Athens Journal of Sciences 2023, 10: 1-10
https://doi.org/10.30958/ajs.X-Y-Z

Teaching Mathematics with Visuals

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This article is about research done on the ways of using visuals in teaching introductory mathematics. Research showed that the use of visuals improves the learning of mathematics. However, more research is needed to explore which math concepts need to be addressed, how to choose visuals for that purpose, and how to incorporate them in teaching mathematics. The analysis shows that the effective use of correctly chosen visuals in teaching introductory mathematics positively impacts students' attitudes toward mathematics, and enhances their learning, engagement, and motivation.

Keywords: *analytical thinking skills, instructional strategies with visuals, introductory mathematics, student engagement, and motivation*

Introduction

The acquisition of analytical thinking skills is an essential component of learning introductory mathematics. However, many students struggle with this aspect of mathematics education, which can lead to negative attitudes toward the subject. One potential solution is the use of visuals to support the development of analytical thinking skills. Visuals have been found to be effective in promoting mathematical thinking, problem-solving, and reasoning skills.

This article explores the use of visuals in developing students' analytical thinking skills, by exploring how sticks and abacus can be effectively used in teaching arithmetic. The purpose of this study is to investigate the impact of using visuals in mathematics instruction on the development of analytical thinking skills. The article begins with a review of relevant literature, including theoretical frameworks and previous research on the use of visuals in education and their impact on analytical thinking skills development.

The article concludes with thoughts, recommendations, and discussions for using visuals to support the development of analytical thinking skills. The significance of this study lies in its potential to inform instructional practices that promote student success in mathematics, especially in the development of analytical thinking skills.

Previous Research on Visuals in Education

In recent years, there has been increasing interest in the use of visuals to support learning in various subject areas. This interest is driven by the recognition

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that visuals can enhance students' understanding, comprehension, and retention of concepts, as well as their problem-solving and reasoning skills.

Previous research has demonstrated the effectiveness of visuals in improving student learning outcomes in mathematics. For example, John Hattie's meta-analysis of educational research studies found that the use of visuals had a moderate to large effect size on student achievement across all academic subjects. Hattie's study, which analyzed data from over 800 meta-analyses, concluded that the use of visual aids such as diagrams, graphs, charts, and pictures can help students better understand complex concepts and retain information more effectively. Additionally, the study found that teachers who incorporate visual aids into their lessons can help students develop stronger critical thinking skills and improve their problem-solving abilities (Hattie 2009).

Moyer-Packenham, Westenskow, and Jordan conducted a meta-analysis of the effects of visual representations on learning mathematics and found that visual representations had a positive effect on learning outcomes (Moyer-Packenham et al. 2016). Similarly, Suh, Moyer-Packenham, and Westenskow investigated the effects of visuals on problem-solving in elementary school mathematics and found that visuals improved students' problem-solving skills (Suh et al. 2016).

Theoretical frameworks, such as cognitive load theory, also support the use of visuals in mathematics instruction. Cognitive Load Theory (CLT) suggests that instructional materials should be designed in a way that reduces the cognitive load on learners' working memory. The use of visuals in mathematics instruction is one way to achieve this goal, as visuals can help learners better understand and process mathematical concepts (Kalyuga 2009). This can help students to process information more effectively and efficiently, leading to improved learning outcomes.

Other studies have investigated the impact of specific types of visuals on learning outcomes in mathematics education. For example, Tarmizi and Sweller (1988) found that the use of worked examples and diagrams improved learning outcomes in algebra.

Overall, these studies suggest that the use of visuals can be an effective strategy for improving learning outcomes in mathematics education. By providing students with visual representations of mathematical concepts and relationships, visuals can help students to make sense of complex information and develop the analytical thinking skills necessary for success in mathematics and other academic disciplines.

However, despite the potential benefits of using visuals in mathematics education, there is still a need for further research on the impact of visuals on analytical thinking development, especially in introductory mathematics students. This study seeks to address this gap in the literature by investigating the effectiveness of visuals in developing analytical thinking skills in mathematics education.

Purpose of the Study

The purpose of this study is to investigate the impact of using visuals in mathematics instruction on the development of analytical thinking skills in students. The study seeks to answer the following research question: What is the impact of using visuals in mathematics instruction on the development of analytical thinking skills in students?

The study aims to contribute to the understanding of how visuals can be used to support the development of analytical thinking skills in mathematics education. By investigating the impact of visuals on analytical thinking skills development, the study seeks to provide insights into instructional practices that promote student success in mathematics, especially in the development of analytical thinking skills.

Significance of the Study

The significance of this study lies in its potential to inform instructional practices that promote student success in mathematics, especially in the development of analytical thinking skills. Analytical thinking skills are crucial for success in mathematics and other academic disciplines, as well as for real-world problem-solving and decision-making. However, many students struggle with the development of analytical thinking skills, which can lead to negative attitudes toward mathematics and reduced academic performance.

The study's findings on the impact of visuals on analytical thinking skills development can provide valuable insights into how educators can support students' analytical thinking skills development. By identifying effective instructional strategies, such as the use of visuals, educators can enhance students' mathematical thinking and problem-solving abilities, leading to improved academic performance and increased confidence in mathematics.

Overall, the study's findings on the impact of visuals on analytical thinking skills development in mathematics education have the potential to make a significant contribution to improving students' academic outcomes and attitudes toward mathematics.

Theoretical Framework

This study is informed by several theoretical frameworks related to the use of visuals in mathematics education and the development of analytical thinking skills.

The first theoretical framework is the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000), which emphasizes the importance of visualization in mathematics learning. The NCTM advocates for the use of visuals to help students make sense of mathematical concepts and relationships.

The second theoretical framework is cognitive load theory, which suggests that the use of visuals can help to reduce cognitive load.

The third theoretical framework is the conceptual change theory, which suggests that students' prior knowledge and beliefs can influence their learning of new concepts. The use of visuals can help students to develop new conceptual frameworks that are more accurate and better aligned with mathematical concepts and relationships.

The fourth theoretical framework is the problem-based learning theory, which emphasizes the importance of problem-solving and inquiry-based learning in mathematics education. The use of visuals can support problem-based learning by providing students with visual representations of problems and supporting their problem-solving skills.

Overall, these theoretical frameworks suggest that the use of visuals in mathematics education can support students' learning and development of analytical thinking skills. By reducing cognitive load, supporting conceptual change, and promoting problem-based learning, visuals can help students to make sense of mathematical concepts and relationships and develop the analytical thinking skills necessary for success in mathematics and other academic disciplines.

Use of Visuals in Developing Analytical Thinking Skills

The use of visuals can support the development of the analytical thinking skills of students in several ways.

First, visuals can support the development of spatial reasoning skills, which are essential for mathematical problem-solving. Visuals such as diagrams, graphs, and models can help students to visualize mathematical relationships and concepts, leading to improved spatial reasoning abilities (Clements and Sarama 2011).

Second, visuals can support the development of mathematical communication skills by providing a common visual language for students to communicate their mathematical thinking. This can help students to develop their ability to explain their thinking and ideas, leading to improved communication skills (Lannin and Barker 2015).

Third, visuals can support the development of metacognitive skills by encouraging students to reflect on their thinking and problem-solving strategies. Visuals can be used to prompt students to think about their thinking and reflect on their problem-solving processes, leading to improved metacognitive skills (Fyfe and McNeil 2018).

Overall, the use of visuals can support the development of analytical thinking skills in students by promoting spatial reasoning, critical thinking, mathematical communication, and metacognitive skills. By using visuals in mathematics education, teachers can help students to develop the analytical thinking skills necessary for success in mathematics and other academic disciplines.

The Impact of Visuals on Analytical Thinking Development

The methodology devised in Marikyan (2019) does not require students to spend hours on rote learning and memorization. The research showed that the only effective way of learning mathematics is through understanding. The net effect of learning mathematics through understanding is the development of students' analytical thinking skills. The research also showed students started being interested in mathematics, enjoyed learning it, and considerably progressed in learning mathematics (Marikyan 2019).

In mathematics, students are required to work with formulas and solve word problems. For both, and especially for solving word problems students need to use their analytical thinking skills. On the other hand, solving word problems develops analytical thinking skills. Therefore, teaching mathematics should nurture analytical thinking skills among students (Marikyan 2013).

However, unfortunately, there are required tests that all students have to pass, preferably with high scores. The aftermath is that the focus of learning mathematics becomes high test scores, and classrooms are being converted into "test preparation centers," then memorization becomes the fastest way of preparing students for the test. However, memorization only helps students to remember the topics to pass the test (Casbarro 2003). The net effect is that students are being checked and graded on memorization. Unfortunately, memorization may require a considerable amount of time for some students. Those students will be forced to memorize mathematics that does not make sense to them, since they do not understand it. Those students will make the same amount of effort to learn mathematics as to learn a poem by heart in a foreign language. Needless to say, this may create math anxiety among those students. Another group of students is those who are interested in understanding mathematics, but no one explains it to them because the teacher spends time mainly on preparing students for required tests with high grades. The frustration for these students is the lack of time and support, and they become discouraged from learning mathematics.

Another example is a child from a YouTube video who didn't want to go back to school because the teacher on Monday had said that 6 + 2 = 8, on Tuesday she told that 4 + 4 = 8, and on Wednesday she said that 5 + 3 = 8. Then the child continues, saying that she'll wait until the "teacher makes up her mind." Sounds funny, but on a serious note, the teacher failed to explain why 6 + 2 = 8, 4 + 4 = 8, and 5 + 3 = 8. This example shows that the child has analytical thinking skills which were not put to use in learning mathematics. She compared what the teacher taught without explaining and concluded that the teacher does not know what she was talking about. It is obvious that it could be very helpful to use visuals in explaining to the child why 6 + 2 = 8, 4 + 4 = 8, and 5 + 3 = 8.

Below are two scenarios to compare. Both explain why 6 + 2 = 8, 4 + 4 = 8, and 5 + 3 = 8.

Scenario 1. The teacher puts some sticks on the table and asks students to count 6 sticks and put them on a side. Then students count 2 sticks and add them to the other 6 sticks, and count all sticks again. Then using the same technique, the teacher teaches that 4 + 4 = 8, and 5 + 3 = 8. The result will be that the students memorize that 6 + 2 = 8, 4 + 4 = 8, and 5 + 3 = 8. However, the students will not understand why they all are equal to 8, as the child in my above example.

Later on, the commutative property of addition will be introduced to students using the formula (1). The formula is not helpful because they had never used it before while adding numbers. Plus, students, having problems using variables will just ignore the formula, and the commutative property of addition will be never used in learning mathematics.

Scenario 2. The teacher puts 8 sticks on the table and asks students to count them. Now the students know that they have 8 sticks. Next, students are asked to take 6 sticks and put them on a side. Then students count the remaining 2 sticks. Students learn that taking away 6 sticks from 8 leaves with 2 sticks. Then the teacher asks them to add the 2 sticks to the 6 sticks, learning that 6 + 2 = 8. Next, the teacher asks students to separate 6 and 2 sticks and then add the 6 sticks to the 2 sticks, that is, 2 + 6 = 8. This teaches students that 6 + 2 = 8 and 2 + 6 = 8, teaching the concept of the commutative property of addition (1). The same activity can be repeated with 4 and 4, 5 and 3, and 1 and 7. Through this activity, students will learn the concepts of addition, subtraction, and the commutative property of addition (1).

$$a + b = b + a \qquad (1)$$

Explaining the commutative property of addition using sticks, at an early age, will prevent students from getting accustomed to performing addition through memorization.

Then using the same technique, the teacher teaches that 4+4=8, 5 + 3 = 8, and 7 + 1 = 8. The result will be that the students will learn that 6 + 2 = 4 + 4 = 5 + 3 = 7 + 1 = 8, and the commutative property of addition. The students will understand why they all are equal to 8.

The comparison of these two scenarios shows that although both scenarios used the same objects, the results were very different. This shows the importance of how visuals were used.

The associative property of addition (2) sometimes is more difficult to explain. Let us try to explain it using the same sticks.

$$(a + b) + c = a + (b + c) \qquad (2)$$

Scenario 3. The teacher puts the same 8 sticks on the table and asks students to divide them into three groups. Then students are asked to count the sticks in each group. Next, the teacher asks the students to combine the first and the second groups, then add the third group. The students know that the result is 8. Next, the teacher asks students to separate the same groups, combine the second and the third groups, and add them to the first group. From this activity, students will learn that combining three groups in any order will result in the same answer.

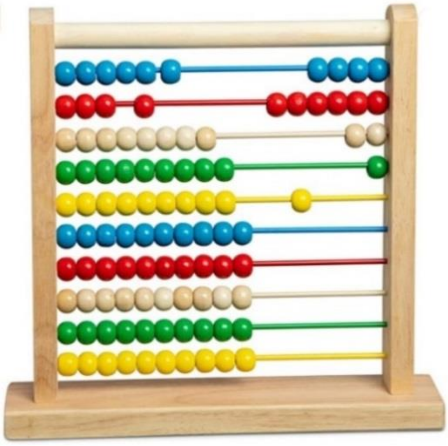
In the first grade in Armenia, children were required to have a box of 6-inch-long plastic sticks of the same color. They were used just for a few weeks to understand the concepts of addition and subtraction, and the commutative and associative properties of addition.

Using sticks, not other objects helped not to connect addition or subtraction with any particular objects. The sticks were of the same color which helped not to connect addition or subtraction with any particular color. Both are important for a smooth transition from concrete thinking to abstract thinking. Another net effect from using sticks of the same size and of the same color, later on, helps with the understanding of variables. This discussion is out of the scope of this article.

The first graders were also required to have an abacus. In fact, those two were the only visual aids used in teaching mathematics in Armenia.

The abacus in Figure 1 looks nice and may attract a child's attention. However, this abacus can only be used to count from 1 to 10 and maybe can be used for adding small numbers, the sum of which is 10 or less.

Figure 1. *Colorful Abacus*



The abacus used in Armenia in the first grade was not as colorful as the one in Figure 1. The beads of that kind of abacus are of two colors. In each row, there are ten beads. The middle two beads of each row are of a contrasting color. In Figure 2 the middle two beads are black. The middle two beads are the fifth and sixth beads. That is, in each row (except the row with 4 beads) there are 4 white, 2 black, and another 4 white beads. This setup of beads helps students to minimize

Figure 2. *An Abacus Used by First Graders in Armenia*

Scenario 4. There is no need to count for moving 1 or 2 beads to the left. After moving 3 beads to the left, they see that out of 4 white beads only one is left, that is, 4 – 3 = 1. Once the students understand that 3 + 1 =4, they will move 3 beads to the left without counting 1, 2, 3, but will move the beads leaving 1 white bead,

therefore, eliminating counting while moving 3 beads. To move 4 beads to the left is not a brainer; those are the 4 white beads. To move 5 beads, students know that there are 4 white beads, therefore, they need to move one more bead, which is a black bead to get 5 beads. Then they see that half of the beads are now on the left side and the other half is on the right. Then students learn, that 5 + 5 =10, 5 × 2 = 10, and also, 10 ÷ 2 = 5. To move 6 beads to the left, they will move 5 beads and one more black bead to the left. The students learn that 5 + 1 = 6. Seeing 4 white and 2 black beads on the left, and 4 white beads on the right. Now they also learn that 4 + 2 = 6, 10 – 6 = 4, and 10 – 4 = 6. Again, no need to count the beads. While moving 7 beads, they will move the 4 white beads, the 2 black beads, and 1 more white bead. They learn that 6 + 1 = 7, 4 + 2 + 1 = 7. Seeing 3 white beads on the right side, they also learn that 10 – 7 = 3 and 10 – 3 = 7. This means, that for moving 7 beads to the left, they need to leave 3 white beads and move the rest to the left. Similarly, to move 8 or 9 beads to the left, they will move all but 2 or 1 bead, correspondingly.

Scenario 5. This abacus can also be used to teach place values. The third from the bottom is for ones, above which is the row of tens. Then comes the row of hundreds. The first bead of the next row is black, to help visually to easily find the row of thousands.

The row with 4 beads is a placeholder, similar to the decimal point. The lower two rows are for tenths and hundredths. For example, to show 0.75, 7 beads from the second from the bottom row and 5 beads from the last row will be moved to the left. (Tenths and hundredths were not covered in the first grade.)

The knowledge gained in Scenario 4, is applicable to all rows. That is, to calculate 5000 + 3000 students move 5 beads and 3 beads from the row of thousands to the left. Seeing 2 beads left on the right side they will write, 5000 + 3000 = 8000. If the students perform the same operation on the paper, they will write 0 + 0 = 0, 0 + 0 = 0, 0 + 0 = 0, 5 + 3 = 8 (See Figure 3).

The knowledge gained while using an abacus, students can use while performing calculations without an abacus.

Figure 3. *Step for Performing Addition on a Paper*

Step 1	Step 2	Step 3	Step 4
5000	5000	5000	5000
+	+	+	+
3000	3000	3000	3000
0	00	000	8000

The top row represents ten-thousands. How this abacus can be used in teaching mathematics?

Scenario 6. The teacher asks students to calculate 7 + 5 using the abacus. Students move 7 beads to the left. Then they move the remaining beads in the row, counting 1, 2, 3. Now all beads on the row of ones are on the left side, which means that 7 + 3 = 10. Students will move all 10 beads to the right and will move 1 bead from the row of tens to the left and will continue moving beads from the row of ones to the left, counting, 4, 5. Seeing 1 bead from the row of tens and 2 beads from the row of ones on the left, the student will know that 7 + 5 = 12. Besides learning that 7 + 5 = 12, the students learned that 5 = 3 + 2, more importantly, they learned that 12 is equal to one 10 and 2 ones.

Scenario 7. Similarly, to calculate 47 + 58, the students move 7 beads from the row of ones, and 4 beads from the row of tens to the left. Next, to add 58, the students move the remaining 3 beads from the row of ones, getting all ten beads on the left side. Moving the 10 beads to the right and moving 1 bead from the row of tens to the left, the students see that 47 + 3 = 50. That is 50 – 47 = 3. Then students move 5 beads from the row of ones to the left. They know that 47 + 8 = 55. The last step is to add 50, for which they move 5 beads from the row of tens to the left, getting all 10 on the right side. They move the 10 beads to the right and move 1 bead from the row of hundreds to the left. 47 + 58 = 105 because on the left there is 1 bead from the row of hundreds and 5 beads from the row of ones. While calculating 47 + 58, students also learned that 10 beads from the row of tens, make a hundred, that is, 10 × 10 = 100.

Scenarios 4 – 7 show that using this type of abacus in teaching mathematics helps students to learn some challenging concepts of mathematics. It also teaches how addition and subtraction are connected.

Discussion & Conclusion

The results of the study indicate that the use of visual aids, sticks and the abacus in teaching mathematics leads to a significant improvement in students' learning of mathematics, improving students' understanding and retention of learned mathematical concepts. The net effect of this improvement leads to the development of their analytical thinking skills.

Another result of this study is that the use of these visuals can help to make abstract or complex concepts more concrete and accessible to students and can also help to promote deeper learning and critical thinking skills.

Overall, these findings suggest that the use of these visuals can be an effective strategy for developing students' analytical thinking skills in mathematics. However, it is important to note that the specific types of visuals used, as well as the instructional strategies used to incorporate them in teaching, play an important role in their effectiveness.

This study did not explore how other visuals can be used in combination with other instructional strategies to further enhance students' analytical thinking skills in mathematics.

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