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Nonlinear Optimization of Weekly Training Plans for Competitive Tennis Players

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Training design in competitive tennis requires balancing performance improvement with injury prevention. Athletes must allocate limited weekly training time across multiple activities while managing nonlinear physiological responses. This study develops a constrained nonlinear optimization model that incorporates a concave performance function with diminishing returns and a convex injury-risk function representing overload effects. The model includes linear constraints on total training time, high-intensity workload, and minimum recovery. Optimality conditions are derived using the Karush–Kuhn–Tucker (KKT) framework, and numerical solutions are obtained using MATLAB's fmincon solver with a Sequential Quadratic Programming (SQP) algorithm. The results show that optimal training allocations balance technical drills, moderate high-intensity work, and sufficient recovery. The total weekly time constraint is consistently binding, while recovery often exceeds its minimum requirement. Sensitivity analysis demonstrates that increasing performance emphasis leads to higher high-intensity training and reduced recovery. It is concluded that nonlinear optimization provides a structured framework for weekly training design in tennis. The model highlights the importance of balancing performance gains and injury risk while respecting realistic training constraints.

Keywords: *Training Load Optimization, Athlete Workload Management, Injury Risk Modeling, Performance Optimization, Tennis Training, Recovery Optimization.*

Introduction

Training design in competitive sports requires balancing performance improvement with injury prevention. In tennis, athletes must distribute limited weekly training time across multiple activities, including match play, technical drills, strength training, conditioning, and recovery. Each of these components contributes differently to performance outcomes and physiological stress, creating a complex decision-making problem for athletes and coaches. Longitudinal athlete monitoring studies have shown that training volume and workload management significantly influence performance adaptation and fatigue accumulation (Rishiraj & Niven, 2018). Mathematical and analytical approaches have increasingly been applied to sports performance evaluation and decision-making. It has also been applied in other sport contexts such as football performance evaluation and team contribution analysis (Leela et al., 2023).

The relationship between training load and performance is inherently nonlinear. Performance improvements typically exhibit diminishing returns, where initial training produces substantial gains, but additional workload yields progressively

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smaller benefits. At the same time, injury risk increases at an accelerating rate as training intensity and volume rise, particularly when high-intensity sessions are excessive or insufficient recovery is provided (Kreher & Schwartz, 2012). Recovery activities play a critical role in mitigating fatigue and reducing injury risk, further complicating the allocation of training time.

Because of these competing effects, designing an effective weekly training schedule requires balancing multiple objectives under practical constraints. Athletes operate within limited time availability while needing to maintain sufficient recovery and avoid excessive workload. These constraints make it difficult to determine an optimal distribution of training activities using intuition alone. This study addresses this problem by developing a mathematical framework for allocating weekly training time across different activities. The model captures the nonlinear relationship between performance and workload, incorporates injury risk considerations, and includes realistic constraints on total training time, high-intensity training, and recovery. The objective is to identify training allocations that achieve a balance between performance improvement and injury prevention.

Problem Description and Assumptions

Weekly Training Components/Variables

A weekly training plan for a competitive tennis player is represented using five primary activity categories: high-intensity on-court training (x_1), low-intensity technical drills (x_2), strength training (x_3), conditioning (x_4), and recovery (x_5). Each of these components contributes differently to performance development and physiological stress.

High-intensity on-court training, which includes match play and competitive drills, plays a critical role in developing tactical awareness and match readiness. However, it also imposes significant physical and mechanical stress on the athlete (Deconstructing Stigma, 2025). Racquet sports require balancing technical skill development with physical conditioning demands that influence competitive performance (Layton & DeBeliso, 2017). Low-intensity technical drills contribute to skill refinement with relatively lower injury risk. Strength training and conditioning improve physical capacity and resilience but can contribute to fatigue when performed excessively. Recovery activities, such as mobility work and therapeutic interventions, are essential for reducing accumulated fatigue and supporting adaptation (Kreher & Schwartz, 2012). Athletic adaptation processes are inherently nonlinear and depend on the interaction between workload and physiological response (Venkata, 2014).

All five variables are continuous and nonnegative:

$$x_i \geq 0, \quad i = 1,2,3,4,5$$

Usage of hours is consistent with how coaches plan workloads. Later, for narrative interpretation, these hours can be discussed as approximate “days or sessions” of training (e.g., 3 hours of recovery aligning with about one recovery day).

Modeling Assumptions

Recent sports analytics studies have demonstrated the usefulness of quantitative modeling frameworks for evaluating complex athletic systems and performance interactions (Leela et al., 2024). To keep the model presented here tractable while still realistic, the following assumptions are made:

1. Weekly horizon and steady state. The schedule is optimized over one representative week. Interactions across multiple weeks (e.g., tournament cycles) are not modeled here.
2. Separable training effects. Performance benefit and injury risk are represented as additive contributions from each activity type. This assumes marginal effects depend mainly on the hours in that activity.
3. Smooth nonlinear responses. Performance gains and risk changes vary smoothly with hours, meaning small hour changes do not cause discontinuous jumps in outcomes.
4. Fixed parameters. Relative weights for performance benefit and injury risk are treated as constant during the week, reflecting a stable training philosophy during that planning period.
5. No discrete scheduling limits. The model does not explicitly enforce “integer days.” Instead, hours are optimized continuously, then interpreted in practice.

These assumptions allow the model to capture key training dynamics while remaining mathematically manageable.

Constraints

Three main linear constraints structured the planning problem:

1. Weekly training budget (total hours limit). The total training time is limited by the athlete’s available schedule and physiological capacity

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq T_{max}$$

This represents time availability due to class, travel, injury management, or fatigue tolerance.

2. Cap on high-intensity on-court load.

$$x_1 \leq H_{max}$$

High-intensity match-like training is the most stressful component. Even elite athletes typically cannot exceed a certain weekly volume without risk of overload (Deconstructing Stigma, 2025).

Minimum recovery requirement.

$$x_5 \geq R_{min}$$

Recovery is treated as a mandatory workload; without an enforced minimum, the optimizer might push recovery unrealistically low to chase performance.

All constraints are written in the nonlinear-programming convention $g_i(x) \leq 0$, which is required for inequality-constrained NLP and MATLAB *fmincon* (Hu, 2018). All decision variables are constrained to be nonnegative, reflecting that training time cannot be negative.

Parameter Choices

The model parameters are selected to reflect realistic weekly training volumes for competitive tennis players. A total weekly training limit of 18 hours is assumed, along with a maximum of 8 hours of high-intensity training. The minimum recovery requirement is set at 3 hours per week under baseline conditions (UCHealth, 2025), with higher values considered in sensitivity. Relative weights are assigned to reflect the importance of different training activities in contributing to performance and injury risk. High-intensity training is associated with both strong performance benefits and elevated injury risk, while recovery contributes primarily to reducing fatigue and mitigating injury risk (Kreher & Schwartz, 2012).

Baseline constraints reflect common competitive weekly volume:

- $T_{max} = 18\ hours/week.$
- $H_{max} = 8\ hours/week.$
- $R_{min} = 3\ hours/week\ (\approx one\ recovery\ day)$

A second, safer recovery scenario uses:

- $R_{min} = 5\ hours/week\ (\approx two\ recovery\ days)$

Relative weights in the objective are chosen so that on-court time contributes strongly to performance, while overload in high-intensity, conditioning, and strength contributes strongly to injury risk.

The Model parameters are given in Table 1. Parameters were selected to be realistic for a competitive player and to illustrate the nonlinear tradeoff structure of the model. The weekly time budget $T_{max} = 18$ hours and high-intensity cap $H_{max} = 8$ hours reflect typical feasible training volume, while recovery minimum R_{min} was tested at both baseline (3 hours/week) and safer (5 hours/week) policy levels. Performance weights p_i emphasize the higher competitive value of on-court work, whereas quadratic risk penalties $\beta_1, \beta_3, \beta_4$ discourage overload in high-stress categories. Sensitivity experiments varied α to represent conservative ($\alpha = 0.5$), balanced ($\alpha = 1.0$), and aggressive ($\alpha = 2.0$) coaching priorities.

Table 1. *Model Parameters Used in Numerical Experiments*

Symbol/name	Value(s) used	Units	Meaning/role in model
T_{max}	18	hours/week	Maximum total weekly training time (time-budget constraint)
H_{max}	8	hours/week	Maximum high-intensity on-court hours (cap on x_1)
R_{min}	3(baseline), 5(safer case)	hours/week	Minimum required weekly recovery volume (lower bound on x_5)
α	0.5, 1.0, 2.0	-	Risk-performance tradeoff weight. Smaller = more risk-averse; larger = more performance-driven.
$p = [p_1, p_2, p_3, p_4, p_5]$	[4, 2.5, 1.5, 1.5, 1.0]	-	Performance benefit weights for each training category in $P(x) = \sum \quad p_i ln(1 + x_i)$.
β_1	0.08	-	Quadratic risk weight for high-intensity on-court training $x^2{}_1$.
β_3	0.08	-	Quadratic risk weight for strength training $x^2{}_3$.
β_4	0.08	-	Quadratic risk weight for conditioning training $x^2{}_4$.
ρ	0.6	-	Recovery benefit weight in risk term $-\rho ln(1 + x_5)$.

Mathematical Model

Performance Benefit

Performance improvements from training show diminishing returns. A smooth concave form is:

$$P(x) = \sum_{i=1}^{5} \quad p_i ln(1 + x_i), \text{ with } p_i > 0.$$

Marginal benefits:

$$\frac{\partial P}{\partial x_i} = \frac{p_i}{1+x_i} > 0, \quad \frac{\partial^2 P}{\partial x^2{}_i} = -\frac{p_i}{(1+x_i)^2} < 0$$

This formulation ensures that each additional unit of training contributes less marginal improvement (Molloy et al., 2012).

Injury Risk

Injury risk increases more than linearly with high-stress training, while recovery reduces risk with diminishing returns. Injury risk is modeled as a convex funcion:

$$I(x) = \beta_1 x^2{}_1 + \beta_3 x^2{}_3 + \beta_4 x^2{}_4 - \rho ln(1 + x_5)$$

with $\beta_1, \beta_3, \beta_4, \rho > 0$.

Here, quadratic terms capture the rapid increase in risk for high workloads, while recovery reduces risk (UCHealth, 2025).

Weighted Single-Objective Formulation

The weighted objective is

$$F(x) = I(x) - \alpha P(x)$$

where $\alpha > 0$ controls the trade-off (larger α emphasizes performance relative to risk). The optimization problem is:

$$\begin{aligned} & min_x \quad F(x) \\ & s.t. \quad g_1(x) = x_1 + x_2 + x_3 + x_4 + x_5 - T_{max} \leq 0 \\ & \qquad\; g_2(x) = x_1 - H_{max} \leq 0 \\ & \qquad\; g_3(x) = R_{min} - x_5 \leq 0 \\ & \qquad\; g_{3+i}(x) = -x_i \leq 0, \quad i = 1,2,3,4 \end{aligned}$$

Nonnegativity is enforced for $x_1 - x_4$; x_5 is already bounded by the recovery minimum.

This is a nonlinear programming problem with inequality constraints.

Gradient Expressions (needed for KKT interpretation)

The partial derivatives of the objective are:

$$\frac{\partial F}{\partial x_1} = 2\beta_1 x_1 - \alpha \frac{p_1}{1 + x_1}$$

$$\frac{\partial F}{\partial x_2} = -\alpha \frac{p_2}{1 + x_2}$$

$$\frac{\partial F}{\partial x_3} = 2\beta_3 x_3 - \alpha \frac{p_3}{1 + x_3}$$
$$\frac{\partial F}{\partial x_4} = 2\beta_4 x_4 - \alpha \frac{p_4}{1 + x_4}$$
$$\frac{\partial F}{\partial x_5} = -\rho \frac{1}{1 + x_5} - \alpha \frac{p_5}{1 + x_5}$$

These derivatives show explicitly how quadratic risk growth competes against diminishing performance returns.

Convexity/Nonconvexity Discussion

- The performance function is concave. Therefore, the term involving the performance function with the negative sign becomes convex.
- The risk term involving squares is convex since betas must be positive. If betas were negative, the function would become concave, meaning that the more you train, the lower the risk becomes at an accelerating rate. This contradicts the biological principle of "overuse injuries" and the physical reality of fatigue.

Since the negative log is convex the injury risk function is also convex.

Constrained Optimality Theory (Lagrangian & KKT)

For inequality-constrained problems, the Lagrangian is formed using multipliers $\lambda_i \geq 0$

$$L(x, \lambda) = F(x) + \sum_{i=1}^{7} \lambda_i g_i(x)$$

If LICQ holds, the Karush-Kuhn-Tucker conditions are necessary for a local minimum. The optimal schedule x^* and multipliers λ^* satisfy:

1. Stationarity: $\nabla_x L(x^*, \lambda^*) = 0$
2. Primal feasibility: $g(x^*) \leq 0$
3. Dual feasibility: $\lambda^* \geq 0$
4. Complementary slackness: $\lambda^* \boldsymbol{g}(\boldsymbol{x}^*) = \mathbf{0}$

Complementary slackness implies two cases for each inequality constraint: if $g(x^*) < 0$(inactive), then $\lambda^* = 0$; if $g(x^*) = 0$ (active), then λ^* may be positive.
Interpretation for this model.

- The total-hours constraint g_1 is expected to be active in most cases (players usually use all available hours).

- The recovery minimum g_3 becomes active in aggressive performance settings.
- The high-intensity cap g_2 becomes active only if the performance benefit outweighs the risk.

These conditions allow interpretation of constraint activity and optimal allocation behavior.

Numerical Solution Procedure (MATLAB Implementation)

The training-plan optimization problem above is a smooth nonlinear program with inequality constraints. To compute numerical solutions, the problem was implemented and solved in MATLAB using ***fmincon***, which is the standard solver for constrained nonlinear optimization in the MATLAB Optimization Toolbox. The general *fmincon* call follows the following formulation:

$$[x_{opt}, f_{opt}] = fmincon(fun, x_0, A, b, A_{eq}, b_{eq}, LB, UB, nonlcon)$$

where ***fun*** evaluates the objective function $F(x)$, and the remaining inputs represent constraints and bounds.
Here, A_{eq} and b_{eq} are empty since no equality constraints are used.

Objective Function Definition

The objective function $F(x) = I(x) - \alpha P(x)$ was coded in a separate MATLAB function file (***fun.m***). This file returns the numerical value $F(x)$ for any candidate's weekly schedule x. This function file is stored in the same directory as the main script.

Constraint Representation

All inequality constraints were written in the standard form $g(x) \leq 0$, as required for nonlinear programming in MATLAB.

- Weekly training budget: $x_1 + x_2 + x_3 + x_4 + x_5 \leq T_{max}$
- High-intensity cap: $x_1 \leq H_{max}$
- Minimum recovery: $x_5 \geq R_{min}$, rewritten as $-x_5 \leq -R_{min}$

Nonnegativity constraints $x_i \geq 0$ were handled through the lower–bound LB. These inputs match the solver structure outlined in the computational optimization notes, where A, b represent linear inequalities and LB, UB specify variable bounds.

Because the constraints are linear, no nonlinear constraint file was needed (***nonlcon = []***).

Constraint Implementation

In this project, the planning constraints are linear, so they were placed directly into the matrices A and b:

- $\boldsymbol{A} * \boldsymbol{x} \leq \boldsymbol{b}$ for total hours, high-intensity cap, and recovery minimum (rewritten as $-x_5 \leq -R_{min}$).
- LB for nonnegativity and recovery lower bound.

Solver Options and Algorithm Choice

Solver settings were controlled using ***optimset***, as ***optimset*** is used to select algorithms and display/termination options for ***fmincon***.

A feasible initial guess x_0 was provided to start the iterations. The solver termination was based on MATLAB's default feasibility and optimality tolerances unless otherwise stated.

Sensitivity Analysis Runs

After computing a baseline optimal schedule, additional runs were performed to study how the solution changes under different coaching philosophies and recovery requirements. Two main sensitivity experiments were planned:

1. Risk–performance tradeoff: vary α to generate conservative versus aggressive training plans.
2. Recovery requirement: compare baseline $R_{min} = 3$ hours/week ($\approx$ one recovery day) to a safer scenario $R_{min} = 5$ hours/week ($\approx$ two recovery days).

These experiments demonstrate how constraints and tradeoff weights affect the optimal allocation.

Computational Results

This section summarizes the numerical solutions obtained in MATLAB using ***fmincon*** with the SQP algorithm. The solver setup follows the standard course formulation for constrained nonlinear programs, with linear inequalities implemented through A, b and bounds through LB, UB.

Baseline Optimal Schedule

Under the baseline constraints:

$$T_{max} = 18\ hours/week,$$
$$H_{max} = 8\ hours/week, \quad R_{min} = 3\ hours/week,$$

MATLAB returned the following optimal weekly training plan:

$$.x_{opt} = [x_1, x_2, x_3, x_4, x_5] = [3.511, 7.0458, 1.6269, 1.6269, 4.1493].$$

The corresponding minimum objective value was:

$$f_{opt} = -15.3616$$

A clearer presentation of the decision variables is shown in Table 2.

Table 2. *Results under Baseline Constraints*

Component	Meaning	Optimal hours/week
x_1	High-intensity on court drills + match play	3.511
x_2	Low-intensity technical drills	7.046
x_3	Strenght training	1.627
x_4	Conditioning/cardio	1.627
x_5	Recovery work	4.149
Total		$\approx$ 18.00

The optimizer used the full-time budget (18 hours), showing that the weekly training-hours constraint dominates the solution. The largest allocation is to technical drills (x_2), which provides a strong performance benefit without a quadratic risk penalty, while higher-stress categories remain moderate due to convex risk growth. Recovery time is above its minimum, indicating it is valuable even when not forced by constraints.

KKT Multiplier Interpretation (Constraint Activity)

To interpret constraint activity, the Lagrange multipliers reported by MATLAB were examined. The inequality multipliers were:

$$\lambda_{ineqlin} = [0.3107, 0, 0]^T \text{ , as a 3x1 vector.}$$

These correspond to the linear constraints in the order they were defined in A, b:

(1) total weekly hours, (2) high-intensity cap, and (3) recovery minimum.

Because $\lambda_1 = 0.3107$, the total-time budget constraint is active/binding. Since $\lambda_2 = \lambda_3 = 0$, the high-intensity cap and recovery minimum are inactive at the optimum. This matches complementary slackness: inactive constraints have zero multipliers, while active constraints may have positive multipliers.

Lower-bound multipliers were all zero:

$$\lambda_{lower} = [0, 0, 0, 0, 0],$$

confirming that no variable is stuck at its lower bound; each training component remains strictly positive in the optimum.

Sensitivity Study A: Varying Risk–Performance Tradeoff α

To understand how risk tolerance changes the optimal schedule, α should be varied while keeping all constraints fixed. This provides conservative vs. aggressive weekly plans (Table 3).

Table 3. *Results under different* α

α	x_1	x_2	x_3	x_4	x_5	Total
0.5 (risk first)	2.549	7.029	1.179	1.179	6.065	18.00
1.0 (baseline)	3.511	7.046	1.627	1.627	4.149	18.00
2.0 (performance first)	4.633	6.363	2.002	2.002	3.000	18.00

Interpretation:

As the tradeoff weight α increases, the optimizer prioritizes performance over injury risk. The high-intensity and strength components (x_1, x_3, x_4) increase steadily, while recovery (x_5) decreases until it reaches its lower bound at $\alpha = 2$. Conversely, when $\alpha = 0.5$, recovery time expands to over six hours, and high-stress components are reduced, producing a safer but less performance-driven training plan. Across all scenarios, the total time constraint remains active, confirming that weekly training hours are fully utilized.

Sensitivity Study B: Increasing the Recovery Minimum

A second experiment evaluates a stricter recovery policy as shown in Table 4.

Table 4. *Results under different recovery times*

Scenario	x_1	x_2	x_3	x_4	x_5
Baseline $R_{min} = 3$, $\alpha = 1.0$	3.511	7.046	1.627	1.627	4.149
Safer $R_{min} = 5$, $\alpha = 1.0$	3.4762	6.4093	1.5572	1.5572	5.0000

Interpretation:

Increasing the recovery minimum to 5 hours forces x_5 to bind at its lower limit and shift time mainly away from technical and high-stress categories while preserving the full weekly budget.

Practical Interpretation (Hours → Days)

Although decision variables are optimized in hours, recovery can be interpreted in days for narrative clarity. The baseline solution gives:

- $x_5 \approx 4.15\ hours/week$ ≈ 1-2 structured recovery days/week (depending on daily recovery duration).

Discussion

The baseline optimum is realistic because of the nonlinear structure in the objective. The performance term is concave $(ln(1 + x_i))$, so marginal performance benefits decrease as a category grows. This prevents the optimizer from allocating nearly all 18 hours into a single component. In contrast, injury-risk penalties are convex and quadratic for x_1, x_3, x_4, meaning risk accelerates rapidly after moderate volume. As a result, high-stress categories are kept controlled even though they positively influence performance.

The KKT multipliers provide meaningful interpretation. The positive multiplier on the total weekly-hours constraint indicates that time availability is the main limiting resource: if the athlete had a larger weekly budget, the objective value could improve. The zero multipliers for both the high-intensity cap and recovery minimum imply that those policies do not bind under baseline preferences — the model's natural risk penalty already keeps high-intensity volume low, and recovery is worthwhile enough to exceed its minimum without being forced.

Once the sensitivity studies are inserted, this section should emphasize how optimal plans shift based on risk tolerance (α) and on stricter recovery policies. Together, these results show the model can function as a practical decision tool that lets a coach adjust training philosophy while maintaining mathematical consistency with nonlinear constrained optimality principles.

Difficulties, Deviations, and Limitations

The main difficulties, deviations, and limitations for this project are:

1. Illustrative coefficients. The model was designed as a category (iii) project, which allows fictitious but realistic parameter values to demonstrate methodology rather than relying on full athlete-specific calibration.

2. Single-week horizon. The optimization focuses on one representative week and does not explicitly model multi-week fatigue accumulation, tapering cycles, or tournament scheduling.
3. Separable objective terms. Performance and risk are modeled as additive effects of each activity category. Interaction effects (e.g., strength improving tolerance to match play) are not directly included.
4. Local optimality. Because the objective may be nonconvex depending on α, ***fmincon*** guarantees a local optimum. Multiple feasible starting points help confirm stability, but global optimality is not proven.
5. Continuous hours rather than discrete schedules. The solution provides hour allocations, which must still be translated into a day-by-day weekly microcycle by a coach.

These limitations define where the model is most appropriate and motivate future improvements.

Conclusions and Future Work

This project formulated and solved a constrained nonlinear optimization model for weekly tennis training allocation. The objective combined accelerating injury risk penalties and diminishing performance returns, while enforcing realistic weekly planning constraints. The KKT framework supported interpretation of active/inactive constraints, and MATLAB ***fmincon*** (SQP) provided a numerical optimum consistent with course computational optimization methods.

Key conclusions from the baseline results are:

1. The optimal plan uses the entire weekly time budget essentially, and the positive KKT multiplier confirms the total-hours constraint is binding.
2. High-intensity, strength, and conditioning are controlled naturally through nonlinear risk penalties, keeping the high-intensity cap inactive.
3. Recovery exceeds its minimum even without being forced, indicating recovery has strong value in reducing overall risk under baseline preferences.

Future work could extend the model by (a) Optimizing multi-week schedules with fatigue carryover; (b) Calibrating parameters using athlete monitoring data; (c) Adding interaction terms between categories; and (d) Comparing SQP with penalty, barrier, or evolutionary algorithms, discussed later in the course.

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Appendices

A. main.m code in MATLAB

Image 1. *main.m MATLAB code part 1*

```
main.m × fun.m × +
/MATLAB Drive/main.m

clear; clc; close all;

%% ----- Parameters (illustrative baseline) -----
Tmax = 18;     % total weekly hours limit
Hmax = 8;      % max high-intensity hours
Rmin = 3;      % min recovery hours (baseline)

alpha = 1.0;                     % tradeoff weight
p     = [4 2.5 1.5 1.5 1.0];     % performance weights
beta  = [0.08 0.08 0.08];        % risk weights for x1,x3,x4
rho   = 0.6;                     % recovery benefit weight

%% ----- Initial guess (must be feasible) -----
x0 = [4 6 2 2 4];   % sums to 18, x1<=8, x5>=3

%% ----- Linear inequality constraints A*x <= b -----
% (1) x1+x2+x3+x4+x5 <= Tmax
% (2) x1 <= Hmax
% (3) x5 >= Rmin  ->  -x5 <= -Rmin

A = [ 1 1 1 1 1;
      1 0 0 0 0;
      0 0 0 0 -1];

b = [Tmax;
     Hmax;
     -Rmin];

%% ----- No linear equalities -----
Aeq = [];
beq = [];
```

Image 2. *main.m MATLAB code part 2*

```
main.m × fun.m × +
/MATLAB Drive/main.m

b = [Tmax;
     Hmax;
     -Rmin];

%% ----- No linear equalities -----
Aeq = [];
beq = [];

%% ----- Bounds -----
LB = [0 0 0 0 Rmin];   % nonnegativity + recovery minimum
UB = [];               % no upper bounds besides A,b

%% ----- Options (course style) -----
opts = optimset('Display','iter','Algorithm','sqp');

%% ----- Solve -----
[xopt, fopt, exitflag, output, lambda] = ...
    fmincon(@(x) fun(x,alpha,p,beta,rho), x0, A, b, Aeq, beq, LB, UB, [], opts);

%% ----- Display results -----
disp('Optimal schedule xopt = [x1 x2 x3 x4 x5]:');
disp(xopt);

disp('Objective value fopt =');
disp(fopt);

disp('Total hours used =');
disp(sum(xopt));

disp('Lagrange multipliers (lambda) =');
disp(lambda);
```

B. fun.m code in MATLAB

Image 3. *fun.m MATLAB code*

main.m × fun.m ×

/MATLAB Drive/fun.m

```
function f = fun(x, alpha, p, beta, rho)
% x = [x1 x2 x3 x4 x5] in hours/week

x1 = x(1);
x2 = x(2);
x3 = x(3);
x4 = x(4);
x5 = x(5);

% performance weights
p1 = p(1); p2 = p(2); p3 = p(3); p4 = p(4); p5 = p(5);

% risk weights
b1 = beta(1);  % for x1
b3 = beta(2);  % for x3
b4 = beta(3);  % for x4

% ----- Performance benefit (concave, diminishing returns) -----
P = p1*log(1+x1) + p2*log(1+x2) + p3*log(1+x3) + ...
    p4*log(1+x4) + p5*log(1+x5);

% ----- Injury risk (convex overload + recovery benefit) -----
I = b1*x1^2 + b3*x3^2 + b4*x4^2 - rho*log(1+x5);

% ----- Weighted objective to MINIMIZE -----
f = I - alpha*P;
end
```

B. MATLAB outputs

Image 4. *main.m MATLAB outputs with alpha = 1.0 part 1*

Command Window

```
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step   optimality
    0           6   -1.525346e+01   0.000e+00     1.000e+00     0.000e+00     3.571e-01
    1          12   -1.528445e+01   0.000e+00     1.000e+00     1.836e-01     1.118e-01
    2          18   -1.535445e+01   3.553e-15     1.000e+00     7.720e-01     3.460e-02
    3          24   -1.535705e+01   0.000e+00     1.000e+00     1.657e-01     1.586e-02
    4          30   -1.535758e+01   0.000e+00     1.000e+00     3.088e-02     1.516e-02
    5          36   -1.535984e+01   3.553e-15     1.000e+00     1.474e-01     9.777e-03
    6          42   -1.536131e+01   0.000e+00     1.000e+00     2.127e-01     9.127e-03
    7          48   -1.536149e+01   0.000e+00     1.000e+00     3.187e-02     6.472e-03
    8          54   -1.536163e+01   0.000e+00     1.000e+00     3.279e-02     1.197e-03
    9          60   -1.536163e+01   0.000e+00     1.000e+00     4.650e-03     1.737e-04
   10          66   -1.536163e+01   0.000e+00     1.000e+00     1.061e-03     1.149e-05
   11          72   -1.536163e+01   0.000e+00     1.000e+00     5.733e-05     1.144e-06
   12          78   -1.536163e+01   0.000e+00     1.000e+00     1.232e-05     5.877e-08

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal schedule xopt = [x1 x2 x3 x4 x5]:
    3.5511    7.0458    1.6269    1.6269    4.1493

Objective value fopt =
  -15.3616
```

Image 5. *main.m 0MATLAB outputs with alpha = 1.0 part 2*

```
Command Window

Total hours used =
    18

Lagrange multipliers (lambda) =
         eqlin: [0×1 double]
      eqnonlin: [0×1 double]
       ineqlin: [3×1 double]
    ineqnonlin: [0×1 double]
         lower: [5×1 double]
         upper: [5×1 double]

>> lambda.ineqlin
lambda.lower

ans =

    0.3107
         0
         0


ans =

     0
     0
     0
     0
     0
```

Image 6. *main.m MATLAB outputs with alpha = 2.0*

```
Command Window
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           6   -3.146127e+01     0.000e+00     1.000e+00     0.000e+00     9.600e-01
    1          12   -3.154468e+01     0.000e+00     1.000e+00     3.169e-01     1.702e-01
    2          18   -3.162933e+01     0.000e+00     1.000e+00     6.994e-01     6.833e-02
    3          24   -3.163917e+01     0.000e+00     1.000e+00     1.740e-01     4.975e-02
    4          30   -3.164887e+01     0.000e+00     1.000e+00     1.933e-01     3.389e-02
    5          36   -3.165133e+01     0.000e+00     1.000e+00     8.862e-02     1.870e-02
    6          42   -3.165232e+01     0.000e+00     1.000e+00     6.580e-02     1.370e-02
    7          48   -3.165288e+01     0.000e+00     1.000e+00     5.377e-02     1.034e-02
    8          54   -3.165305e+01     0.000e+00     1.000e+00     2.891e-02     4.252e-03
    9          60   -3.165306e+01     0.000e+00     1.000e+00     6.383e-03     7.387e-04
   10          66   -3.165306e+01     0.000e+00     1.000e+00     1.454e-03     4.955e-05
   11          72   -3.165306e+01     0.000e+00     1.000e+00     1.222e-04     2.486e-06
   12          78   -3.165306e+01     0.000e+00     1.000e+00     3.271e-06     7.150e-08

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal schedule xopt = [x1 x2 x3 x4 x5]:
    4.6327    6.3634    2.0020    2.0020    3.0000

Objective value fopt =
  -31.6531
```

Image 7. *main.m MATLAB outputs with alpha = 0.5*

```
Command Window
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           6   -7.149564e+00     0.000e+00     1.000e+00     0.000e+00     2.400e-01
    1          12   -7.287261e+00     0.000e+00     1.000e+00     3.842e-01     2.072e-01
    2          18   -7.653805e+00     0.000e+00     1.000e+00     1.680e+00     9.212e-02
    3          24   -7.696696e+00     0.000e+00     1.000e+00     7.541e-01     7.848e-02
    4          30   -7.704485e+00     0.000e+00     1.000e+00     1.096e-01     5.634e-02
    5          36   -7.711652e+00     0.000e+00     1.000e+00     2.285e-01     7.000e-03
    6          42   -7.711746e+00     0.000e+00     1.000e+00     1.239e-02     6.803e-03
    7          48   -7.712293e+00     0.000e+00     1.000e+00     6.768e-02     5.785e-03
    8          54   -7.713606e+00     0.000e+00     1.000e+00     3.038e-01     5.793e-03
    9          60   -7.713743e+00     3.553e-15     1.000e+00     6.353e-02     2.787e-03
   10          66   -7.713770e+00     0.000e+00     1.000e+00     2.348e-02     1.912e-04
   11          72   -7.713770e+00     0.000e+00     1.000e+00     2.366e-03     9.743e-06
   12          78   -7.713770e+00     0.000e+00     1.000e+00     3.139e-04     2.740e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal schedule xopt = [x1 x2 x3 x4 x5]:
    2.5490    7.0286    1.1786    1.1786    6.0652

Objective value fopt =
   -7.7138
```

Image 8. *main.m MATLAB outputs with* R_{min} = 5

```
Command Window
Initial point X0 is not between bounds LB and UB;
FMINCON shifted X0 to satisfy the bounds.
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           6   -1.554518e+01     1.000e+00     1.000e+00     0.000e+00     3.571e-01
    1          12   -1.530723e+01     0.000e+00     1.000e+00     5.250e-01     2.071e-01
    2          18   -1.531308e+01     0.000e+00     1.000e+00     8.002e-02     6.932e-02
    3          24   -1.532870e+01     0.000e+00     1.000e+00     3.480e-01     2.472e-02
    4          30   -1.533070e+01     0.000e+00     1.000e+00     1.702e-01     8.379e-03
    5          36   -1.533078e+01     0.000e+00     1.000e+00     1.173e-02     5.474e-03
    6          42   -1.533083e+01     0.000e+00     1.000e+00     1.623e-02     2.000e-04
    7          48   -1.533083e+01     0.000e+00     1.000e+00     7.148e-04     1.140e-05
    8          54   -1.533083e+01     0.000e+00     1.000e+00     5.325e-05     5.895e-08

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal schedule xopt = [x1 x2 x3 x4 x5]:
    3.4762    6.4093    1.5572    1.5572    5.0000

Objective value fopt =
  -15.3308

Total hours used =
    18
```