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The Growth of Teacher's Mathematical Knowledge for Teaching as Participating in a Primary School Teacher's Professional Learning Community

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Teacher's Professional Learning Community (PLC) should effectively operate through sharing teaching resources, professional dialogues, and collaboration to reduce pupil's learning achievement gaps and make teaching close to their learning experiences through providing learning scaffolding. This study adopts a qualitative research method to investigate the change of participants' Mathematics Knowledge for Teaching (MKT) of a primary school teacher PLC which has been running for three years. The three research subjects are experienced teachers and none of whom are mathematics or science majors. According to the framework of the MKT (Hill et al. 2008), the qualitative data which include PLC meeting videos, lesson observation sheets, interviews, and learning feedback are analyzed and triangulated by the researchers and other mathematics educators. The results show that PLC may help teachers improve their MKT. At the beginning of the PLC, the discourse was mainly related to the teacher's Knowledge of Special Content Knowledge (SCK) and Knowledge of Content and Teaching (KCT). It reveals that the participants ought to be energized in SCK and KCT, and the PLC activities should be specially arranged in these two aspects. After the continuous professional dialogue and teaching practices, the teacher's KCC, Knowledge of Content and Student (KCS), and Special Content Knowledge (SCK) are improved most significantly, which also promotes the student learning achievements.

Keywords: *mathematics teaching, teacher professional development, teacher professional learning community, mathematics knowledge for teaching*

Introduction

Teachers should strengthen the connection with their students' knowledge and skills to enhance their learning. Mathematics education of teachers' professional knowledge follows Shulman's seminal idea about pedagogical content knowledge (PCK) (Shulman 1986). Shulman introduced that pedagogical content knowledge inflects pupil's learning. PCK assumes teachers' ability to design effective instruction and skills to contribute to students' learning (Hill et al. 2008). Ball et al. (2008) have pioneered the consideration of Mathematical Knowledge for Teaching (MKT). MKT is an analytical tool to measure teachers' mathematical knowledge. Many researchers considered MKT to study mathematics teachers' knowledge in order to improve their teaching and promote students' effectiveness. Most of the

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primary teachers in Taiwan do not major in mathematics or science, but they all have to teach mathematics, even the teachers who possess weak abilities in mathematics teaching. Thus, they have to make plans to improve their own teaching of mathematics. A teacher professional learning community (PLC) should effectively operate through sharing teaching resources, professional dialogues, and collaboration to reduce pupils' learning achievement gaps and make teaching close to their learning experiences through providing suitable learning scaffolding. A powerful experience has happened in the classes that are utilizing and sharing teaching reflection which can help teachers to improve their teaching (Putnam and Borko 2000). Many Taiwan teachers lack sound mathematical understanding and skills. They need to have more support and resources for improving their teaching.

Recently, PLC has emerged as a support community to help teachers grow in their teaching practice. The partners have the same demands in PLC. When teachers have identified with PLC, they need to improve their curriculum, teaching and students' learning which is the correct teaching development (Stigler and Hiebert 1997). Teachers need to know how to promote their students to achieve, and identify what conditions are most likely to facilitate their mathematical learning. A teacher in PLC should effectively operate through sharing teaching resources to assess their teaching for understanding their students' ability for learning. According to the framework of the MKT (Hill et al. 2008), they would lead to a greater understanding of the constructs of mathematical knowledge for teaching. The study discussed teachers' knowledge using the MKT framework in the PLC. In the PLC, teachers prepare to preserve programs and share their teaching and resources to develop teachers' professional ability for achievement. In past studies, most of the qualitative data collection includes PLC meeting videos, lesson observation sheets, interviews, and learning feedback, which are analyzed and triangulated by the researchers and other mathematics educators. The research question is as follows: What kind of teachers' change in MKT do the members of a primary school teacher PLC have? We hope to get some suggestions for teachers regarding how to facilitate their teaching effectively in the PLC.

Literature Review

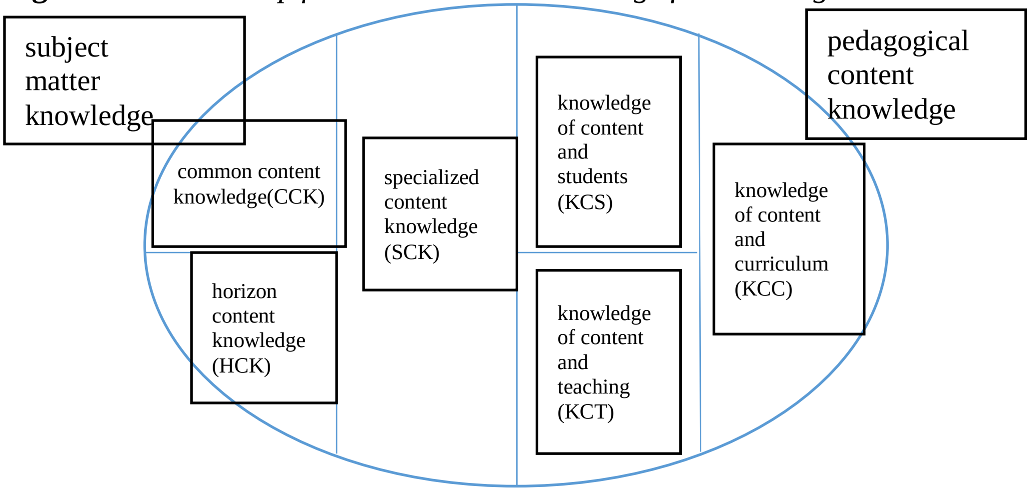
Mathematical Knowledge for Teaching

Shulman (1986, 1987) defines PCK within seven domains including: content knowledge; knowledge of subject matter; knowledge of educational aims; goals and purposes; knowledge of other content; general pedagogical knowledge; knowledge of learners; and curriculum knowledge. PCK was explained in two dimensions. First, it characterizes teacher knowledge containing teachers' representation of ideas and the ability to help students connect mathematics ideas (e.g., Ball 1988, Stein et al. 1990). Second, PCK discusses a teacher's understanding to know students' common preconceptions and misconceptions in different ages and backgrounds. PCK is canonical in developing a deeper understanding of the teacher's content knowledge and pedagogical knowledge.

Ball et al. (2008, p. 399) define MKT as a theory that encapsulates mathematical knowledge needed to perform the recurring tasks of teaching mathematics, noting that they have adopted a flexible conception of “needs” that allows for the perspective, habits of mind, and sensibilities that matter for the effective teaching of the content.

MKT assesses the knowledge by teachers in their teaching process, which includes pedagogical content knowledge and subject matter knowledge in the construct of mathematical knowledge for teaching (Hill et al. 2007) That is, knowledge of how making mathematical understanding of students and knowledge of students’ conception and misconceptions (Shulman 1986). The relationship between pedagogical content knowledge and subject matter knowledge can be seen in Figure 1. The concept of MKT appears by studying records in mathematics teaching, and identifying teachers’ mathematical knowledge, reasoning, and insight (Ball and Bass 2003, Hill et al. 2005).

Figure 1. *Domain Map for Mathematical Knowledge for Teaching*



Source: Ball et al. 2008.

Loughran et al. (2012) explain that PCK builds on teachers’ personal experiences and own conceptions—particularly their expertise with individual idiosyncrasies and important differences that are influenced by the teaching experience, content, and context. Ball et al. (2008) defined MKT as a practice-based theory that encapsulates the mathematical knowledge needed to perform the recurrent tasks of teaching mathematics. There are six portions of the oval that are a proposed standard of MKT. The right side associates with Shulman’s (1986) proposed PCK that contains KCS, KCT and KCC. KCS is content knowledge intertwined with knowledge of how students think about, know and learn mathematical knowledge content. The teachers can be diagnosed with students’ errors as a partial or a complete explanation for selecting their answer for mathematical reasoning (Hill et al. 2008). KCS is the teacher’s ability to know how making lessons better designed and foresee possible alternative conceptions of students and plan how to help them go past those conceptions which requires substantial knowledge of the students. Teachers know what works for students and support the development of their understanding (Chua, 2018). KCT is the knowledge of content and teaching that is teaching design in practices and combines knowing about students and mathematics. KCT is proof that includes strategies of representing, explaining, or connecting proof ideas and responding to students' contributions (Lesseig, 2016). KCC is the knowledge of content and curriculum that describes somewhat Shulman's conception of curriculum knowledge. The left side is the subject matter knowledge that is divided into common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). CCK is intrinsically defined as the mathematical knowledge and skill that is used in many other professions or occupations for mathematics. CCK means pure mathematical knowledge (Carrillo et al., 2011). CCK includes recognizing proof through which mathematical knowledge is verified, established and communicated (Lesseig 2016). SCK allows teachers to work in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules, procedures, examinations and understanding unusual solution methods to solve problems (Hill et al. 2005). HCK means an awareness of the relationship between mathematics topics and the curriculum. The MKT measures teachers' abilities to use mathematical knowledge in practice in the teaching processes. The six domains of the MKT are important tools that allow scholars to study factors of how various professional development activities can help develop it. The research on MKT is used to provide a powerful tool for evaluating knowledge used by mathematics teachers in their practice (Nettles et al. 2011).

Professional Learning Community

The PLC means the core task of formal education that is deep learning, not teaching (DuFour 2004, Hargreaves 2007). Stoll et al. (2006, p. 5) defines the PLC as, "an inclusive group of people, motivated by a shared learning vision, supporting and working with each other, finding ways, inside and outside their direct community, examining on their practice and together learning new and better approaches that will enhance all pupils' learning." Kruse et al. (1995) consider characters of the PLC that include: (1) reflective dialogue that helps teachers improve and promote teaching discussion; (2) focus on student learning that is a goal of PLC's activities to improve students' learning; (3) interaction among teachers or deprivation of practice that can engage teachers in sharing ideas, learning and helping; (4) collaboration that is happening when teachers share their teaching strategies, skills and growth; and (5) shared values and norms: partners reach a consensus for mission, value and specifications to build their professional behavior.

There are seven components of effective learning involved in CTL: constructivism, questioning, inquiry, learning community, modeling, reflection and authentic assessment (Yerizon and Putra 2021). Teachers need appropriate environments for their professional growth that can be used effectively to improve creative thinking, critical thinking and teaching skills (Hord 1997, 2004). PLCs should be a place where the principal and teachers are all learners and distributed leadership positively (Hargreaves and Fink 2006). Teachers have the responsibility to promote their teaching skills and students' achievement. The shared personal practice contributes to the development of teachers' professional learning and supports a professional learning community (Hord 1997, Pickering and Garrod 2007). Darling-Hammond et al. (2017) identify seven characteristics of effective PD: (1) is content focused; (2) incorporates active learning utilizing adult learning theory; (3) supports collaboration, typically in job-embedded contexts; (4) uses models and modeling of effective practice; (5) provides coaching and expert support; (6) offers opportunities for feedback and reflection; (7) is of sustained duration. To achieve this shared purpose, participants are encouraged to be involved in the process of developing a clear vision how their collaboration must contribute to their students' learning and effective teaching. They build collective leadership and commitments that clarify the responsibility of teachers' contributions to their teaching and students' learning. Stylianides (2007) suggests that teachers' strong mathematical knowledge for teaching proves their ability to structure opportunities through arguments for their students, which helps teachers improve their weak teaching skills and mathematical knowledge understanding. As through the PLC's operation, teachers need to construct mathematical proofs. The PLC is an entire professional continuum system that supports and links to teachers' experiences in preparation and induction, as well as to teaching standards and evaluation (Darling-Hammond et al. 2017). The PLC is a continuous improvement process and helps teachers to continue their growth in teaching. Teachers are not completely alone on this because partners will help each other and solve teaching problems in the classroom.

The broadening research on PLC is used to provide more information about making MKT a powerful tool for evaluating teachers' mathematical knowledge. The element of MKT framework specializes in the work of teaching practices. The MKT measure that represents classroom, school process and teachers' ability to use mathematical knowledge in the classroom practice (Charalambous 2008). The MKT framework may be described in three ways: (1) as open-ended discussion which allow for the exploration of teachers' reasoning about mathematics and students' thinking; (2) as materials that are used to inform teachers' professional development; (3) as examples of what the mathematical knowledge teacher have to use in teaching (Fauskanger et al. 2012). However, there has been little research focused on examining how teachers' MKT is operationalized in the PLC.

Methodology

The study utilized a qualitative study design that described three teachers' knowledge of MKT in the PLC over the last three years. The research subjects were three primary school teachers in Taiwan. Table 1 presents their background information and pseudonyms. All subject teachers earned master's degrees, but no one majored in mathematics or science. Their teaching experiences are ranged from 15 to 18 years. The teacher PLC had operated for three years while a professor was invited to guide partners to improve their teaching. The purpose of this study discussed participants in the PLC and what teachers' knowledge of MKT over the last 3 years. Subject teachers had dialogue and discussion for teaching 6~8 times and choose to teach a lesson every semester. The study was designed to capture a set of qualitative data of teachers' mathematics knowledge for interviews, lesson observation sheets, reflection and teaching feedbacks.

Table 1. *Background Information of Participants*

Pseudonyms	Gender	Education	Background	Teaching Subject	Teaching Years
AT	Female	M.Ed.	D.P.E	Chinese, Mathematics	18
BT	Female	M.Ed.	Teacher class	Chinese, Mathematics	17
CT	Female	C.A.C.S.	Finance	Chinese, Mathematics	15

Data Collection

Shulman (1986) proposes that teaching requires unique subject-matter-related knowledge, classroom observation became a primary method used to explore this idea. For this reason, the study discussed the participants' professional development for the last three years. The data sources included PLC meeting videos, lesson observation sheets, interviews, and teaching feedback which were analyzed and triangulated by the researchers and other mathematics educators. Before teaching, each teacher in the PLC needed to review lesson design ideas and how to teach and assess students' learning. Other teachers shared their teaching, skills and resources, and advised the teachers with more ideas on how to teach. PLC meeting and classroom observation were video recorded and transcribed verbatim. Semi-structured interviews are widely used in qualitative research. All data can provide limited insight into teachers' MKT, four different types of semi-structured interviews were executed to understand what teachers know and reasons for their teaching actions: (1) background interview- participants ask questions related to their teaching backgrounds, teaching design and mathematical knowledge; (2) pre-observation interview-focus on teachers' planning of the lesson to be observed; (3) post- observation interview-understand each teacher's reflection on the lesson; (4) teaching feedbacks-the participants revisit the lesson and issue the reasons for their teaching decisions and process. The pre-observation and post- observation interviews are carried out in combination with each observation.

Data Analysis

In order to capture the nature and dynamic process of the MKT components in the PLC for the last three years, data were analyzed through two approaches: (1) in-depth analysis of the explicit MKT (Ball et al. 2008); (2) analysis of the MKT elements of teachers' change process. Gencturk (2012) indicates the teachers' MKT improved and increased efficiently change in the quality of their interview, lesson design, mathematical agenda, task choices, and classroom situation.

According to the analysis of interview, observation and PLC meeting data that reveal teachers' beliefs, mathematical knowledge, teaching skills, and instructional practices. In-depth analysis of explicit MKT, was first identified within the documents from the video recordings and interviews, which gave a detailed description of MKT and what the teacher did and how many times the MKT components appeared. The data were used to answer the first research question. Then we analyzed the MKT elements of teachers' change process in the PLC for the last three years. This method assesses the relative extent to which teachers talk about different aspects of the MKT framework within each teacher's data set. In order to integrate the process of the MKT components in a clear way, we adopt an enumerative approach through the in-depth analysis of the explicit MKT (LeCompte and Preissle 1993). Every MKT component was identified in all of the six portions that the mathematical knowledge needed to perform the recurrent tasks of teaching mathematics. The data were coded by two authors to establish the reliability of parsing MKT coding. Inter-reliability was achieved that all agreements were resolved through discussion. This analysis provided direction for further qualitative analysis and supported the identification of the topic through the PLC activities.

Results

Identification of the MKT Domain Classification

Having profiled the theoretical and empirical basis for MKT, we developed the notion further for the proposed measurement. Table 2 shows one to three items from the MKT's six domains that each item was defined (Hill et al. 2008, Ball et al. 2008). Throughout the early conceptualization of these items we discussed how to classify the MKT items. We adopted the classification to measure a teacher's development work and a basis for future discussions about the nature of the teacher's knowledge (Hill et al. 2008). Knowledge of content and students (KCS) combines prior knowledge about students and mathematics. When the teacher chooses the examples, he needs to predict students' ability and assign the task, whether they will find it easy or hard. In order to analyze the teachers' KCS, we define that teachers can intertwine knowledge of how students think about, know, learn mathematical knowledge content (KCS-1), and foresee possible alternative conceptions of students (KCS-2). Knowledge of content and teaching (KCT) combines teaching and mathematics. Teachers design the mathematical task that requires mathematical knowledge for a particular sequence of content for instruction and evaluates the representation used to teach a specific idea and identify different methods and procedures. KCT separate the third components: KCT-1 means the teacher decides to begin the teaching and learning sequence, KCT-2 evaluates the quality of the mathematical presentation, and KCT-3 identifies different mathematical solution. KCC-1 means that a teacher connects the knowledge of content and curriculum. The common content knowledge (CCK) define it as the mathematical knowledge and skills when the teacher writes on the board that he need to use correctly terms and notations. From our data as shown in Table 2, we indicate that CCK-1 means pure mathematical knowledge and CCK-2 diagnose students' wrong answer. SCK is the mathematical knowledge and skills unique to teach. The teacher looks for the students' error pattern and misconceptions. So SCK-1 is used to analyze students' misconceptions and SCK-2 accurately represents mathematical ideas and provides mathematical explanations for common rules. HCK means an awareness of the relationship in mathematics topics and curriculum.

Table 2. *MKT's Domain Classification*

Mkt	Domain	Items	Content
PCK	KCS	KCS-1	Intertwine with knowledge of how students think about, know, learn mathematical knowledge content
		KCS-2	Foresee possible alternative conceptions of students
	KCT	KCT-1	Decide begin of teaching and learning sequence
		KCT-2	Evaluate the quality of the mathematical presentation
		KCT-3	Identify different mathematical solution
	KCC	KCC-1	Connect the knowledge of content and curriculum
SMK	CCK	CCK-1	Pure mathematical knowledge
		CCK-2	Diagnose students' wrong anwser
	SCK	SCK-1	Analyze students' misconceptions
		SCK-2	Accurately represent mathematical ideas and provide mathematical explanations for common rules
	HCK	HCK-1	Relationship in mathematics topics and curriculum

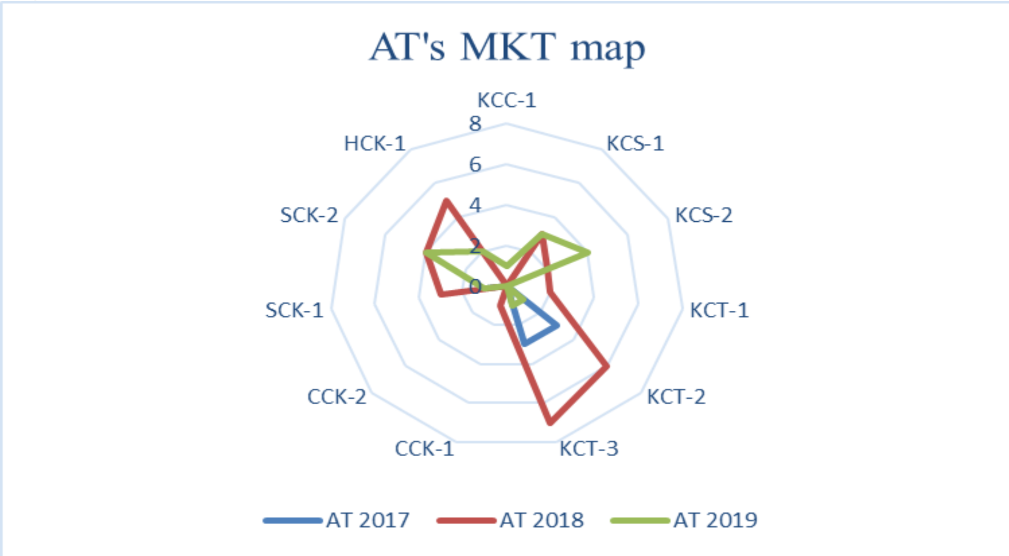
Source: Hill et al. 2008.

Our analysis of the MKT map expressed in the radar chart that revealed the participant's MKT for the last three year and process. The MKT map showed the MKT's domain classification that had a clear vision of the participants. The teachers' MKT map presented their professional development in the PLC that was summarized in Figures 2 to 4. Each teacher's MKT was composed of the frequency for three years. Each item was evident in the subsequent data that we could find out each teacher's teaching change process in the PLC. Each teacher's MKT map differed for their teaching and participation in the PLC. We realized the teachers kept changing through the PLC's sharing and diagnosis of teaching skills and mathematical knowledge to promote the individual teacher's professional development. Besides, we could diagnose the teachers' weak ability and which items could help them to improve their teaching. The analysis was presented to the participants who could realize how to promote their mathematics teaching and correct mathematical misconceptions.

Collecting and Using Data to Prove our Classification

The study collected three participants in the PLC in order to study teachers' mathematical teaching. We argued the six domains of the MKT that could help us to discuss teachers' mathematical knowledge and teaching skills. All data proved whether teachers' growth in our measure is sensitive to their teaching about how students learn mathematics. In the PLC, teachers examined students' computational work for errors, conceptions, explained those errors, and discussed how they remedy them in teaching. They described which problems students used to solve various types of problems and viewed in students solving problems and examined their work.

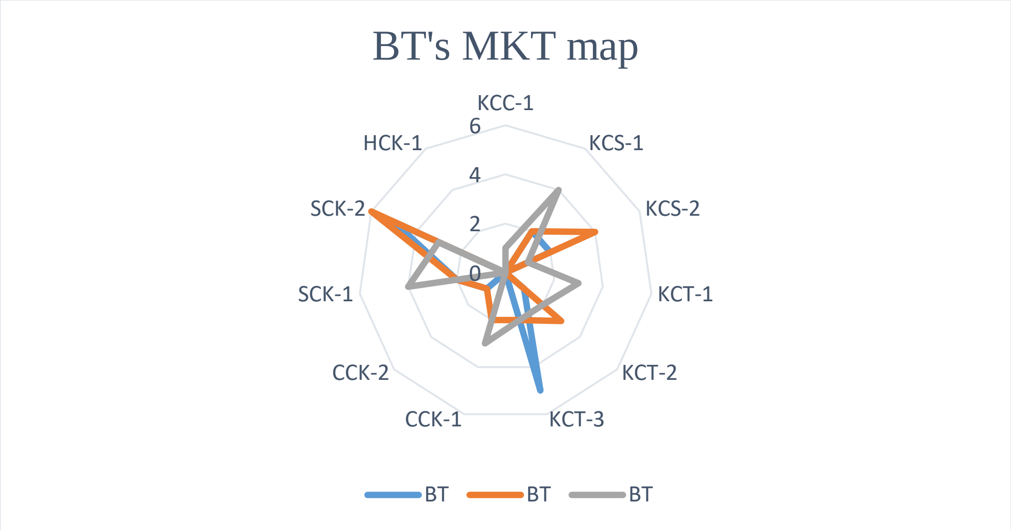
Figure 2. *AT's MKT Map During the Years 2017-2019*



At is a grades 1-2 teacher. At the beginning of the PLC, AT's data was more focused on KCT-2 and KCT-3. KCT is the knowledge of content and teaching that is teaching design in practices and combines knowing about students and mathematics. In 2018, she was aware that her teaching was insufficient that she evaluated the quality of the mathematical presentation (KCT-2), identified different mathematical solutions (KCT-3), analyzed students' misconceptions (SCK-1), accurately represented mathematical ideas and provided mathematical explanations for common rules (SCK-2) and relationship in mathematics topics and curriculum (HCK). AT has a clear vision of the PLC lens that helped her know how to improve her teaching. In the third year, AT focused on intertwining with knowledge of how students think about, know, learn mathematical knowledge content (KCS-1), foresee possible alternative conceptions of students (KCS-2) and accurately represented mathematical ideas and provide mathematical explanations for common rules.

In AT's reflection, because she had mathematics problems in practice, she would find out solutions to help students. AT tried to use more mathematical presentations in practice and students were engaged in activities in which they made the observation, counting activity, problem posing and compared those results with mathematical conceptions. For example, AT taught the topic of "time" using the real clock and calendar that helped the students to realize the concept of "time". In order to help the students to understand the mathematics of geometry, AT designed to understand the meaning of operations that illustrated in a three-layer box through stacking blocks, stratification and drawing out the picture in their vision. We discussed her teaching progress and students' performance that the result was shown. Most students could draw each layer box and counted the total box that could help the students understand which part of the layer box was not seen. Some students cut different directions in longitudinal sections or cross-sections, but they could find out the same results. The results showed the proofs for KCT-2, KCT-3. Evidence of KCT often took the form of teachers describing specific teaching strategies. AT shared her teaching idea that could make students understanding how to find out the invisible box and try to count every box in the second graders. Those proofs of KCS were shown. AT would like to challenge when she had a difficult task. For example, she shared her ideas in which they measured the classroom's window length in the first grade, explanation by students what they thought and measurement methods. Evidence of AT focused on intertwining the knowledge of how students think about, know, learn mathematical knowledge content (KCS-1), with foreseeable possible alternative conceptions of students (KCS-2) that accurately represent mathematical ideas and provide mathematical explanations for common rules. One student could not find the invisible box that AT used in the stacking blocks example to help her to see and understand how many boxes were in the figure. These discussions elicited both CCK-2 and SCK-1. In the PLC, she discussed the elements of mathematical teaching ideas, skills and students' representation made visible.

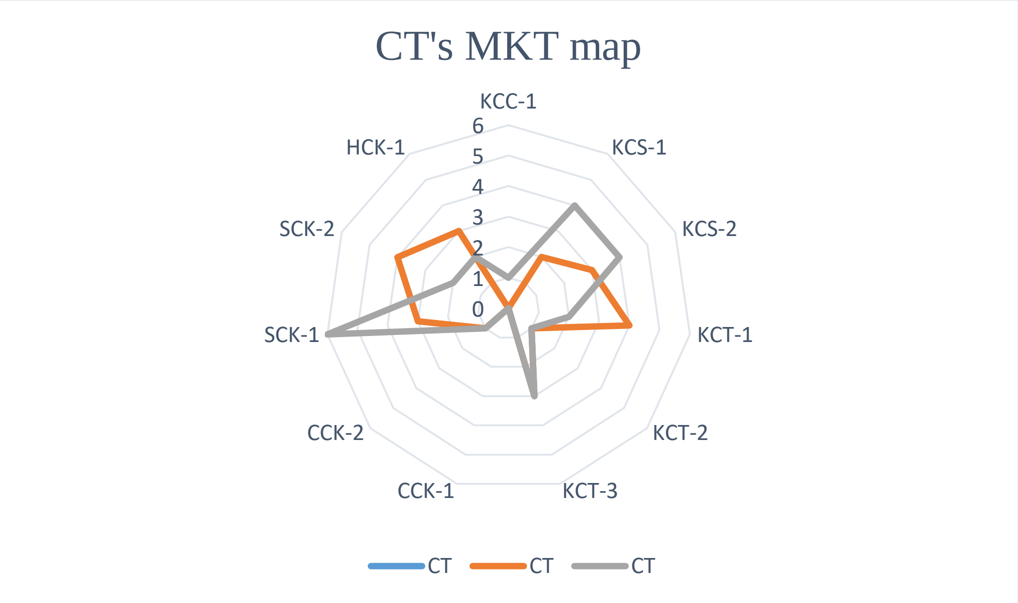
Figure 3. *BT's MKT Map During the Years 2017-2019*



BT was a grades 4~6 teacher. She had positive attitude toward participating in the PLC. She would like to share her mathematical teaching idea and students' misconceptions to help her solve mathematical teaching problems. BT used more representations in mathematical teaching.

In 2017, BT showed SCK-2, KCT-3, KCS-1 and KCS-2. Before teaching, BT used to consider her students' mathematical conceptions to design curriculum and teaching materials. That matches SCK-2. In the second year, BT invested continuously in foreseeing possible alternative conceptions of students (KCS-2), accurately represent mathematical ideas and provided mathematical explanations for common rules (SCK-2) and evaluated the quality of the mathematical presentation (KCT-2). BT improved her quality of teaching and her students noticed. The students reflected BT's improvement of teaching skills and understood the students' misconceptions and preconceptions. BT's teaching made the students enjoy mathematical learning and promoted their achievement for three years. This conclusion encouraged participants invest in the PLC's activities on their initiative. They realized that teachers must be to understand the problems in students' learning to improve teaching.

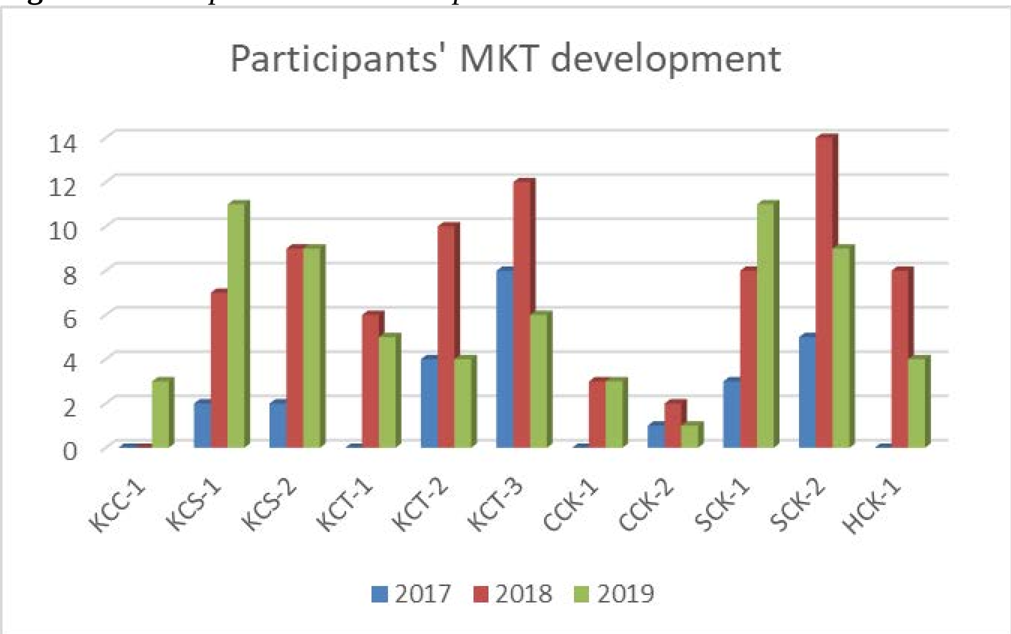
Figure 4. *CT's MKT Map during the Years 2017-2019*



CT was mobilized to this school in 2018, hence she just had two years in the PLC. CT effectively has invested in sharing her teaching, correcting mathematics errors and developing richness of teaching mathematics. CT's map (Figure 4) revealed that CT paid attention to accurately represented mathematical ideas and provided mathematical explanations with common rules (SCK-2), analyzed students' misconceptions (SCK-1), decided to begin teaching and learning sequence (KCT-1), evaluated the quality of the mathematical presentation (KCT-2) and relationship in mathematics topics and curriculum (HCK) in 2018. CT showed more presentations of SCK-1, KCS-1, KCS-2, KCT-3 that revealed CT's understanding of mathematics teaching and willingness to change her teaching mind to prove the teaching practices. SCK-1 had the most amount of the MKT elements. For example, CT analyzed students' misconceptions in counting as shown below: students counted three numbers forward and backward: 20, □, 22. Someone could not find out □ what the number is. CT used number cards to help the first graders understand counting number forward and backward. The proof showed her KCT-1. CT tried to analyze students' misconceptions (SCK-1) and use different mathematical solutions (KCT-3) to clear students' misconceptions. Those helped her identify correct mathematical knowledge. Before teaching, CT would intertwine with knowledge of how students think about, know, learn mathematical knowledge content (KCS-1) and foresee possible alternative conceptions of students (KCS-2). She prepared to understand students' mathematical knowledge background to choose her teaching style and curriculum. Even though CT had only participated in the PLC for two years, she kept moving on developing her teacher professional growth and changed her mind to improve mathematical teaching.

In teaching progress, we found that teacher's MKT was determined by their mathematical knowledge and students' learning. This present study also empirically supported the assertions by showing those three teachers. As shown in Figures 2 to 4, AT, BT, CT demonstrated a more coherently structured MKT map for three years. According to Figure 5, all of the items of participants' MKT development were increased. Evidenced in codes for PLC meetings, classroom teaching and learning, occurred in HCK-1, SCK-1-2, KCT-1-3, and KCS-1-2.

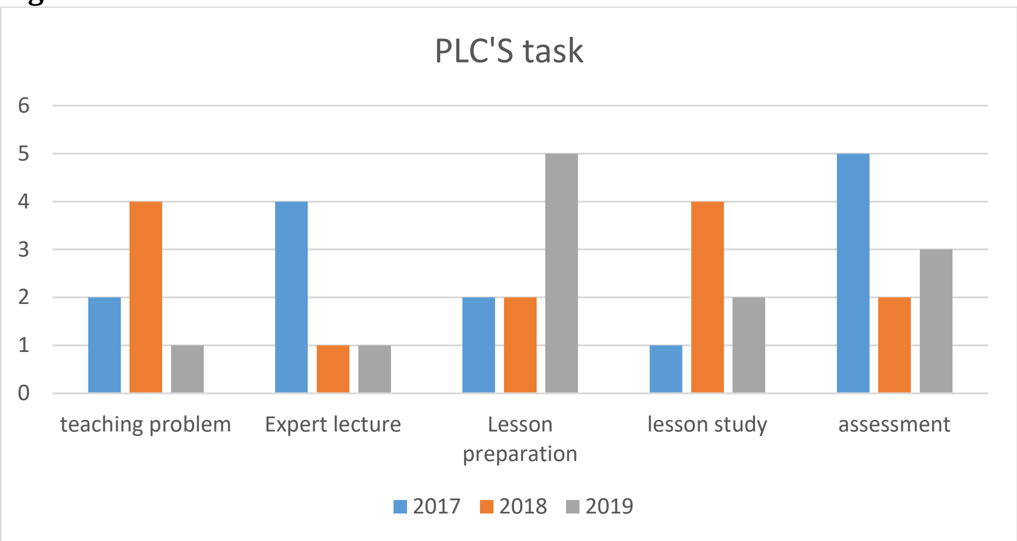
Figure 5. *Participants' MKT Development*



The reason the teachers shared their mathematical problems in the PLC, the expert teacher and professor had advises and resources to help them, they accepted their suggestions then tried those strategies in teaching later. After they tried and tested, the suggestions were proved to be effective. The teachers showed their reflections and the students made progress. The professor and expert teacher encouraged the participants to keep on trying different mathematical knowledge and teaching skills. Figure 5 shows the participants' MKT development for three years. At the beginning of the PLC, the discourse was mainly related to SCK-1, SCK-2, KCT-2, KCT-3 and CCK-1. It reveals that the participants ought to be energized in SCK, KCT, CCK and the PLC activities should be specially arranged in these aspects. After the continuous professional dialogue and teaching practices, the teacher's KCC-1, KCS-1-2, KCT-1-3, SCK-1-2, HCK-1 are improved most significantly, which also promotes the change of the teachers' teaching skills and student learning achievements. It was contributed to the teacher professional development.

As shown in Figure 6, there were five tasks in the PLC. The tasks included teaching problem; expert lecture; lesson preparation; the lesson study and assessment. Because MKT's framework contains the teachers' teaching idea and mathematical knowledge, so assessment is considered in this study. At the beginning of the PLC, the professor and expert teacher provided expert lectures to enrich the teachers' mathematical knowledge and backgrounds. The teachers' mathematical foundations would be set up, because they did not major in science or mathematics. Then the teachers revealed their mathematical problems in the classroom, professor and expert teacher diagnosed the mathematical errors, corrected mathematical knowledge and presented teaching skills to help the participants to understand how to teach. The participants tried the new methods and corrected their mathematical knowledge in practice. Their students were making more progress and liked mathematics more. The researchers have used MKT framework to assess the teachers' mathematical knowledge during classroom teaching and development in teacher preparation programs (Stylianindes and Ball 2008, Steele and Rogers 2012, Stylianindes and Stylianides 2009). The former study was empirically grounded in PLC and investigated the MKT of proof (Lesseig 2016).

Figure 6. *PLC's Task*



In order to verify the teachers' MKT development, we tried to understand the teacher's ideas and teaching skills through the lesson preparation and the lesson study. At the beginning of the semester, all participants shared their lesson preparations, curriculum, and mathematical skills. These tasks could help teachers sort out their mathematical knowledge, teaching progress and evaluate students' learning tools. Lesson study is a common method in promoting teacher professional development. Through lesson study, it will prove the effectiveness of teachers' lesson design, teaching and students' learning. More empirical studies in the PLC are needed to understand mathematical teaching orientations concerning MKT components and the whole construct of MKT in the context of teaching practice. Our PLC tasks provided efficient strategies to promote teachers' professional development.

Discussion

This study adopts a qualitative research method to investigate the change of participants' MKT of a primary school teacher PLC. The results show that PLC may help teachers improve their MKT. Shulman introduced that pedagogical content knowledge inflect pupil's learning. PCK assume teachers' ability to design effective instruction and skills to contribute to students' learning (Hill et al. 2008). Utilizing the in-depth discourse within the PLC meetings, collaborative lesson preparation, peer lesson observation, and analyzing exam items, the participants could transfer, the sharing resources to their own teaching practices, and manage to learn actively. In Taiwan, many teachers are not majoring in science and mathematics. They used to use e-book by the curriculum vendor to teach mathematics. Teachers have weak abilities in lesson design and mathematical knowledge.

In order to promote teachers' efficient actions in mathematical teaching, we adopted the tasks including teaching problem, expert lecture, lesson preparation, lesson study and assessment. At the beginning of the PLC, the discourse was mainly related to the KCC and KCT. It reveals that the participants ought to be energized in KCC and KCT, and the PLC activities should be specially arranged in these two aspects. After the continuous professional dialogue and teaching practices, the teacher's KCC, KCS, and SCK are improved most significantly, which also promote the student learning achievements. This suggestion has been supported by other empirical studies (e.g., Hill et al. 2008). The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations (Shulman 1986, p. 7). As shown in BT's MKT map, she held a strong didactic direction to support Shulman's study. In this issue, the MKT map approach was developed that could explore various research questions about MKT. The MKT map can be used as a reflection tool to identify which components they need to improve for teaching more effectively. Through the PLC, teachers have more partners to share their teaching idea and solve their mathematical teaching problems practice. We have more data to realize teachers' MKT map that can explore more directly concerning mathematical knowledge domains. The result can suggest the orientations to the participants' professional development.

Conclusions

An effective teacher has a sense of the potential that their students possess and encourages their students to excel by providing the motivation to push the student to make sustained efforts when needed (Stronge 2018). In order to improve mathematics teaching from the result of our analysis of the MKT map, we have several conclusions. First, PLC is helpful for teachers to promote their teaching. Through the PLC, teachers were diagnosed by other participants; they focused on teaching skills and correctly mathematical knowledge to help them solve problems in practice. Second, SCK-2 was a max item that reveals the participants would like to accurately represent mathematical ideas and provide mathematical explanations for common rules. Teachers would like to change their minds on mathematics teaching and discuss how to achieve effective teaching, even not in the PLC period that shows collaboration happening to participants. Third, we discuss three teachers participating in the PLC for three years. If there are more participants in the PLC, they share the same vision to improve mathematical teaching that can elevate other primary school teachers. If the research methods can explore more PLC, it must be help for more teachers in teaching mathematics. Forth, the PLC operation will be work that suggests participants including professors or professional except teachers. The professor can provide more resources, mathematical knowledge and theories to help other participants' development. In summary, our study suggests MKT map prove theoretically productive and empirical studies to help more mathematics teachers. The result informs our understanding of the MKT map in practice and influence teacher professional development in the PLC.

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