of Case 3a Depicts the Optimal Height where the Love Reached Before Depreciating After Three Years of Togetherness where* α *=0.25*

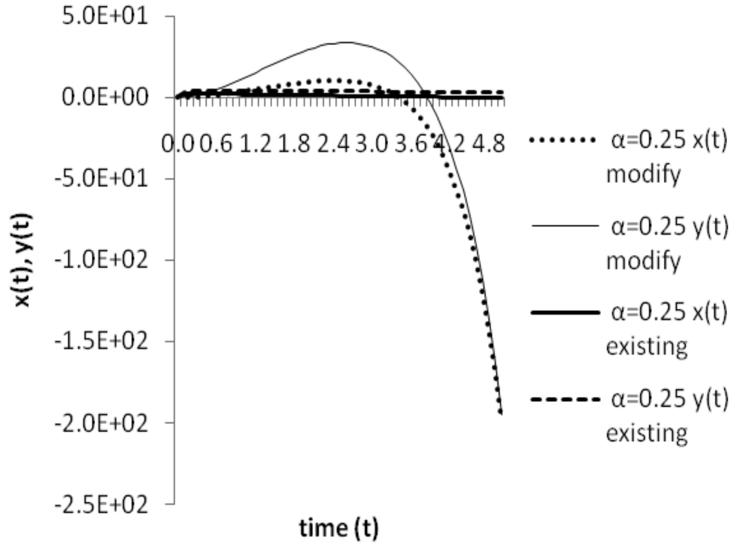
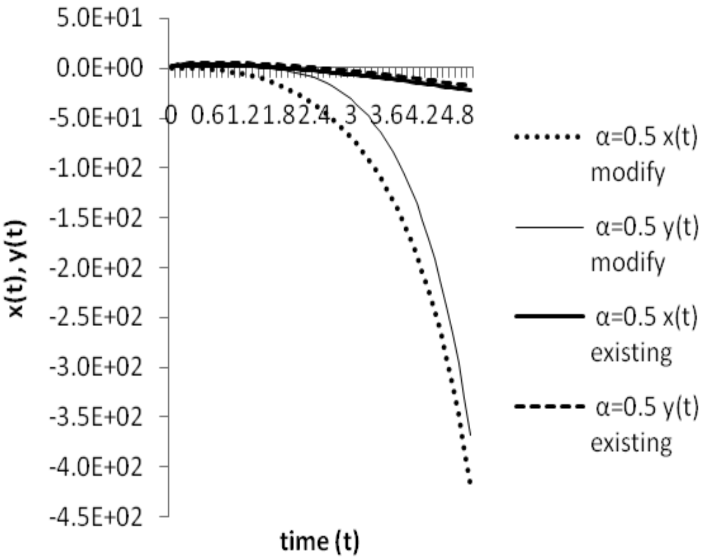


Figure 4b. *Time Series Plot of Case 3b Depicts Drop in the Love Life of the Couple due to Increase in the Impact Factor Memory where* α *=0.5*



Discussion

Case 1

This case implies that the love story between case 1a enters into the dynamics of antagonist, when two lovers first meet at time t=0, we say they are completely indifferent to each other (Rinaldi and Gragnani 1998). The parameters used were from the analysis of lover 1 while Tables 1 and 2 showed the data from the analysis for order 0.50 and 0.25 respectively. Figure 2a allows us to deduce the qualitative shape of the solutions, the time response of the love between individual moves faster and faster until it reaches the optimal height. The love depreciated after three years of togetherness and then moved to negative feelings. Comparing the existing and modified model shows that the trajectory for the existing model for the case reached the optimal height almost at the same time and the love did not start from the state of indifference but for the modified model; the love started from the state of indifference in accordance with Rinaldi and Gragnani (1998) where they are just trying to know each other and the husband love (y(t)) for the wife (x(t)) moved at a faster rate than that of x(t) before the feeling towards each other crept to the negative feelings with order 0.25.

Figures 2a and b allow us to deduce the qualitative shape of the solutions, lovers 1 were together for few years before divorce and they started experiencing troubles within a year in their marriage. The plots showed that when the impact memory factor increased (the moment they allowed their past to take over their present), right from the beginning of their love life they have no feeling towards each other. The love depreciated within a year of their togetherness and moved to negative feelings. Comparing the existing and modified model, it shows that the trajectory for the existing model for the love did not start from the state of indifference but for the modified model, it shows the systematic dropped in the love life of both couple. The love started from the state of indifference with order 0.25, 0.05 and 0.75.

Case 2

In this case, the parameters used were from the analysis of lover 3. Iterative decomposition method was used in solving the problem, the numerical results from Tables 3 and 4 were plotted and the graphs were also interpreted.

Figures 3a and b allow us to deduce the qualitative shape of the solutions, the time response of the love between individual moves faster and faster until it reaches the optimal height. At the optimal height, the love dropped a little before picking again and followed a parabolic part. After few years of their togetherness, the love depreciated and moved to negative feelings. Comparing the existing and modified model shows that the trajectory for the existing model for the couple did not start from the state of indifference (the state when they first met each other) but for the modified model; the love started from the state of indifference where they are just trying to know each other but the love for each other later creeped to the negative feelings with order 0.25, 0.50 and 0.75.

Case 3

In this case, the parameter values used were from the analysis of couple 8 while Tables 5 and 6 showed the data from the analysis for order 0.25 and 0.50 respectively. Iterative decomposition method was used in solving the problem, the numerical results were plotted and the graphs were similarly interpreted.

Figure 4a allows us to deduce the qualitative shape of the solutions, the time response of the love between individual moves faster and faster until it reaches the optimal height. The love depreciated after few years of togetherness and moved to negative feelings. Figure 4b depicts rapid drop in the love life of the lovers due to increase in the impact factor memory. The plot compares the existing and modified model time response of the lover in their marriage with order 0.5.

Conclusions

In the present work, linear love dynamic model was modified by introducing nonlinear terms to become nonlinear fractional order differential equations. The model was used to interpret the dynamics of most marriages. According to Sprott (2005) he reported that simple linear models of happiness can produce surprisingly complex dynamics, much of which rings true to common experience. When there are periodic external events, simple nonlinearities can produce chaos. The existence of chaos in relationship implies a degree of unpredictability. Numerical simulations were shown to illustrate the effectiveness and applicability of this design. The integer-order love model is extended to a fractional-order pattern through the comparison between the existing and modified model, which is found to be more suitable to depict the marriage behaviour. It is noteworthy that the conception of IFM proposed here is to denote a measurement of how influenced an individual is by his/her past experiences. When one’s IFM is low, his/her past experiences may have little influence on his/her present and future life; while when IFM is high, it might be difficult for him/her to escape from past experiences, in despite of nightmares or sweet memories.

Moreso, this research was carried out to look into the reason behind the increasing divorce rate among young lovers these days which is causing societal decadency making it unbearable for friendly environment. However, the work also focuses on analyzing data with the aid of questionnaires and to interpret the result with the aid of phase plots, time series plots and charts. Finding solutions to the problems young lovers are facing is a problem with required solution that the current research work is attempting to un-ravel to allow reasonable stability to enhance marriage so that the socio-economic growth and development in the nation can be improved upon.

Also, fractional order model has been used in this work because of its memory effect compared to integer-order model. The model describes the dynamical behaviour of the couples in their marital affair. The dynamics of the proposed system was studied numerically using Iterative Decomposition Method with different order. The method is easily applied as the terms of the approximating series are easy to compute. It was demonstrated via numerical simulations by showing that fractional order model of marriage can exhibit irregular pattern using an appropriate set of parameters gotten from analysis of the survey results. From the time series and phase plots acquired using computer simulation, it was recognized that there is a clear difference in the results of modified and existing model. The modified model shows the trajectory of the couple from the state of indifference which implies that the status of love between the couples can be affected by various parameters like opinions of parents, maltreatment, loneliness etc. As the impact factor memory increases, it also affects their togetherness making the love between them to decay easily.

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