# Mathematical Model of HoloLens 2 Goggles Kinematics for AR Support of Flight Simulator

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The paper describes how the kinematics model of HoloLens 2 goggles mounted on the user's head is used for converting the reference frame fixed to the flight simulator with the reference frame fixed to the goggles. The constant correspondence between the two has to be kept to properly map the internal 3D space relative to the position of the device with the real 3D space of the flight simulator cockpit in which HoloLens 2 produces holographic objects. Such conversions are prerequisites of SLAM (Simultaneous Localization and Mapping) algorithms which are used by AR devices to build a map of an environment, and, at the same time, this map is used to compute the goggle's own position and orientation. In the paper, we also present the application of HoloLens 2 goggles in the WrightBroS project to support flight simulator users (pilots and/or technicians) in their training and maintenance procedures.

**Keywords**: augmented reality (AR), slam, kinematics, hololens 2, flight simulators

### Introduction

The advances in flight simulators technology focus on the use of architectures supported by Augmented Reality (AR) as opposed to currently available architectures which are based solely on Virtual Reality (VR). This approach has a potential to significantly broaden the market of professional products by introducing "learn as you go" paradigm to train pilots and technical staff in operation, maintenance and servicing of these innovative products. The complete platform referred to as "LEARN AS YOU GO" is composed of the following applications:

- General image recognition engine a special programming tool for AR.
- Cockpit procedures app AR manual for pilots application recognizes
  the cockpit of the aircraft and guides the pilot through the cockpit
  procedures in the pre-flight preparation mode, during all emergency
  procedures etc.
- Virtual maintenance and service app application guiding technicians of the aircraft or flight simulator through the database of components, repairing procedures and logistic operations.

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This concept, when deployed in construction of flight simulators requires manufacturing of knowledge-based smart diagnostic products capable for detection of malfunctioning elements and presenting to the user step-by-step interactive instructions for making system functional. In addition, a constant alignment between two 3D spaces (internal space of AR goggles and external space of the flight simulator) needs to be kept for proper displaying of holographic images. In the paper, we focus on this latter approach by using cockpit flight simulator as an example of an electro-mechanical product characterized by complex human-machine interfaces in which a pilot is supported in his/her activities by presenting him/her holographic images created by HoloLens 2 googles. One of the prerequisites of how to achieve this alignment is described by mathematical model of HoloLens kinematics transformation. The aim of this paper is to apply the well-known mathematical formalism in the context of Augmented Reality device (HoloLens 2 goggles) working in the flight simulator cockpit (Boeing 737).

Following this introductory section, the rest of the paper contains the Literature review section (in which state-of-the-art of advances in flight simulators, augmented reality, and SLAM algorithms is presented) and Methodology section where the kinematics model of the goggles is given. Then, Results and Discussion section presents how results of the modelling are used in the AR actual system and discusses what additional components are essential for the final solution. The Conclusions, Acknowledgements and References are closing the article.

### **Literature Review**

The enhancement of air traffic is a priority in aviation development. As new technologies are implemented, the demands on the crew's theoretical and practical skills, especially those of the pilots, are increasing. Because of their ability to mimic the virtual reality of flights, flight simulators have proven to be a powerful part of pilot training. Virtual reality, thanks to modern technology, faithfully recreates real flights, removing concerns and uncertainties among pilots, airlines, aircraft manufacturers, and regulatory agencies about its usage in training (F012-02 2009). Adopting flight simulators into pilot training lowered hazards and improved training quality, while also improving overall flying safety and lowering training and aircraft operating costs. According to research, aircraft incidents are mostly due to human errors, the pilot's inabilities, or a pilot is caught off guard by adversity and responds in an effective manner (Safety Regulation Group 2002). However, the results differ depending on the scenarios. These errors lead to a series of events that causes flight incidents.

Flight simulation uses a technique called flight simulator, to simulate the flight of an airplane and the surroundings in which it flies. In addition, it mimics the model that controls how an aircraft flies and how it responds to flight control applications and its external elements like air density, turbulence, precipitation, wind shear, cloud, etc. In this regard, augmented reality (AR) is a potential technology for developing enhanced interfaces with interactive and wearable

visualization systems to apply new techniques for displaying documents as digital data and graphical databases (de Crescenzio et al. 2011).

The progress in real pilot training comes down to realism, and a large part of that is the motion tracking technology utilized in both live and simulated aircraft applications. Motion tracking is the act of establishing the spatial and temporal relationships between moving objects and the robot or between moving and stationary objects (Wang et al. 2007). To add to the visuals to move somewhat, (as they would when a pilot moves their head or changes their viewpoint in a genuine cockpit), you want to know the position furthermore, the direction of their head in 3D space. To accomplish this, a helmet or even a pair of glasses can be used to track the pilot's head throughout their range of motion in the cockpit.

Currently, several motion capture systems are found on simultaneous localization and mapping (Sturm et al. 2012). Positional tracking in virtual reality (VR) determines the exact location of head-mounted displays, controllers, other objects, and body parts in Euclidean space. Because the goal of virtual reality is to simulate real-world experiences, it's essential that positional tracking is exact and precise so that the illusion of three-dimensional space is maintained (Lang 2013).

Since its inception in the 1980s, Simultaneous Localization and Mapping (SLAM) has become a significant field that reaches across many disciplines, particularly in the fields of intelligent systems and robotics. Many people feel that solving the SLAM problem will open a world of possibilities for autonomous robots (Cadena et al. 2016). The development in tackling SLAM challenges has been swift and exciting, with several interesting implementations of SLAM approaches. To solve mapping and localization difficulties, some academics had been investigating the use of estimation-theoretic methods (Durrant-Whyte and Bailey 2006). A SLAM system must be able to estimate the robot's pose (position and orientation) in the world while also creating a map of the environment using data collected by a collection of sensors attached to the robot (Azuma et al. 2001). To navigate through a previously unknown world or to modify a map of a previously known environment, the SLAM method employs a sophisticated array of computations, algorithms, and sensory inputs (Maxwell 2013).

The foundation for scene understanding is initiating the spatial and temporal links between a robot, fixed items, and moving objects in a scene. Because of ambiguity and unobservable states in the real world, localization, mapping, and moving object tracking are challenging. Motion sensors like odometry and inertial measurement units, as well as perception sensors like cameras, radar, and laser range finders, are noisy. Without the use of additional sensors positioned on the moving items, the goals, or controlling inputs, of the mobile robots, remain unobservable (Smith et al. 1990). Due to the obvious usual mistake in predicted vehicle location, while a mobile robot drives across an unfamiliar area taking relative observations of landmarks, the estimations of these landmarks must relate to each other (Bailey and Durrant-Whyte 2006).

To build stable values through repeated exploration of the world, SLAM must ideally converge over time, which is a highly stochastic process because all calculations and outputs are expressed as probabilities. The robot keeps track of its estimated (probabilistic) position while navigating independently across an

unknown environment by comparing it to a global static frame, typically situated at the starting location or a previous pose frame. It evaluates the environment around it using its sensory processes and builds a probabilistic model of the constituent parts in relation to a fixed global frame of reference.

Critically, at the map's convergence qualities or steady-state behavior. As such, it was commonly anticipated at the time that the predicted map errors would not converge and instead would follow a random walk with limitless error accumulation. Given the computational complexity of the mapping problem and the lack of knowledge about the map's convergence behavior, researchers instead focused on a series of approximations to the consistent mapping problem solution, which assumed or even forced the correlations between landmarks to be drastically reduced or eradicated, effectively reducing the full filter to a series of disconnected landmark to vehicle filters (Leonard and Durrant-Whyte 1992).

SLAM scales quadratically with the number of landmarks on a map in its most basic version. This scale could be a significant constraint in the usage of SLAM approaches in real-time applications. There have been various techniques explored to lessen this uncertainty. These techniques comprise linear-time state augmentation, information form sparsification, partitioned updates, and submapping procedures.

The accurate association of landmark observations with landmarks recorded on the map is another important hurdle to overcome in the application of SLAM technologies. An inaccurate association can cause the SLAM algorithm to fail catastrophically. When a robot returns to an earlier mapped territory after a long journey, data association is very crucial; this is known as the "loop closure" issue. Batch validation approaches that utilize limitations inherent in the SLAM formulation, appearance-based methods, and multi-hypothesis procedures are some of the current data association methods employed in SLAM (Bailey and Durrant-Whyte 2006).

The major benefit of SLAM is that it does not require any artificial infrastructure or prior topological knowledge of the surroundings. A solution to the SLAM problem would be invaluable in a variety of applications where the absolute position or precise map information is unavailable, such as autonomous planetary exploration, subsea autonomous vehicles, autonomous airborne vehicles, and autonomous all-terrain vehicles in mining and construction, to name a few (Dissanayake et al. 2001).

Augmented Reality (AR) systems based on Simultaneous Localization and Mapping (SLAM) have been gaining popularity due to their promise to give users an immersive and participatory experience. By showing 3D virtual items registered in a user's natural environment, it enables users to interact with both real and computer-generated objects (Bajura and Neumann 1995). Based on the development of Simultaneous localization and mapping (SLAM) technology, users can walk about in actual space and engage with virtual objects because of this (Chi et al. 2020). Since a home user may not carefully move the AR device and the real environment may be complicated, a range of tough conditions (e.g., fast motion, powerful rotation, substantial motion blur, dynamic interference) may be easily encountered for AR applications in practice (Jinyu et al. 2019). AR

approaches seek to provide users with information that is spatially consistent with the seen scene. They show this information by enhancing a camera-captured view with graphical objects that are suitably aligned with real-world 3D structures. Real-time estimation of the camera's 3D position and orientation (often referred to as the pose) in relation to the real-world objects to be enhanced is a crucial AR technology deriving from computer vision. For AR applications, the camera posture must be estimated in real-time, precisely, and without drift, and the surroundings must be represented using a metric scale. To initialize or maintain the metric scale while solving odometry progressively, current AR systems must use additional range sensors, which is their principal bottleneck. However, a potential solution lies within the SLAM algorithm which relies on visual tracking (Laviola et al. 2017).

# Methodology

The kinematic model of HoloLens 2 goggles is derived from kinematics of a rigid body described for example by Bestaoui Sebbane (2012) and it is used here in the context of Augmented Reality where a match between virtual 3D space of the goggles needs to be constantly aligned to the real 3D space of external world (in our case flight simulator cockpit). The model of dynamics is omitted as the inertia of the goggles is negligible for movements of the head of the user. Thus, all functional dimensions and degrees of freedom of the goggles are mathematically described here only as kinematics. This method also includes a description of the object's workspace, positional capabilities, and limits. Depending on the axis around which the rotations are performed, there are different Euler angles  $(\emptyset, \theta, \psi)$  where:

- $\psi$ : Yaw (around vertical axis z, head left and right movement)
- $\theta$ : Pitch (around horizontal axis y, head up and down movement)
- $\phi$ : Roll (around horizontal axis x, head left and right movement)

By using these angles, we denote the goggles-fixed reference frame orientation as a vector:

$$\eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{1}$$

Further, we introduce a  $3\times3$  rotation matrix  $R(\eta_2)$ , in which each column is a unit vector provided in terms of the navigation axes. Such rotation matrix is referred to as the direction cosine matrix and is used to transformation of the goggles-fixed reference frame coordinates relative to the external reference frame coordinates. From mathematics of rotations it follows that:

$$R(\eta_2) = R_z(\psi)R_y(\theta)R_x(\phi)$$
, where:

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$$R_{x}(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}, R_{y}(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 0 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, R_{z}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(2)$$

Hence, after computation we get:

(3)

$$R(\eta_2) = \begin{bmatrix} \cos\psi \cos\theta & -\sin\psi \cos\phi + \cos\psi \sin\theta \sin\phi & \sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi \\ \sin\psi \cos\theta & \cos\psi \cos\phi + \sin\psi \sin\theta \sin\phi & -\cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi \\ -\sin\theta & \cos\theta \sin\phi & \cos\theta \cos\phi \end{bmatrix}$$

By using cosine direction matrix, we are able to convert coordinates of any point in the goggles-fixed reference frame to the corresponding coordinates in the flight simulator-fixed reference frame.

For this, we denote:  $\eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  as a vector of coordinates of some point in the

simulator-fixed frame, and  $\eta'_1 = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$  as a vector of corresponding coordinates in

the goggles-fixed frame. Then,  $\eta_1 = R(\eta_2)\eta_1$ . Similarly, the linear velocities of

the selected point in goggles-fixed frame, given by: 
$$v_1 = V = \begin{bmatrix} \frac{d}{dt}x' \\ \frac{d}{dt}y' \\ \frac{d}{dt}z' \end{bmatrix}$$
, are

transforming to linear velocities in simulator-fixed frame according to:

$$\frac{d\eta_1}{dt} = R(\eta_2)V \tag{4}$$

Note, that a set of rotation matrices  $R(\eta_2)$  forms a special orthogonal group SO(3) defined as

$$SO(3) = \{ R \in \Re^{3 \times 3}, R^T R = 1_{3 \times 3}, det(R) = 1 \}.$$
 (5)

The rotation matrix and the displacement vector are both combined into one matrix called the homogeneous transformation matrix (this representation is referred to as homogeneous matrix formulation).

Let  $R(\eta_2) = [R_1(\eta_2) \ R_2(\eta_2) \ R_3(\eta_2)]$ , where  $R_i \in \Re^3$  (6) Then, the homogeneous matrix formulation allows for unique representation of the orientation and position of HoloLens 2 goggles by using  $A_M$  matrix:

$$A_{M} = \begin{bmatrix} R(\eta_{2}) & \eta_{1} \\ 0_{3\times 1} & 1 \end{bmatrix} = \begin{bmatrix} R_{1}(\eta_{2}) & R_{2}(\eta_{2}) & R_{3}(\eta_{2}) & \eta_{1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(7)

Set of matrices  $A_M$  is a special Euclidean group SE (3).

$$SE(3) = \left\{ A_M \in \Re^{4 \times 4} \middle| A_M = \begin{bmatrix} R(\eta_2) & \eta_1 \\ 0_{3 \times 1} & 1 \end{bmatrix}, R(\eta_2) \in SO(3), \eta_1 \in \Re^3 \right\}$$
(8)

It describes transformations in three dimensions with six degrees of freedom (6DoF). Six degrees of freedom transformations, including three position vector coordinates (identifying, for example, the location of the center of mass) and three angles (e.g., Euler angles or yaw, pitch, and roll) are used to uniquely parameterize a 3×3 rotation matrix—to define the pose of a three-dimensional rigid body such as HoloLens 2 goggles in our case.

Note, that the special Euclidean group, also known as the special Euclidean transformations group, forms a Lie group as follows. Let us define S(t) as:

$$S(t) = A_m^{-1}(t) \frac{dA_M(t)}{dt} = \begin{bmatrix} R_1^T & -R_1^T \eta_1 \\ R_2^T & -R_2^T \eta_1 \\ R_3^T & -R_3^T \eta_1 \\ 0_{3\times 3} & 1 \end{bmatrix} \begin{bmatrix} \frac{dR_1}{dt} & \frac{dR_2}{dt} & \frac{dR_3}{dt} & \frac{d\eta_1}{dt} \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} R_1^T & -R_1^T \eta_1 \\ R_2^T & -R_2^T \eta_1 \\ R_3^T & -R_3^T \eta_1 \\ 0_{3\times 3} & 1 \end{bmatrix} \begin{bmatrix} \frac{dR}{dt} & \frac{d\eta_1}{dt} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} R^T \frac{dR}{dt} & R^T \frac{d\eta_1}{dt} \\ 0_{3\times 1} & 0 \end{bmatrix}$$
(9)

Let  $v_2 = \Omega = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$  are angular velocities in a goggles-fixed reference frame,

Then: 
$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} \frac{d\phi}{dt} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \frac{d\theta}{dt} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{d\psi}{dt} \end{bmatrix}$$

(10)

Hence,

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$$\begin{bmatrix} w_{x} \\ w_{y} \\ w_{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi & \cos\theta \\ 0 & -\sin\phi & \cos\phi & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{d}{dt} \phi \\ \frac{d}{dt} \theta \\ \frac{d}{dt} \psi \end{bmatrix} \text{ or equivalently: } \begin{bmatrix} \frac{d}{dt} \phi \\ \frac{d}{dt} \theta \\ \frac{d}{dt} \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi & \cos\theta \\ 0 & -\sin\phi & \cos\phi & \cos\theta \end{bmatrix}^{-1} \begin{bmatrix} w_{x} \\ w_{y} \\ w_{z} \end{bmatrix}$$
(11)

Now, let consider a vector that is tangential in the specific location to a curve drawn by some point, say center of a mass, of the goggles in a SE(3) configuration space. This vector is a tangent vector of a curve.

Note also that: 
$$R^T \frac{dR}{dt} = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & w_x \\ -w_y & w_x & 0 \end{bmatrix} = SK \begin{pmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \end{pmatrix} = SK(v_2),$$
 (12)

Hence, given a curve  $C(t): [-T,T] \to SE(3)$ , a set  $S(t) = \frac{dC(t)}{dt}$  (of tangent vectors at arbitrary configuration of a HoloLens 2 given by  $A_M(t)$  is a lie algebra se(3) with elements  $S(t) \in se(3)$  given by:

$$S(t) = \frac{dC(t)}{dt} = A_M^{-1}(t) \frac{dA_M(t)}{dt} = \begin{bmatrix} 0 & -w_z & w_y & R_1^T \frac{dn_1}{dt} \\ w_z & 0 & -w_x & R_2^T \frac{dn_1}{dt} \\ -w_y & w_x & 0 & R_3^T \frac{dn_1}{dt} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
(13)

Now, let us define matrix 
$$J(\eta_2)$$
 as: 
$$J(\eta_2) = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}^{-1} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix}$$
, then

$$\frac{d\eta_{2}}{dt} = \begin{bmatrix} \frac{d}{dt} \phi \\ \frac{d}{dt} \theta \\ \frac{d}{dt} \psi \end{bmatrix} = J(\eta_{2}) \begin{bmatrix} w_{x} \\ w_{y} \\ w_{z} \end{bmatrix} = J(\eta_{2}) \Omega \text{ and } \frac{d\eta_{1}}{dt} = \begin{bmatrix} \frac{d}{dt} x \\ \frac{d}{dt} y \\ \frac{d}{dt} z \end{bmatrix} = R(\eta_{2}) \begin{bmatrix} \frac{d}{dt} x' \\ \frac{d}{dt} y' \\ \frac{d}{dt} z' \end{bmatrix} = R(\eta_{2}) V \tag{14}$$

Thus, we have the following 6D representation of the kinematic relationship of HoloLens 2:

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$$\begin{bmatrix}
\frac{d\eta_1}{dt} \\
\frac{d\eta_2}{dt}
\end{bmatrix} = \begin{bmatrix} R(\eta_2) & 0_{3\times 3} \\ 0_{3\times 3} & J(\eta_2) \end{bmatrix} \begin{bmatrix} V \\ \Omega \end{bmatrix} = RV$$
(15)

Quaternions offer an intriguing alternative to Euler angles for describing the rotation of bodies in a space and have several advantages over them, including the absence of discontinuities and gimbal lock as well as mathematical simplicity. Therefore, quaternions are an excellent tool for describing how rigid bodies change when numerous rotations and translations are considered in increasingly complicated systems (Abaunza et al. 2018).

The Euler parameters of a rotation about the axis  $\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ 

by angle  $\mu$  have representation of a unit quaternion q such that:

$$q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \cos\frac{\mu}{2} \\ \sin\frac{\mu}{2}n_x \\ \sin\frac{\mu}{2}n_y \\ \sin\frac{\mu}{2}n_z \end{pmatrix}, 0 K \le \mu \le 2\pi$$

$$(16)$$

Thus, configuration space  $SO(3) \times R^3$  is replaced here by  $S^3 \times R^3$ , where:

$$S^3$$
 is a unit hypersphere in  $R^4$ :  $S^3 = \{x \in R^4 \text{ such that } ||x||_2 = 1\}$ 

## **Results and Discussion**

The results of aforementioned transformations between goggles (virtual 3D space) and flight simulator (real 3D space) reference frames and the application in actual AR system are based on the research program of the WrightBroS project that is aimed to develop a prototype of a new technology professional flight simulator to demonstrate the deployment of augmented reality (AR) in such environment (see Figure 1).

Figure 1. AR Support of Flight Simulator





Source: WrightBroS Project.

For functioning of the whole system, not only the presented above transformations between internal HoloLens 2 reference frame and the flight simulator-fixed reference frames need to be implemented (for matching these two spaces), but also the scenario player of the information provided with the goggles should be integrated. Bach et al. (2021) have described the architecture of such scenario player as a hierarchy of finite state machines. Integration of the scenario player with HoloLens 2 is still under development.

## **Conclusions**

In the paper, the kinematic model of HoloLens 2 goggles movements (corresponding to the user head movements) has been considered. It is a theoretical basis for matching the internal (virtual) 3D space of the goggles, which move with the movement of the user's head, with the external (stationary) 3D space of the flight simulator. Application of this model to an AR system based on HoloLens 2 goggles on a head of a person working in a flight simulator environment can help in better mapping of the real space of the flight simulator with its representation in the AR system.

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