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Metacognitive Failures of Preservice Mathematics Teachers ın Problem Solving

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Metacognitive success is one of the factors that positively affects problem solving skills. Identifying metacognitive failures in the problem-solving process is also important in recognizing the factors that will inhibit metacognitive success. In this study, it is aimed to reveal metacognitive failures of pre-service mathematics teachers in the process of given mathematical problems. The present research on investigated metacognitive failures of pre-service teachers in the process of problem solving is modelled as case study. Data collection was carried out in clinical interviews were conducted with the preservice teachers who were predicted to obtain rich data in accordance with the purpose, using the "think aloud" interview technique, among these pre-service teachers. As a result of the analysis of the data and field notes obtained from the clinical interview voice recordings, 8 different metacognitive failure behaviors were encountered; including "metacognitive mirage" two times, "metacognitive blindness" three times and "metacognitive vandalism" three times.

Keywords: *metacognitive failure, preservice teacher, mathematics education*

Introduction

In contemporary education system, it is important for the students to reach the right information by doing research, to use this information, to manage their own mental process and to gain high level mental skills. In order to be successful individuals in academic and social field, students must have be aware of their own learning style and develop appropriate learning strategies. Metacognition is also important as the knowledge acquired by the student about his own learning. In the previous studies with students in elementary school, the use of metacognitive skills in problem solving process have been investigated in quantitative and qualitative ways (Aydemir and Kubanç 2014, Jacobse and Harskamp 2012, Swanson 1992, Şengül and Işık 2014). According to the results obtained from these studies, the students with metacognition successfully exhibited metacognitive behaviors in the process of problem solving. On the other hand, the relationships between metacognition and problem solving success have been also conducted with secondary and university students (Bakioğlu et al. 2015, Başol et al. 2014, Kapa 2001,Kiremitçi 2011, Yıldırım and Ersözlü 2013). According to the results of these studies, a significant relationship has been found between metacognition and problem solving success. In addition, gaining metacognitive skills to students increased the success in problem solving.

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In contrast to above researches, in this study, it is aimed to reveal metacognitive failures of pre-service mathematics teachers in the process of given mathematical problems. In mathematical metacognitive processes; Metacognitive failure situations have not been studied in different forms, except for not using useful information and lack of control behavior (Goos 2002, Stacey 1992). As explained below, defining metacognitive failures is important in mathematical problem solving processes. Also, nearly any study has been conducted about this subject in literature except Goos (1997, 2002), Ng (2010), Stillman (2011), Huda et al. (2016, 2018), Surya (2019) and Faradiba and Alifiani (2020) in which the term "metacognitive failure" is defined.

Literature Review

Metacognition

Conceptualizations of metacognition have expanded over time within educational psychology and mathematics education. Despite the extensive theoretical and empirical literature on metacognition, there is no consensus definition of the construct. Metacognition was first defined by Flavell (1976) as "one's knowledge concerning one's own cognitive processes and products or anything related to them" (p. 232). Flavell (1976, 1979) described several aspects of metacognition, including metacognitive knowledge and metacognitive experiences, as well as the monitoring, regulation, and orchestration of cognitive processes. Soon after Flavell (1976) introduced the term, metacognition, Brown (1978) reviewed existing research on related phenomena, describing several aspects that later came to be recognized as aspects of metacognition, including planning, checking and monitoring, and knowing when and what you know. In summary, metacognition refers to the psychological structures, knowledge, events and processes that are involved in the control, modification and interpretation of thinking itself (Wells and Hatton 2004). A good understanding of the concept of metacognition is possible by explaining its relationship with the concept of cognition. Cognition is defined as the functions and working processes of the brain used in the mental activities of individuals such as attention, perception, understanding, interpretation, discrimination, making sense of information, and reasoning (Bacanlı 2002, Ömeroğlu and Kandır 2005). During the realization of these tasks in cognitive processes, the responsibility for managing the tasks belongs to metacognition. Metacognition refers to a series of processes that an individual uses to monitor ongoing cognition in order to effectively control his or her behavior (Desoete and De Craene 2019, Rhodes 2019, Veenman et al. 2006). Metacognitive activities that occur in the individual occur either before cognitive activities or during cognitive activities. Metacognition includes information about the strategies that an individual uses to fulfill cognitive tasks, as well as self-monitoring and evaluation skills while performing these tasks (Desoete and Veenman 2006, Schoenfeld 2016). When the studies on metacognition are examined (Aşık and Sevimli 2015, Özsoy 2011), metacognition is structurally under two headings as metacognitive knowledge and metacognitive control.

Metacognitive knowledge refers to an individual's knowledge, strategy, beliefs, and cognitive awareness of a task or problem situation. Knowing the solution strategies and solution ways of how to solve a problem correctly, that is, knowing the procedures of a task and being aware of the situation of being able to do this task is within the scope of metacognitive knowledge. Although Flavell (1979) suggested that person, task and strategy variables constitute metacognitive knowledge, Paris et al. (1984) argued that metacognitive knowledge can be organized as declarative, procedural and situational knowledge. These three types of knowledge have often been discussed and expanded upon in later metacognitive studies. In general, metacognitive knowledge refers to individuals' awareness of their own knowledge of their strengths and weaknesses, and includes tasks, strategies, and knowledge related to the achievement of a particular task.

Metacognitive control is defined as an individual's ability to use his/her metacognitive knowledge to fulfill cognitive tasks and to manage cognitive processes (Desoete et al. 2019, Flavell 1979, Özsoy 2008). Metacognitive control skills were examined under four headings: estimation, planning, monitoring and evaluation. It shows that metacognitive control skills have an important place in the learning processes of the individual, as he monitors and evaluates the cognitive processes of the individual and can organize these processes by choosing appropriate strategies according to different situations.

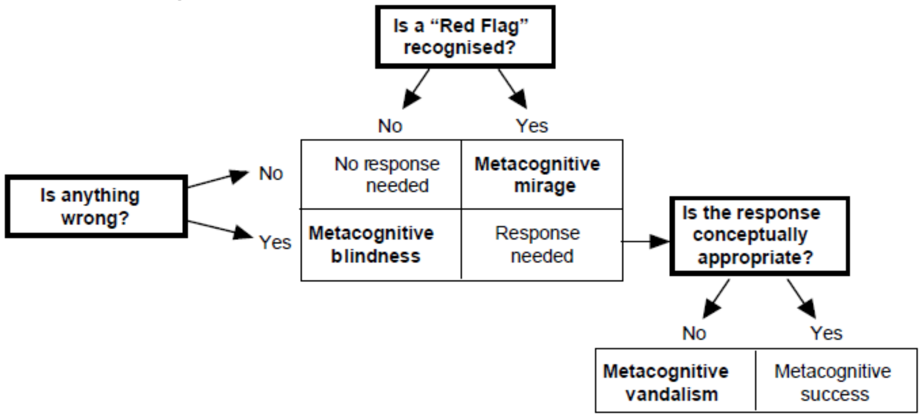
The frameworks outlined by researchers in educational psychology are very similar to the processes of mathematics problem solving as described by Polya (1945), and Garofalo and Lester (1985). The works of these mathematics researchers and educators contextualize regulation by studying how students regulate their thinking during problem solving situations. Previous studies investigating the impact of metacognition on problem solving have shown that people with metacognitive skills perform better in problem solving environments (Balcı 2007, Özsoy 2008, Pilten and Yener, 2010, Bagçeci et al. 2011, Oğraş 2011, Memnun and Akkaya 2009, Kanadlı and Sağlam 2013, Aydurmuş 2013, Azak 2015, Yıldız and Güven 2016, Kaplan et al. 2016, Demir 2016, Lester 1994, Lester et al. 1989). However, as much as the presence of metacognitive behaviours is crucial for favourable problem solving outcomes (Hessels and Hessels-Schlatter 2010), researchers have found that the quality of the nature of metacognitive interactions (Stillman and Galbraith 1998) is just as important.

Metacognitive Failure

The term "metacognitive failure" is firstly defined by Goos (1997, 2002). This term indeed is related to "red flags". Metacognitive red flags can occur at critical junctures where the problem solvers are faced with important decision making pertaining to the success or failure of their attempts. Thus, purposeful, conscious, and at times drastic actions (e.g., pausing for reflection, backtracking, re-doing the problem in another way) may be warranted to change problem solving pathways. Nonetheless, subsequent metacognitive regulatory behaviours (or the lack of them) in reaction to red flag situations also play a large role in savaging or sabotaging the problem solving situation. Goos (1997, 2002) identified three red flag situations in her study of group collaborative problem solving process: (a) lack of progress, (b) error detection, and (c) anomalous or strange results. According to Goos (1997, 2002), red flag situations are distinguished from routine monitoring behaviours (e.g., assessment of knowledge, approach, outcomes) which served to confirm that the problem solving process is on the right track.

Goos (2002) identified three types of metacognitive failures displayed by problem solvers in reaction to red flags. These are described by the metaphors of “blindness”, “vandalism”, and “mirage”. Metacognitive blindness occurs when a problem solver did not notice his or her likelihood of impending failure in solving the problem, opting for instance to continue with an inappropriate approach. Metacognitive vandalism comes into play when problem solvers decide to take destructive action to deal with a deadlock situation (e.g., changing the conditions of the problem so as to suit the fixed mindset of the problem solver). Metacognitive mirage takes place when problem solvers mistakenly change course of actions upon perception of difficulties which in fact do not exist. These metacognitive failures types is schematized in Figure1.

Figure 1. *Metacognitive Success and Failure Scenarios*



Source: Goos 2002, p. 9.

Methodology

Research Design

The present research on investigated metacognitive failures of pre-service teachers in the process of problem solvingis modelled as case study, which is a qualitative study method.

The case study explores a case or event that the researcher cannot control based on the “how” and “why” questions (Şimsek and Yıldırım 2011). From this point of view, case study methodology was found suitable with the aim of the current research because the aim of this study is to investigate the behavior of pre-service teachers in the process of problem-solving according to the metaphor defined by Goos (2002) and to reveal these behaviors 'how' and 'why' occurred.

Participants

The study group of the research is 5 second grade preservice teachers from Kastamonu University, Department of Elementary Mathematics Education. The study group was selected in accordance with criterion sampling, which is a purposive sampling method. Purposive sampling enables the studying of cases, which are thought to have rich information (Patton 2002). The criterion for the selection of these pre-service teachers to participate in the present study was they are considered to be sufficient for the problems to be given because they completed the required courses and additionally they took course in "problem solving and posing" last semester.

Data Collection Tools

The data collection tool was prepared for requiring knowledge and reasoning skills. In order to prepare, firstly, some problems about metacognition and metacognitive failure in the literatüre were examined. Two expert opinions were taken from the field of mathematics education to determine whether the data collection tool are valid and reliable. As a result of the evaluation of these opinions, the data collection tool consisting of 4 open-ended problems was prepared.

The data collection tool was applied to 5 pre-service teachers in clinical interview sections. They were asked to explain the problems they had in the solution processes, explain the reason of the strategy they chose and not delete them when they thought they made a mistake, but instead continue explaining why they changed their minds. Clinical interviews give opportunity to "enter the persons' mind" considering individual difference and their mathematical understanding (Newell and Simon 1972). For this purpose; the participants were expected to solve the problems in the text by using 'think aloud' method (McKeown and Gentilucci 2007). When the thoughts of the pre-service teacher are not explanatory, the researcher asked the leading questions like"Can you explain what you understand about the problem?", "Why did you choose this strategy?" and "What do you think about your solution? Can you explain it?". During the clinical interview, voice recording and field notes were taken.

Analysis of Data

For the data analysis; data tool collection, voice recording and field notes was analyzed based on descriptive analysis. In the descriptive analysis, the data is first described systematically and clearly (Çepni 2014). Afterwards, descriptions are explained according to the conceptual framework of the research, interpreted and cause-effect relations are examined and some conclusions are reached (Çepni 2014). In the research; the data were analyzed according to the metacognitive failure structures, which is described by Goos (2002), whose themes were is “metacognitive blindness”, “metacognitive mirage” and is “metacognitive vandalism”. In terms of the confidentiality of the research, clinical interviewed participants in the study were coded as PST.1, PST.2, PST.3, PST.4 and PST.5.

Ethical Considerations

The real names of the pre-service teachers participating in the research were hidden and each of them was given a pseudonym. With the permission of the teacher candidates, the interviews were recorded on a voice recorder. After the interview, the transcription of the records was completed and the transcripts were sent back to them for review and approval.

Findings

In this section, the findings obtained in the research are included. As a result of the analysis of the data and field notes obtained from the clinical interview voice recordings, 8 different metacognitive failure behaviors were encountered; including “metacognitive mirage” two times, “metacognitive blindness” three times and “metacognitive vandalism” three times. As stated in the research problem; in this research, it was aimed to show in detail how and why the pre-service teachers exhibited metacognitive failure behavior in the process of problem solving. Accordingly in the findings of the research, one for each the sample presented and discussed about each different metacognitive failure behavior.

Metacognitive Mirage

The first open-ended question where PST1 has encountered a metacognitive failure is as follows:

> *Problem1: You are given101 identical-looking balls and a two-sided scale. One of the balls is of a different weight, although you don’t know whether it’s lighter or heavier. How many weightings of the scale at least you must use to determine whether it’s lighter or heavier?*

The process of solving this non-routine problem that PST1 had never met during the interview is shown below:

> *PST1: So, it sounds like two. At least ... is the probability that it will be the least? (02: 00)*
> *R: How many times will be measured but for a final result?*
> *(She thinks again on the question)*

While PST1 was first reading the question, she considered the problem as a probabilistic problem but gave up the idea with the use of the exact result of the researcher.

R: Tell me what you understand about the question. (03: 12)
PST1: There are 101 balls. One is different as weight, but the image is the same and we need to measure it with a scale.
R: What is asked?
PST1: How many times we need to be weighed in order to find out for sure. 51… I say then
R: How did you find it?
PST1: I make group of 50 balls divided into two groups. 50 as I have 50 measurements in that way, then the last one of the measurements I've made a measure before I thought I would find this way.
R: Do you want to think again? The question is that you are not asked to find the ball itself, only to find out whether it is heavy or light.

PST1 gave a wrong answer by choosing the wrong strategy at 04:00. While she only needed to determine whether the different balls were heavier or lighter than the others, she tried to determine the ball itself by applying 51 measurements. Again, she gave up the way to solve the problem again by repeating the desired.

PST1: Ok ok! I found it. I can split any ball, for example 50 to 50 measurements. Either 50 different balls or one ball will be left out. For example, the ball outside will be different when it is equal. I take one ball from the group (50 balls in the balance) and put the other ball (the ball outside) and I think I'll find it that way. (07: 30) (new idea)
R: So what happens when it's not in balance?
PST1: Can we tell if it is light or heavy? What if we took one of the pans and put them in 25 and then weigh them… but how can we understand from here? (new idea)
[Thinks about the problem situation for a while. (09: 54)]
PST1: We cannot detect it
R: Why?
PST1: For example, the pan is out of balance. Either there is a light ball here (above the scale of the scale) or heavy (here below the scale of the scale). We can't determine which one.

At 07:00, PST1 put forward a "new idea" that could lead to the solution of the problem. In her strategy on 2 probabilities, she followed a correct way of thinking in case of probability (2 weighing operations to determine if the ball is heavy or light).In the event of a possibility, although she was actually on the right track, she could not decide how she could make a determination from there and felt that her strategy was wrong. After thinking for a while;

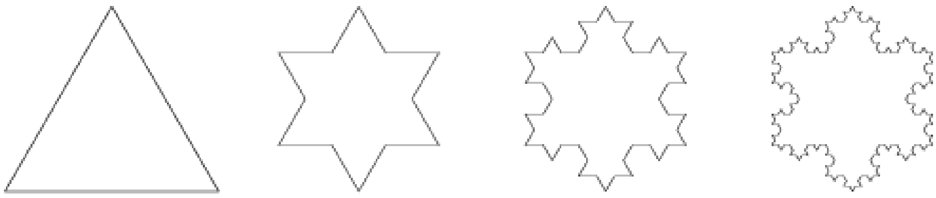
PST1: Then I leave this solution. The first thing I'm saying is 51 because the same thing will come out here.
R: Are you sure about the answer?
PST11: I'm not quite sure, but that's the way I do it.

PST1 thought that she had encountered an anomalous result that is "red flag" "contradictory" warning sign that she had been dealing with a problem she had not encountered before and was not sure of what she understood and knew about the problem. The strategy she was using was unreasonable and did not meet the conditions of the problem. She abandoned the useful strategy to solve the problem. Afterwards, she failed to provide the correct answer to the problem by continuing her old strategy which was wrong. PST1 exhibited a “metacognitive mirage” by detecting a red flag warning sign that does not exist even though there was no error in the problem solving and therefore abandoning a strategy suitable for the solution in the solution process.

Metacognitive Blindness

In this section, the solution process of PST2 in a pattern problem (Figure 2) is discussed. In the question, Koch snowflake pattern was used and the shapes and fields formed in the first two steps were given and the area of the next step and the number of triangles formed in the iteration of k and the area of one of these triangles were asked.

Figure 2. *Koch Snowflake Pattern*



Source: Goos 2002, p. 9.

The solution process of PST2 in the shown pattern problem is as follows: When PST2 encountered the problem, he thought about the problem for about 3 minutes.

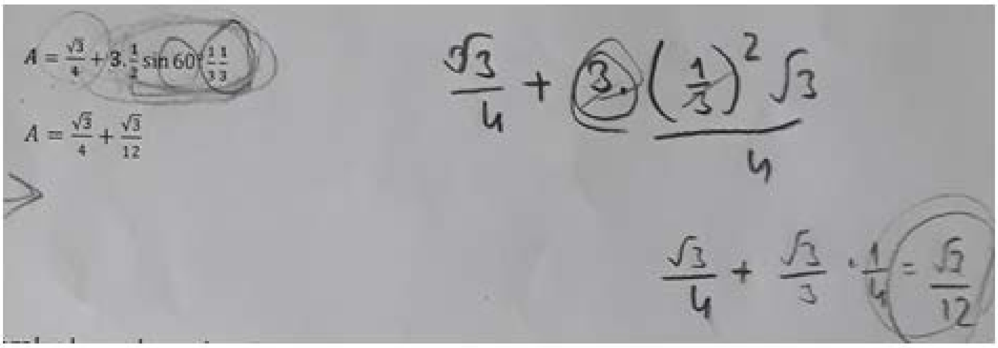
R: What do you understand from the question? Can you tell me?
PST2: It thought as a pattern. In step 3 this field will already be. Other than that, I need to find the last addition. [After a little more thought at 04:00.]
PST2: It would be ridiculous, but I didn’t understand where this 12 came from. (Number of triangles added in Step 2)
R: You need to find it.

When he first read the question, he noticed that there was a pattern question and that the area of the previous step was preserved from the text given in the question. But because he didn’t carefully examine what was given in the steps in the question, he didn’t realize that 12 in step 2 was the last 12 added triangles. If he had studied more carefully what was given in step 1, he could have noticed that three triangles were added and the coefficient 3 in the field formula would speed up the solution process.

PST2: “$a^2 \sqrt{3} / 4$” is already the formula of the area of the equilateral triangle. So here he is I thought it could be used, I did not fully understand why it used the sinus formula.
R: You can do whatever you want.

PST2, by applying the area formula, has reached the same conclusion as given in the question (Figure 3).

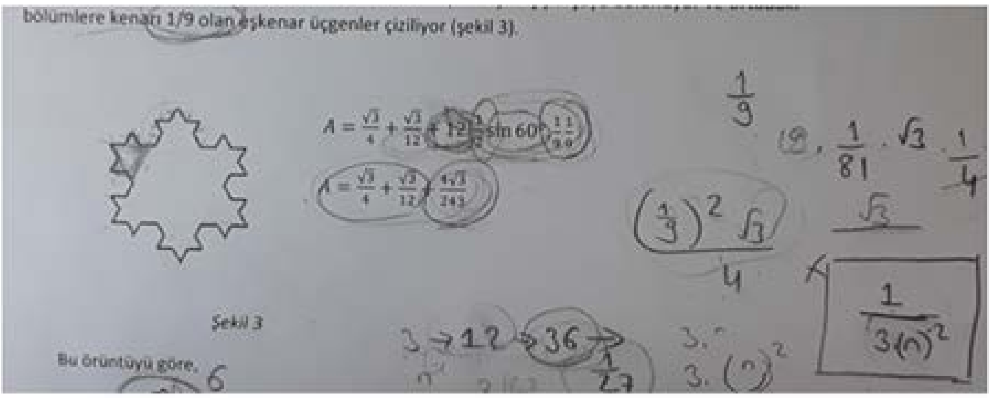
Figure 3. *The Calculations of PST2*



PST2: The pattern will now be 1/3, 1/9 and 1/27. In other words, $1/(3.n)^2$is used in the pattern (07: 00)
R: What is this formula you found?
PST2: The formula in which the pattern advances If $a = 1/3^n$ then I think 1/27 would be in the third step.

When calculating one edge of the triangles formed in each step (a = (1/3) ^ n) PST2 made an error applying to the field formula (Figure 4). He did not notice that the formula found was incorrect because it did not check whether the general formula it found provided the previous steps (error detection). PST1 then tried to find the area of the shape formed in Step 3.

Figure 4. *The Calculations of PST2*



R: What do you think?
PST2: It had given the same result before (a^ 2 √3 / 4). Like it gives again.
R: How?
PST2: In the previous example, we found equilateral triangles with 1/3 edge. We also find triangles that are 1/9 with the following formula ($(1/9)^2$) √3 / 4) (evaluation)

He applied his formula and saw that the result was different from the one given in the question.

PST2: Oh, that didn't come out, actually.

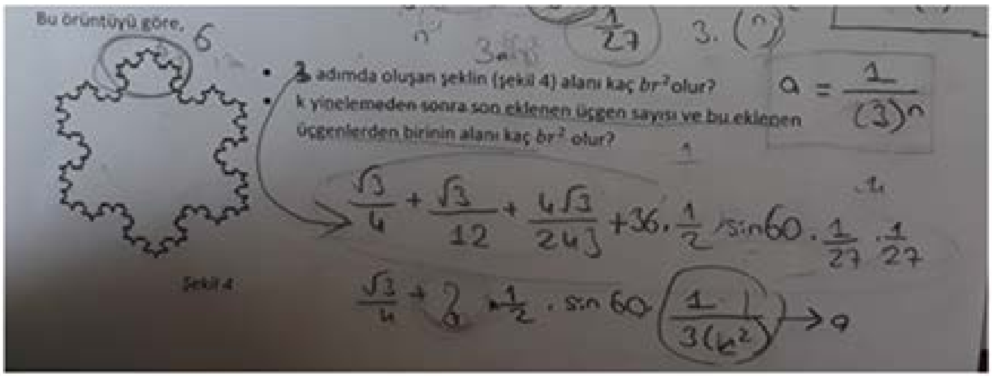
The first time he applied the equilateral triangle field, he reached the correct result because he unknowingly multiplied the area by 3, but when he found the area of a single triangle in Step 2, he encountered an "anomalous result" and realized that he needed to "error detection".

PST2: How many of these triangles do I have that have a triangle whose edge is 1/9? 18 yes I used the number of triangles here (the solution he used for step 1).
R: How did you find 18 triangles? (Evaluation)
PST2: So there were 3 triangles, 3 triangles were made again, I thought it would be 18 because it has 6 corners; but I could not make sense of the relationship between 3 and 18. (evaluation)

After thinking a little [at 17:00];

PST2: Actually, I always thought it was 3 triangles, but it's part of the big triangle, these two triangles are added later. 12 triangles. Then… (doing its operations) .. here (Step 3) 6-12-18… 36 (new idea)
R: How did you find it?
PST2: now that 2 triangles are added to each corner, and that's where the 12 comes from. Here again 2 of these triangles are added to the edge and there are 6 in each corner is 36 I found here. The shape of the shape formed in step is in this way. (Figure 5)

Figure 5. *The Calculations of PST2*



PST2 reassessed the problem solution because he did not find a relationship between the triangles numbers 3 and 18, and realized red flag "error detection" checking for error, he found that the number of triangles added in the figure was actually 12 and showed a successful metacognitive movement. Then, when he calculated the number of triangles formed in step 3, he thought that 48 triangles should be added because he thought that only the last formed triangles were divided into three identical parts, not the whole edge of the shape and 36 triangles would be added. If he had followed the steps in the figures, he would have achieved the right result.

PST2: I'm trying to figure out what these 3 12 and 36 patterns look like, but it doesn't, unfortunately (lack of progress) (22: 00)
After considering the question for a certain time;
PST2: Sorry I couldn't find it
R: Is the number of triangles added?
PST2: Yes. I couldn't find the pattern of the added triangle number.
R: Why? Where could you have made a mistake?
PST2: I don't know I couldn't relate
R: Well you couldn't find the number of triangles added. What could be the area of one of the last added triangles?
PST2: I need to find it here, so I need the coefficient at the beginning (number of added triangles) I can't find it either

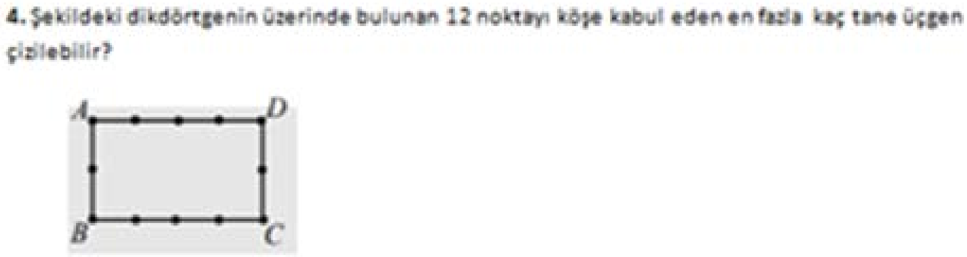
PST2 gave an incorrect answer in the first stage of the problem because he found the number of added triangles 36 in step 2, which he found during the solution process. Then he could not find the number of triangles that would occur in the iteration because he could not find a relationship between the number of added triangles (3 and 12). At this stage, PST2 encountered a "lack of progress" which is red flag but ignored the "error detection" that it had successfully provided in the previous solution steps. If he had noticed this red flag and checked his operation and re-examined the given data (for example the figure in step 3), he would have been able to detect the error at this stage and successfully solve the problem.

In the previous process, PST2 also incorrectly created the field formula of the triangle and did not validate the general formula it obtained and ignored the red flag of error detection. In the second part of the problem, he thought that he could not solve the question because the number of triangles added to find the last added triangle area did not need to be known. It has been observed that PST2 has "metacognitive blindness" behavior due to the fact that it does not notice the red flags that appear at different moments (at 7:00, 17:00 and 22:00) and that it should review the problem process again and therefore fail to solve the problem.

Metacognitive Vandalism

This section discusses the solution process of a combination problem (Figure 6).

Figure 6. *The Combination Problem (Question in English: How Many Triangles can be formed by using 12 Points as One of Their Corners)*

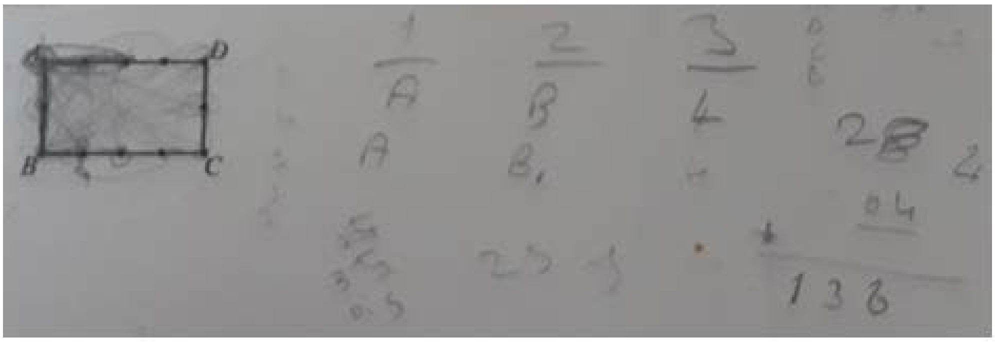


The solution process of PST3 in the illustrated problem is as follows:

PST3: I often find these questions counting quickly, but this may not be the question.
R: Do you want to apply?

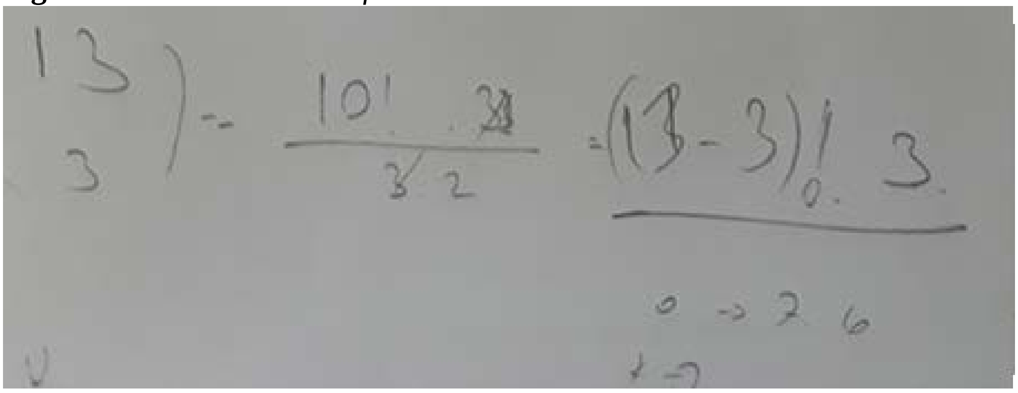
ST3 tried to solve the question by using the counting method, but as he said, the counting method was not an appropriate method for this question. At the end of the solution, the PST3 encountered an “anomalous result” (Figure 7).

Figure 7. *The Calculations of PST3*



R: The result you found is not correct. As you said, the counting method is too long for this question. Can you solve this question any other way?
PST3: I can do anything. I get all three states in combination, but those that don’t work for us (linear dots) take them out of all these three. (new idea)
R: How to do?
PST3: I have 13 points. 3_combinations of 13… something like this (Figure 8)

Figure 8. *The Calculations of PST3*

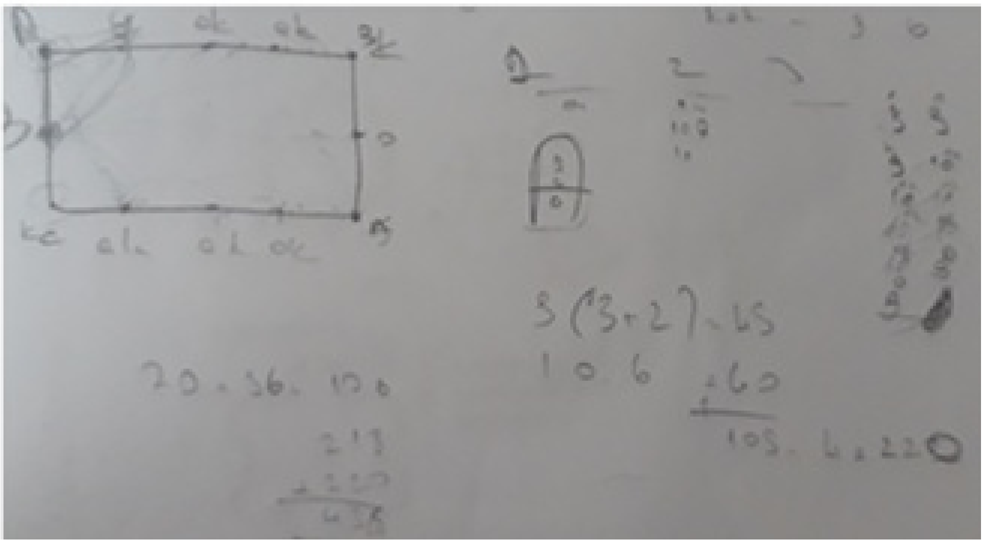


When searching for a new solution for the question, PST3 proposed a "new idea" that could lead him to the right answer, but because he did not examine the question carefully. He took the 12 points given as 13 points and made a wrong application because he remembered the combination formula incorrectly. As the result of his transaction would be a very large number, PST3 was unable to continue the operation and was encountered a lack of progress (at 03: 28).

> *R: Are you sure you are doing the right operation?*
> *PST3: No, I made a mistake somewhere. It wasn't supposed to happen. (reviews) I'll do it again by counting.*

PST3 recognized the red flag "lack of progress" at 03: 22, but when he "error detection" he decided to apply the method he had previously tried and failed instead of trying to understand the question and detecting the errors in the solution. He decided to apply the counting method with reference to different corners this time, after a long period of struggle, 13:55 minutes reached the following solution (Figure 9).

Figure 9. *The Calculations of PST3*



R: Your answer is not correct again. Where could you have made a mistake? Could you use another solution?
PST3: I don't know, but I would try to find it again by counting.

PST3 knew that counting would not be the right strategy when faced with the problem and did not reach the right result when he first used it (Figure 7). Then, the preservice teacher who noticed a "new idea" using the right strategy did not read the question carefully and misused the combination formula once again caused a faulty solution (Figure 8).

Even though he noticed the "lack of process" red flag indicating that something was wrong in his solution, he chose to use the counting method, where the probability of finding the right result was rather poor, rather than giving correct feedback and correcting the wrong points. As a result, it was seen that he noticed red flag but could not give correct feedback and because it was easier for him to solve the problem, he chose the wrong strategy and displayed "metacognitive vandalism".

Conclusion and Implications

As a result of the data analysis obtained from the study, preservice teachers may display some metacognitive failures in problem solving sessions where they over-looked a "red flag" indicating a calculation error (blindness), responded to a lack of progress "red flag" by imposing an irrelevant conceptual structure on the problem (vandalism), and imagined an anomalous result "red flag" in mistakenly rejecting a correct answer (mirage). Especially they mostly show metacognitive blindness when they are weak in metacognitive problem-solving behaviors such as carefully analyzing the instructions given in the problem understanding what is desired in the question and using the right strategy. The metacognitive blindness exhibited by PST2 was caused by ignoring the "error detection" red flags in the second stage. This result is similar to the findings of the studies in the literature (Goos 2002, Huda et al. 2016, Surya et al. 2019). The metacognitive mirage behavior occurs when they work to solve a problem that they have not encountered before, that the solution does not know exactly the appropriate strategy and that they do not have sufficient level of judgment to prove the correctness of their solutions or strategies. In the example of PST1, the candidate thought that there was an "anomalous result" warning sign or red flag in his answer because he was not sure about the solution strategy that could reach the correct result, and exhibited a "metacognitive illusion" by abandoning the useful strategy. This finding is similar to the result of the study conducted by Goos (2002) in which metacognitive successes and failures based on collaboration were examined with high school students. Metacognitive vandalism behavior occurs mostly when they are trying to solve the problem with similar problems in the past by trying to apply the strategy to solve the problem in order to facilitate the solution to the current problem or even though he is not sure of the appropriate strategy because he is not sure of the appropriate strategy. In the PST3 example; although the red flag "lack of progress" caused by the wrong solution strategy was noticed, not being inclined to the correct solution strategy caused the continuation of the strategy that was not suitable for the solution of the problem. As a result, the preservice teacher did not get the correct answer. The result obtained in terms of giving devastating feedback to the "Lack of progress" red flag is Huda et al. (2016) and Goos (2002) can be said to be in line with the findings.

Huda et al. (2018) stated that lack of metacognitive assessment may cause metacognitive failure. In this sense, handling a similar study together with the metacognitive assessment dimension may provide richer results regarding the causes of metacognitive failure. Finally; in the study of Goos (2002), a study in which metaphors of metacognitive failure were introduced, was conducted with collaborative groups. A similar study to be carried out with pre-service teachers can reveal whether the cooperative learning environment will prevent metacognitive failures.

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