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Anisotropic Strange Stars with Nonlinear Equation of State

By Manuel Malaver[*]

The modeling of superdense matter is an interesting research area and in the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars and white dwarfs. Taking local anisotropy into consideration, in this paper we present a new classes of solutions for the Einstein´s field equations in a spherically symmetric spacetime under the influence of an electric field considering a quadratic equation of state with a particular form the metric potential that depends on an adjustable parameter. The obtained solutions can be written in terms of elementary functions, namely polynomials and algebraic functions. The graphs generated show that physical variables such as metric potentials, radial pressure, energy density, charge density, anisotropy, radial speed sound are consistent with realistic stellar models. The results of this research can be useful in the development and description of new models of compact structures.

Keywords: *quadratic equation of state, Einstein-Maxwell system, metric potential, adjustable parameter, compact structures*

Introduction

The search of new solutions for the Einstein-Maxwell field equations is an important area of research because it allows describe compact objects with strong gravitational fields as neutron stars, white dwarfs and quark stars (Bicak 2006, Kuhfitting 2011). Within this context it is appropriate to mention the findings of Delgaty and Lake (1998) who constructed several analytic solutions that can describe realistic stellar configurations and satisfy all the necessary conditions to be physically acceptable. These exact solutions have also made it possible the way to study cosmic censorship and analyze the formation of naked singularities (Joshi 1993).

In the development of the first stellar models it is important to mention the pioneering research of Schwarzschild (1916), Tolman (1939), Oppenheimer and Volkoff (1939) and Chandrasekhar (1931). Schwarzschild (1916) obtained interior solutions that allows describing a star with uniform density, Tolman generated new solutions for static spheres of fluid, Oppenheimer, and Volkoff (1939) studied the gravitational equilibrium of neutron stars using Tolman's solutions and Chandrasekhar (1931) produced new models of white dwarfs in presence of relativistic effects.

A great number of exact models from the Einstein-Maxwell field equations have been generated by Gupta and Maurya (2011), Kiess (2012), Mafa Takisa and Maharaj (2013), Malaver and Kasmaei (2020), Malaver (2017, 2018a), Ivanov

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(2002) and Sunzu et al. (2014). For the construction of these models, several forms of equations of state can be considered (Sunzu 2018). Komathiraj and Maharaj (2007), Malaver (2016), Bombaci (1997), Thirukkanesh and Maharaj (2008), Dey et al. (1998) and Usov (2004) assume linear equation of state for quark stars. Feroze and Siddiqui (2011) considered a quadratic equation of state for the matter distribution and specified particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj (2013) obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel (2012) have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver (2013) generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with polytropic exponent. Malaver and Kasmaei (2020) proposed a new model of compact star with charged anisotropic matter using a cosmological Chaplygin fluid. Tello-Ortiz et al. (2020) found an anisotropic fluid sphere solution of the Einstein-Maxwell field equations with a modified Chaplygin equation of state. More recently, Malaver et al. (2022) obtained new solutions of Einstein's field equations in a Buchdahl spacetime considering a nonlinear electromagnetic field.

The analysis of compact objects with anisotropic matter distribution is very important, because that the anisotropy plays a significant role in the studies of relativistic spheres of fluid (Esculpi et al. 2007, Cosenza et al. 1982, Herrera 1992, Herrera and Nuñez 1989, Herrera et al. 1979, Herrera et al. 1984, Malaver 2014a, Malaver 2014b, Malaver 2014c, Malaver 2015, Malaver 2016, Malaver 2017, Malaver 2018a, Malaver 2018b, Malaver 2018c, Malaver 2018d, Sunzu and Danford 2017, Bowers and Liang 1974). Anisotropy is defined as $\Delta = p_t - p_r$ where p_r is the radial pressure and p_t is the tangential pressure. The existence of solid core, presence of type 3A superfluid (Sokolov 1980), magnetic field, phase transitions, a pion condensation and electric field (Usov 2004) are most important reasonable facts that explain the presence of tangential pressures within a star. Many astrophysical objects as X-ray pulsar, Her X-1, 4U1820-30 and SAXJ1804. 4-3658 have anisotropic pressures. Bowers and Liang (1974) include in the equation of hydrostatic equilibrium the case of local anisotropy. Bhar et al. (2015) have studied the behavior of relativistic objects with locally anisotropic matter distribution considering the Tolman VII form for the gravitational potential with a linear relation between the energy density and the radial pressure. Malaver (2015, 2018d), Feroze and Siddiqui (2011, 2014) and Sunzu et al. (2014) obtained solutions of the Einstein-Maxwell field equations for charged spherically symmetric spacetime by assuming anisotropic pressure.

In this paper we generated new classes of exact solutions for anisotropic charged distribution with a consistent with quark matter. New models have been obtained by specifying a particular form for one of the metric potentials and for the electric field intensity. The paper has been organized as follows: In section 2, we present the Einstein´s field equations. In section 3, we have chosen a particular form for the metric potential and for the electric field intensity in order to obtain the new models. In Section 4, physical requirements for the new models are described. In section 5, the models obtained are physically analyzed. In section 6, the conclusions of the work are presented.

Einstein's Field Equations

We consider a spherically symmetric, static and homogeneous spacetime. In Schwarzschild coordinates the metric is given by:

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (1)$$

where $\nu(r)$ and a $\lambda(r)$ are two arbitrary functions.

Using the transformations, $x = Cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji (1983), the Einstein-Maxwell field equations according to Feroze and Siddiqui (2011) can be written as:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2C} \qquad (2)$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{C} - \frac{E^2}{2C} \qquad (3)$$

$$p_t = p_r + \Delta \qquad (4)$$

$$\frac{\Delta}{C} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{C} \qquad (5)$$

$$\sigma^2 = \frac{4CZ}{x}\left(x\dot{E} + E\right)^2 \qquad (6)$$

ρ is the energy density, p_r is the radial pressure, E is electric field intensity, p_t is the tangential pressure, σ is the charge density, $\Delta = p_t - p_r$ is the measure of anisotropy and dots denote differentiations with respect to x.

With the transformations of Durgapal and Bannerji (1983), the mass within a radius r of the sphere take the form

$$M(x) = \frac{1}{4C^{3/2}}\int_0^x \sqrt{x}\rho(x)dx \qquad (7)$$

In this paper, we asume the following quadratic equation of state

$$p_r = \alpha\rho^2 + \beta\rho \qquad (8)$$

where α, β are arbitrary constant.

The New Anisotropic Models

With the Thirukanesh-Ragel-Malaver ansatz (Malaver 2014a, Horvath and Moraes 2020) we take the form for the metric potential *Z(x)*, which is well behaved and regular in the origin. The electric field intensity *E* is continuous in the interior and finite at the centre of the star. So for *Z(x)* and *E* we have

$$Z(x) = (1 - ax)^n \qquad (9)$$

$$\frac{E^2}{2C} = \frac{ax}{(1 + ax)} \qquad (10)$$

In equation (9) *n* is an adjustable parameter. In this paper, we considered the particular cases *n*=1, 2.

For the case *n*=1, with the equations (9) and (10) in eq. (2) we obtained for the energy density

$$\rho = C\left(\frac{3a(1 + ax) - ax}{1 + ax}\right) \qquad (11)$$

Using eq. (11) in eq. (8) the radial pressure can be written as

$$p_r = \alpha C^2\left(\frac{3a(1 + ax) - ax}{1 + ax}\right)^2 + \beta C\left(\frac{3a(1 + ax) - ax}{1 + ax}\right) \qquad (12)$$

Substituting (12), *Z(x)* and eq. (10) in eq. (3) we have

$$\frac{\dot{y}}{y} = \frac{1}{4C(1 - ax)}\left[\frac{\alpha C^2(3a(1 + ax) - ax)^2}{(1 + ax)^2} + \frac{\beta C(3a(1 + ax) - ax)}{(1 + ax)}\right] + \frac{a}{4(1 - ax)} - \frac{ax}{4(1 - ax)(1 + ax)} \qquad (13)$$

and integrating eq. (13)

$$y(x) = c_1(ax - 1)^{A*}(ax + 1)^B e^{-\frac{C\alpha}{8a(ax+1)}} \qquad (14)$$

where c_1 is the constant of integration

For convenience we have let

$$A* = \frac{-36a^2\alpha C^2 + 12a\alpha C^2 - 12a\beta C - \alpha C^2 - 4aC + 2\beta C + 2C}{16aC} \quad (15)$$

$$B = \frac{12a\alpha C - 3\alpha C + 2\beta + 2}{16a} \quad (16)$$

For the metric functions $e^{2\lambda}$, $e^{2\nu}$

$$e^{2\lambda} = \frac{1}{1-ax} \quad (17)$$

$$e^{2\nu} = A^2 c_1^2 (ax-1)^{2A*} (ax+1)^{2B} e^{-\frac{C\alpha}{4a(ax+1)}} \quad (18)$$

and the anisotropy Δ can be written as

$$\Delta = 4xC(1-ax)\left[\begin{array}{l}\frac{(A^2-A)a^2}{(ax-1)^2} + \frac{2Aa^2B}{(a^2x^2-1)} + \frac{AaC\alpha}{4(ax-1)(ax+1)^2} + \frac{(B^2-B)a^2}{(ax+1)^2} + \frac{Ba\alpha C - a\alpha C}{4(ax+1)^3} \\ + \frac{C^2\alpha^2}{64(ax+1)^4}\end{array}\right] \quad (19)$$

$$- a\left[1 + 2x\left(\frac{Aa}{ax-1} + \frac{Ba}{ax+1} + \frac{C\alpha}{8(ax+1)}\right)\right] + \frac{a + a^2x - 2ax}{1+ax}$$

With *n*=2, the expression for the energy density is

$$\rho = C\left(\frac{6a + a^2x - 5a^3x^2 - ax}{1+ax}\right) \quad (20)$$

replacing eq. (20) in eq. (8), we have for the radial pressure

$$p_r = \alpha C^2\left(\frac{6a + a^2x - 5a^3x^2 - ax}{1+ax}\right)^2 + \beta C\left(\frac{6a + a^2x - 5a^3x^2 - ax}{1+ax}\right) \quad (21)$$

Substituting eq. (21), *Z(x)* and eq. (10) in eq. (3) we obtain

$$\frac{\dot{y}}{y}=\frac{1}{4C(1-ax)^{2}}\left[\frac{\alpha C^{2}\left(6a+a^{2}x-5a^{3}x^{2}-ax\right)^{2}}{(1+ax)^{2}}+\frac{\beta C\left(6a+a^{2}x-5a^{3}x^{2}-ax\right)}{(1+ax)}-\gamma\right]+\frac{2a-a^{2}x}{4(1-ax)^{2}}-\frac{ax}{4(1-ax)(1+ax)}$$

(22)

Integrating eq. (22) we have

$$y(x)=c_{2}(ax-1)^{C*}(ax+1)^{D}e^{\frac{Ex^{3}+Fx+G}{8ac(ax+1)(ax-1)}} \qquad (23)$$

where c_2 is the constant of integration

Again for convenience

$$C*=\frac{-40a^{2}\alpha C+18a\alpha C-20a\beta+\alpha C-4a-\beta-1}{16a} \qquad (24)$$

$$D=\frac{22a\alpha C-\alpha C+\beta+1}{16a} \quad \text{and} \quad E=50a^{5}\alpha C^{2} \qquad (25)$$

$$F=-52a^{3}\alpha C^{2}+2a^{2}\alpha C^{2}-2a^{2}\beta C-a\alpha C^{2}-2a^{2}C+a\beta C+aC \qquad (26)$$

$$G=-2a^{2}\alpha C^{2}+2a\alpha C^{2}-2a\beta C-2ac+\beta C+C \qquad (27)$$

and for the metric functions $e^{2\lambda}$, $e^{2\nu}$ and anisotropy Δ we have

$$e^{2\lambda}=\frac{1}{(1-ax)^{2}} \qquad (28)$$

$$e^{2\nu}=c_{2}^{2}(ax-1)^{2C*}(ax+1)^{2D}e^{\frac{Ex^{3}+Fx+G}{4ac(ax+1)(ax-1)}} \qquad (29)$$

$$\Delta=4xC(1-ax)\left[\begin{array}{l}\frac{\left(C*^{2}-C*\right)a^{2}}{(ax-1)^{2}}+\frac{2a^{2}DC*}{\left(a^{2}x^{2}-1\right)}+\frac{2aC*}{ax-1}\left(\frac{3Ex^{2}+2Fx}{8aC\left(a^{2}x^{2}-1\right)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)^{2}(ax-1)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)(ax-1)^{2}}\right)\\+\frac{\left(D^{2}-D\right)a^{2}}{(ax+1)^{2}}+\frac{Da}{ax+1}\left(\frac{3Ex^{2}+2Fx}{8aC\left(a^{2}x^{2}-1\right)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)^{2}(ax-1)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)(ax-1)^{2}}\right)\\+\frac{6Ex+2F}{8aC\left(a^{2}x^{2}-1\right)}-\frac{3Ex^{2}+2Fx}{4C(ax+1)^{2}(ax-1)}-\frac{3Ex^{2}+2Fx}{4C(ax+1)(ax-1)^{2}}+\frac{\left(Ex^{3}+Fx+G\right)a}{4C(ax+1)^{3}(ax-1)}\\+\frac{\left(Ex^{3}+Fx+G\right)a}{4C(ax+1)^{2}(ax-1)^{2}}+\frac{\left(Ex^{3}+Fx+G\right)a}{4C(ax+1)(ax-1)^{3}}+\left(\frac{3Ex^{2}+2Fx}{8aC\left(a^{2}x^{2}-1\right)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)^{2}(ax-1)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)(ax-1)^{2}}\right)^{2}\end{array}\right]$$

$$-2aC(1-ax)\left[1+2x\left(\frac{Ca}{ax-1}+\frac{Da}{ax+1}+\frac{3Ex^{2}+2Fx}{8aC\left(a^{2}x^{2}-1\right)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)^{2}(ax-1)}-\frac{Ex^{3}+Fx+G}{8C(ax+1)(ax-1)^{2}}\right)\right]$$

$$+2aC-a^{2}Cx-\frac{2aCx}{1+ax}$$

(30)

Conditions of the Physical Acceptability

0 For a model to be physically acceptable, the following conditions should be satisfied (Delgaty and Lake 1998, Malaver 2014a):

(i) The metric potentials $e^{2\lambda}$ and $e^{2\nu}$ assume finite values throughout the stellar interior and are singularity-free at the center *r=0*.
(ii) The energy density ρ should be positive and a decreasing function inside the star.
(iii) The radial pressure also should be positive and a decreasing function of radial parameter.
(iv) The radial pressure and density gradients $\frac{dp_r}{dr} \leq 0$ and $\frac{d\rho}{dr} \leq 0$ for $0 \leq r \leq R$.
(v) The anisotropy is zero at the center *r=0*, i.e. $\Delta(r=0)=0$.
(vi) Any physically acceptable solution must satisfy the causality condition where the radial speed of sound v_{sr}^2 should be less than speed of light throughout the model, i.e., $0 \leq v_{sr}^2 = \frac{dp_r}{d\rho} \leq 1$.
(vii) The interior solution should match with the exterior of the Reissner-Nordstrom spacetime, for which the metric is given by

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \tag{31}$$

Through the boundary *r=R* where *M* and *Q* are the total mass and the total charge of the star, respectively.

The conditions (ii) and (iv) imply that the energy density must reach a maximum at the centre and decreasing towards the surface of the sphere.

Physical Analysis of the New Models

With *n*=1, the metric potentials $e^{2\lambda}$ and $e^{2\nu}$ have finite values and remain positive throughout the stellar interior. At the center $e^{2\lambda(0)} = 1$ and $e^{2\nu(0)} = A^2 c_1^2 (-1)^{2A*} e^{-\frac{C\alpha}{4a}}$. We show that in *r=0* $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ and this makes is possible to verify that the gravitational potentials are regular at the center. The energy density and radial pressure are positive and well behaved between the center and the surface of the star. In the center $\rho(r=0) = 3aC$ and $p_r(r=0) = 9a^2\alpha C^2 + 3\beta Ca - \gamma$, therefore the energy density will be non-negative in *r=0* and $p_r(r=0) > 0$. In the surface of the star $r = R$ and we have $p_r(r=R) = 0$ and $R = \frac{3}{\sqrt{3C - 9aC}}$. For the radial pressure and density gradients we obtain

$$\frac{d\rho}{dr} = \frac{C^2\left(6a^2 r - 2ar\right)}{1 + aCr^2} - \frac{2arC^2\left[3a\left(1 + aCr^2\right) - aCr^2\right]}{\left(1 + aCr^2\right)^2} \tag{32}$$

$$\frac{dp_r}{dr} = \frac{2\alpha C^2\left[3a\left(1 + aCr^2\right) - aCr^2\right]\left[6a^2 - 2a\right]Cr}{\left(1 + aCr^2\right)^2} - \frac{4\alpha C^3\left[3a\left(1 + aCr^2\right) - aCr^2\right]^2 ar}{\left(1 + aCr^2\right)^3}$$
$$- \frac{\beta C\left(6a^2 Cr - 2aCr\right)}{1 + aCr^2} + \frac{2\beta C^2\left[3a\left(1 + aCr^2\right) - aCr^2\right]ar}{\left(1 + aCr^2\right)^2}$$
(33)

In order to maintain of causality, the radial sound speed should be within the limit $0 \leq v_{sr}^2 \leq 1$ in the interior of the star (Delgaty and Lake 1998, Herrera 1992). In this model, we have:

$$0 \leq v_{sr}^2 = \frac{dp_r}{d\rho} = 2\alpha C\frac{\left[3a\left(1 + aCr^2\right) - aCr^2\right]}{\left(1 + aCr^2\right)} + \beta \leq 1 \tag{34}$$

On the boundary *r=R*, the solution must match the Reissner–Nordström exterior space–time as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{35}$$

and therefore, the continuity of $e^{2\lambda}$ and $e^{2\nu}$ across the boundary r=R is

$$e^{2\nu} = e^{2\lambda} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \tag{36}$$

Then for the matching conditions, we obtain:

$$\frac{2M}{R} = \frac{\left(a^2 + 2a\right)C^2 R^4 + aCR^2}{1 + aCR^2} \tag{37}$$

For the case n=2, we have for the metric potentials $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_2^2(-1)^{2C*}e^{-\frac{G}{4aC}}$ and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$ at the centre r=0. Again the gravitational potentials are regular in the origin. The energy density and radial pressure also are positive and well behaved in the stellar interior. In the center $\rho(r=0)=6aC$ and $p_r=36\alpha a^2C^2+6a\beta C$, therefore the energy density will be non-negative in $r=0$ and $p_r(r=0)>0$. In the surface of the star $r=R$ and we have $p_r(r=R)=0$ and $R=\frac{\sqrt{10C\left(a-1+\sqrt{121a^2-2a+1}\right)}}{10aC}$. For the radial pressure and density gradients we obtain

$$\frac{d\rho}{dr}=C\left(\frac{-20C^2a^3r^3+2Ca^2r-2Car}{1+aCr^2}\right)-2C^2ar\left(\frac{-5C^2a^3r^4+Ca^2r^2-Car^2+6a}{(1+aCr^2)^2}\right) \quad (38)$$

$$\begin{aligned}\frac{dp_r}{dr}=&\frac{2\alpha C^2\left[-5C^2a^3r^4+Ca^2r^2-aCr^2+6a\right]\left[-20C^2a^3r^3+2Ca^2r-2Car\right]}{\left(1+aCr^2\right)^2}\\&-\frac{4\alpha C^3\left[-5C^2a^3r^4+Ca^2r^2-aCr^2+6a\right]^2ar}{\left(1+aCr^2\right)^3}-\frac{\beta C\left(-20C^2a^3r^3+2Ca^2r-2Car\right)}{1+aCr^2}\\&+\frac{2\beta C^2\left(-5C^2a^3r^4+Ca^2r^2-Car^2+6a\right)ar}{\left(1+aCr^2\right)^2}\end{aligned} \quad (39)$$

The causality condition implies that

$$0\leq 2\alpha C\left(\frac{6a+a^2Cr^2-5a^3C^2r^4-acr^2}{1+aCr^2}\right)+\beta\leq 1 \quad (40)$$

On the boundary $r=R$, the solution must match the Reissner–Nordström exterior space–time and therefore for the matching conditions, we obtain:

$$\frac{2M}{R}=\frac{aCR^2+\left(a^2+2a\right)C^2R^4-a^3C^3R^6}{1+aCR^2} \quad (41)$$

Figures 1, 2, 3, 4, 5 and 6 present the dependence of ρ, $\frac{d\rho}{dr}$, p_r, $\frac{dp_r}{dr}$, v_{sr}^2 and Δ with the radial coordinate respectively with a=0.1, α=0.25, β=0.5, C=1 for n=1 and n=2.

Figure 1. *Energy Density against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*

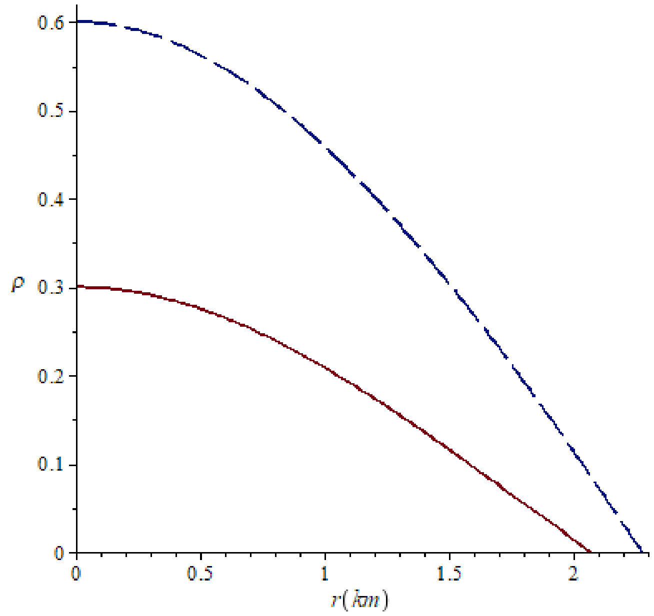


Figure 2. *Density Gradient against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*

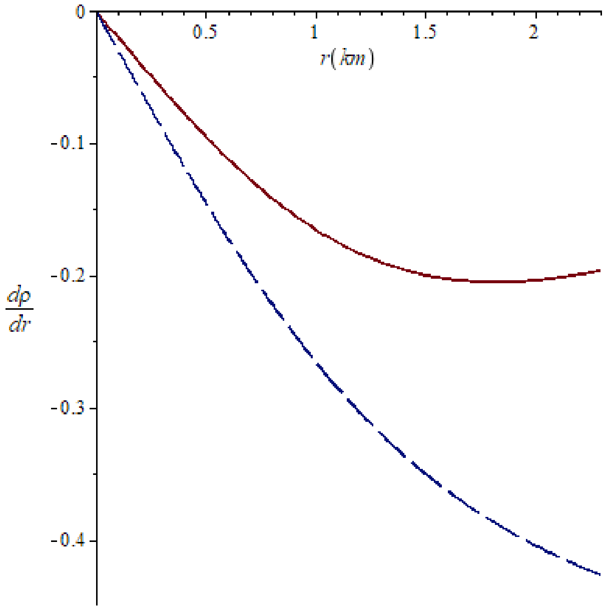


Figure 3. *Radial Pressure against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*

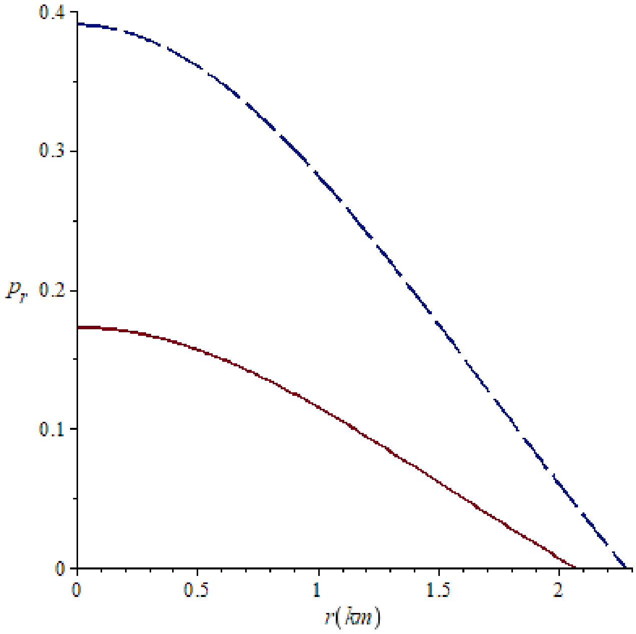


Figure 4. *Radial Pressure Gradient against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*

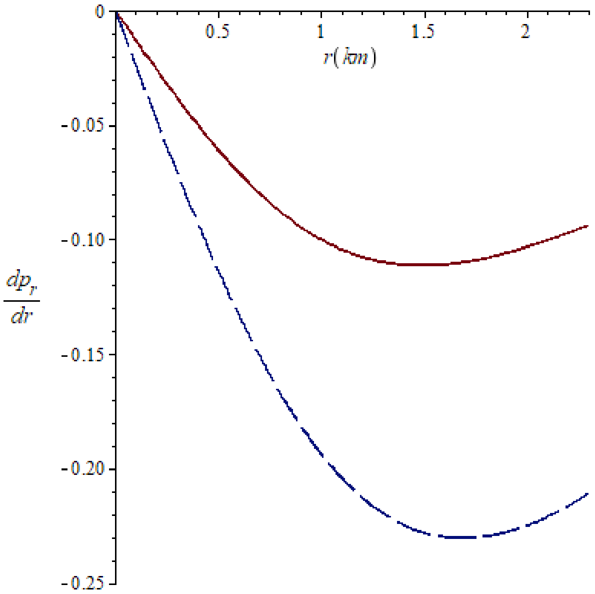


Figure 5. *The Radial Speed of Sound against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*

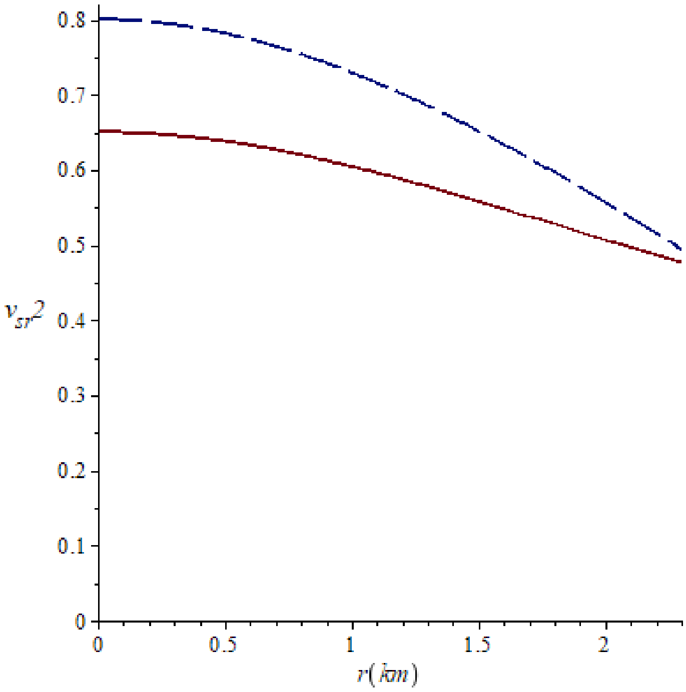
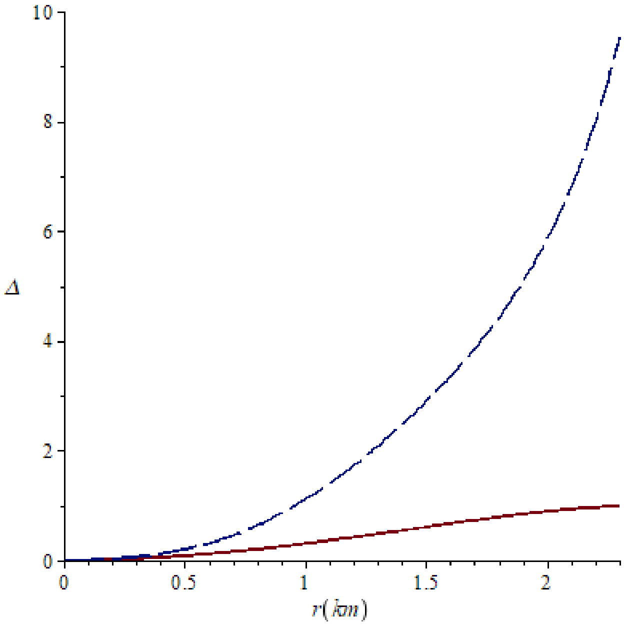


Figure 6. *Anisotropy against Radial Coordinate. It has been Considered that n=1(Solid Line); n=2 (Long-Dash Line)*



In Figure 1 is shown that the energy density remains positive, continuous and is monotonically decreasing function throughout the stellar interior. In Figure 2 it is noted that for the radial variation of energy density gradient $\frac{d\rho}{dr} < 0$ in the two cases studied. The radial pressure showed the same behavior by the energy density, that is, it is growing within the star and vanishes at a greater radial distance and its results are shown in Figure 3. The presence of a quadratic term in the equation of state causes an increase in the maximum values. Again, according to Figure 4, the profile of $\frac{dp_r}{dr}$ shows that radial pressure gradient is negative inside the star for n=1 and n=2. Figure 5 shows that the condition $0 \leq v_{sr}^2 \leq 1$ is maintained throughout the interior of the star and satisfy the causality, which is a physical requirement for the construction of a realistic star (Joshi 1993). The anisotropic factor is plotted in Figure 6 and it shows that vanishes at the centre of the star, i.e., $\Delta(r=0)=0$ (Delgaty and Lake 1998).

Conclusions

In this paper we have generated new models of anisotropic stars considering the Thirukkanesh-Ragel-Malaver ansatz for the gravitational potential and a quadratic equation of state. These models may be used in the description of compact objects in absence of charge and in the study of internal structure of strange quark stars. We show that the developed configuration obeys the physical conditions required for the physical viability of the stellar model. The radial pressure, energy density, anisotropy and all the metric coefficients are regular at the origin and well behaved in the stellar interior.

The constants α and β have been chosen in order to maintain the causality condition and the regularity of metric potentials inside the radius of the star. The new solutions match smoothly with the Reissner–Nordström exterior metric at the boundary $r=R$, because matter variables and the gravitational potentials of this work are consistent with the physical analysis of these stars. It is expected that the results of this research can contribute to modeling of relativistic compact objects and configurations with anisotropic matter distribution.

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