

Early STEAM Education Practice: Application of Graph Theory through Teaching Assistants

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In the age of Society 5.0, which is the concept of a future society developed by the Japanese government, science, technology, engineering, art, and mathematics (STEAM) human resources with the skills to grasp things from multiple perspectives and solve problems will be required. Furthermore, Society 5.0 indicates that the National Institute of Technology (KOSEN) will become the STEAM center for elementary and junior high school students, as part of the efforts to establish a system that supports STEAM education. Since 2019, we have practiced STEAM education as part of “Liberal Arts Special Lectures” for 4th-year students of the main course (1st year of the undergraduate course). In these lectures, the teachers of liberal arts subjects present themes using their specialties, such as mathematics, debate, and economics. Collaborative learning between students from various departments led them to deep learning, which was a fusion of knowledge and creation. However, there are few opportunities to give back to society, particularly a platform to disseminate the acquired mathematics ability. Therefore, we aim to realize early STEAM education and give back to society by creating STEAM teaching materials on graph theory in open courses for junior high school students between the ages of 13 and 15 with help from teaching assistants who studied graph theory in the liberal arts special lectures. Moreover, through a course on graph theory, the course will not only arouse students' interest in mathematics, but also provide them with clues to develop their multifaceted ideas.

Keywords: *society 5.0, graph theory, open course for junior high school students, teaching assistants, liberal arts special lecture*

Introduction

Introduced by Yakman (2008), science, technology, engineering, art, and mathematics (STEAM) education is an approach to learning that uses science, technology, engineering, the arts, and mathematics as access points to guide student inquiry, dialogue, and critical thinking to solve problems in the real world. As an attempt, we have formulated financial education material on simple interest and compound interest from the perspective of STEAM education, i.e., a fusion of economics and mathematics. Moreover, we have used this material in an open course for citizens by utilizing the abilities of liberal arts special course students

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and the 3rd-year students who studied both subjects as instructors. As a result, we have been highly rated by the participants and the students have had a good opportunity to give back their acquired knowledge and ability to society.

As another attempt, this work mainly aims to create STEAM teaching materials on graph theory for open courses for junior high school students and practice early STEAM education; liberal arts special course students are employed as teaching assistants, and the acquired mathematics ability is returned to society.

Graph theory is a mathematical theory about figures consisting of a set of vertices and edges; it has one of its origins in 1736 when Leonhard Euler solved the “Königsberg problem,” which is closely related to the single stroke. In the era of big data, it has become important not only to analyze each individual element but also to analyze how these elements are connected and what properties they have as a whole. Graph theory, which is a powerful weapon for data analysis to be handled in future mathematical learning, is now attracting a great deal of attention.

Additionally, since a graph is easily understood visually and little prior knowledge is required to grasp it, it is suitable as a STEAM learning material for a wide number of generations.

One of the objectives of mathematics in junior high school is "to deepen intuitive views and ideas about figures through activities such as observation, manipulation, and experimentation with plane and spatial figures, and to cultivate the ability to consider and express figures logically". This paper deals with the development and practice of teaching materials for mathematically considering real-world problems with discrete structures, using graphs as examples of plane figures that students are currently studying.

The open course carried out in 2022 is divided into three parts: an introduction to graph theory, including the Königsberg problem and single stroke, applications to social networks, and applications to maximum flow and minimum cut problems. The findings reveal that junior high school students provide high survey ratings, whereas teaching assistants have an invaluable opportunity to give back to society by making use of their acquired skills. The results of a questionnaire demonstrate that our course is effective for learners, and its potential as a STEAM teaching theme is shown. Moreover, the teaching assistants can study graph theory in depth because of the lectures; they can obtain new results and successfully present their research at the 28th KOSEN Symposium in 2023.

Herein, we first describe the graph theory in three parts: an introduction including the Königsberg bridge problem and one stroke one, the connections in society that can be represented by graphs, an algorithm called the Ford-Fulkerson method. Second, the content of our practice is outlined, and we review the scenes of open courses with photos, including a description of the results of the questionnaire obtained from the participants. Finally, we present concluding remarks and future scope.

Graphs

In this section, we introduce the graph and its notation. First, we define graphs, which are the theme of this open course, and define the degree, which is a quantity

used to characterize graphs. Consider the diagram shown in Figure 1.

Remark. The graph we are dealing with here is a set of points and edges, which is different from a function graph.

Example 1. Points P , Q , R , S , and T are called vertices, the lines are called edges, and the entire diagram is called a graph. The degree of a vertex is the number of edges with the vertex as an endpoint. For example, the degree of vertex P is 3, and the degree of vertex Q is 4.

Figure 1. An Example of Graphs

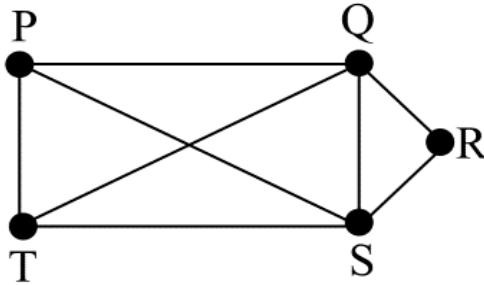


Figure 1 presents a graph. Formally, graph G consists of a finite non-empty set (V) of objects called vertices (the singular is a vertex) and a set (E) of 2-element subsets of V , called edges. Therefore, graph G is a pair (an ordered pair) of two sets (V and E). Thus, some write $G = (V, E)$. Hereinafter, for simplicity, a vertex is referred to as a point. The vertex set of graph G is denoted by $V(G)$, and the edge set of graph G is denoted by $E(G)$. The cardinality of $V(G)$ is the order of G , and the cardinality of $E(G)$ is the size of G . For example, the order and size of the graph shown in Figure 1 are 5 and 8, respectively.

There are many numbers, referred to as parameters, associated with graph G . Knowing the values of specific parameters provides us with information about G but rarely tells us the entire structure of G . We mentioned the best-known parameters: the order and the size. Further, numbers were associated with each vertex of the graph. This is called the degree of a vertex. The degree of a vertex (v) in graph G is the number of edges incident on v and is denoted as $\deg(v)$. For example, for vertices P and Q in the graph shown in Figure 1, $\deg(P) = 3$ and $\deg(Q) = 4$, respectively.

Next, we define connected graphs. Most graphs covered in this open course were connected graphs.

Definition 2. A graph is connected if it cannot be expressed as a union of graphs.

What we have shown in Figure 1 is a connected graph. Graph G is said to be connected if any two vertices (x, y) in G , G have an x - y path (a path in a graph is a finite sequence of edges that joins a sequence of vertices that are all distinct, and x - y path is a path from x to y).

Structure of the Open Course

In this class, we set graph theory as the theme for early STEAM education and provide an opportunity for students who have studied it in Advanced Liberal Arts to teach it as TAs, aiming not only to arouse students' interest in mathematics but also to give them clues to have multiple perspectives. Therefore, the class will be organized as follows.

(a) *Learning contents*

Graph theory and its application.

(b) *Construction*

- Participants

Thirty-four Junior high school students (1st to 3rd grade)

- Leaders

Three teachers and two teaching assistants (TAs) who studied graph theory in a special lecture on the liberal arts

- Time

210 (60 + 60 + 90) min

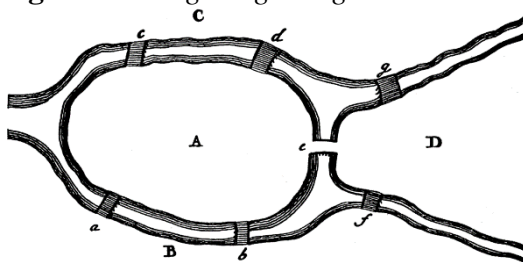
The Content of our Practice

Our practice in graph theory consists of three parts. First, as an introduction to graph theory, we introduce the definition of graphs and topics in graph theory such as the Königsberg bridge and one stroke. Next, we will look at some of the connections in society that can be represented by graphs and some of the things that become clear by doing so. Finally, as an application of graph theory, we introduce an algorithm called the Ford-Fulkerson method.

(a) *Introduction to graph theory*

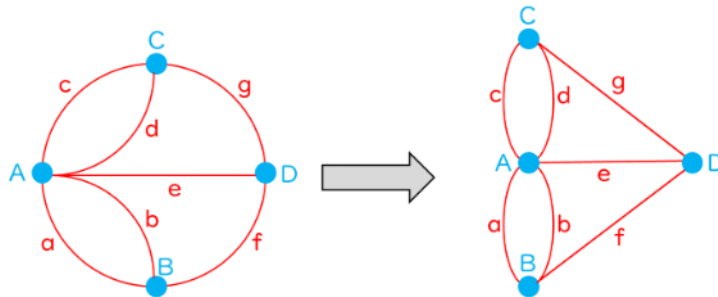
First, we showed a diagram of the Königsberg bridges and asked whether or not you could cross each of the seven bridges shown in Figure 2 once and return to your starting point.

Figure 2. Königsberg Bridge



Next, we explained how the basic idea of topology was used: crossing a bridge was independent of the shape and size of the river, land, and bridge. The graph was constructed by transforming (continuously) the land shown in Figure 2 as points and the bridges connecting lands as lines, as shown in Figure 3.

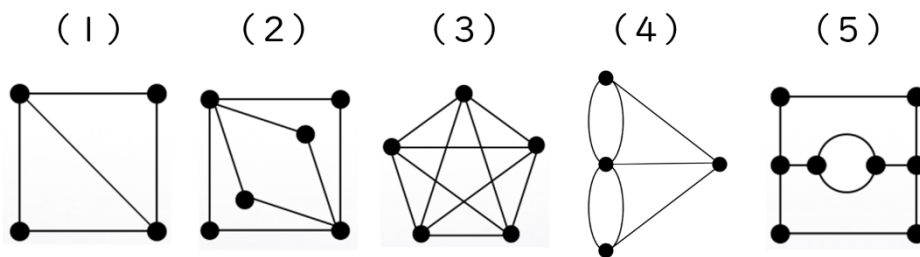
Figure 3. Figure of a Graph



Remark 1. This problem is equivalent to asking whether or not the graph in Figure 3 has an “Eulerian trail” (as defined below). To familiarize the participants with one-stroke writing, the following questions were asked.

Exercise 1. Determine if the following graph can be written in one stroke.

Figure 4. Some Graphs



Further, we defined the odd vertices and gave the problem of examining the relationship between the number of odd vertices and the possibility of one stroke.

Exercise 2. A vertex with an odd degree is referred to as an odd vertex.

- (1) Fill in the table below for the graph in Exercise 1.
- (2) How many odd vertices would make one stroke possible?

Table 1. The Number of Odd Vertices and Possibility of One Stroke

	(1)	(2)	(3)	(4)	(5)
Number of odd vertices					
Possible or not					

After explaining the necessary and sufficient conditions using simple examples, the conditions for writing one stroke were provided.

Theorem 1. A connected graph is one stroke possible if and only if the number of odd points is 0 or 2.

After defining the Eulerian graphs and having the students examine whether the graph in Exercise 1 is an Eulerian graph or not, we provided the necessary conditions for it.

Definition 3. A connected graph (G) is Eulerian if there is a closed trail that includes every edge of G .

Exercise 3. Determine whether the graph in Exercise 1 is an Eulerian graph or not.

Theorem 2. A connected graph (G) is Eulerian only if the degree of each vertex of G is even.

After defining the Hamilton graphs and having the students examine whether the graph in Exercise 1 is a Hamilton graph or not, we provided the necessary conditions for it.

Definition 3. A Hamiltonian cycle is a cycle that visits each vertex precisely once. A graph that contains a Hamiltonian cycle is referred to as a Hamiltonian graph.

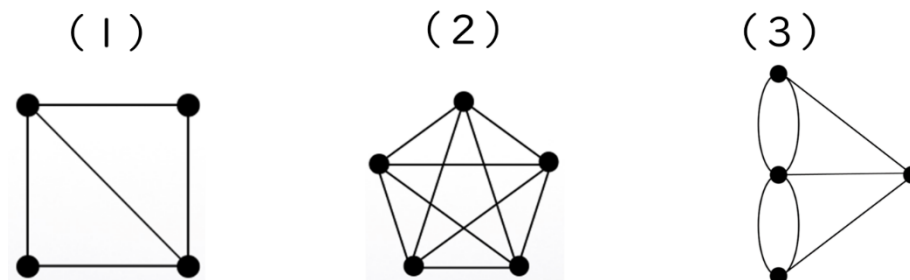
Exercise 4. Determine whether the graph in Exercise 1 is a Hamilton graph or not.

We introduced the “sufficient” condition for a graph to be a Hamiltonian graph and asked students to check that the following graph satisfies this condition.

Theorem 3 (Ore). If G is a simple graph with n (≥ 3) vertices and if $\deg(v)$ and $\deg(w) \geq \frac{n-1}{2}$ for each pair of nonadjacent vertices, v and w , G is a Hamiltonian.

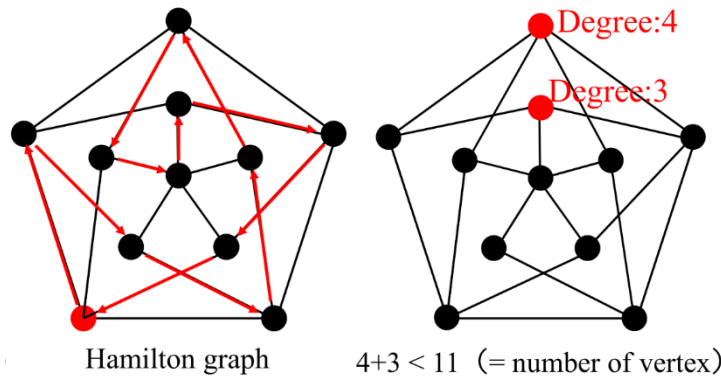
Exercise 5. Verify that the following graph satisfies Ore’s condition.

Figure 5. Some Graphs



We provide an example of a Hamiltonian graph for which the inverse of Ore does not hold and show that the methods for discriminating Hamiltonian graphs are still being studied.

Figure 6. Example of a Graph that is Hamiltonian but does not Satisfy the Ore's Condition

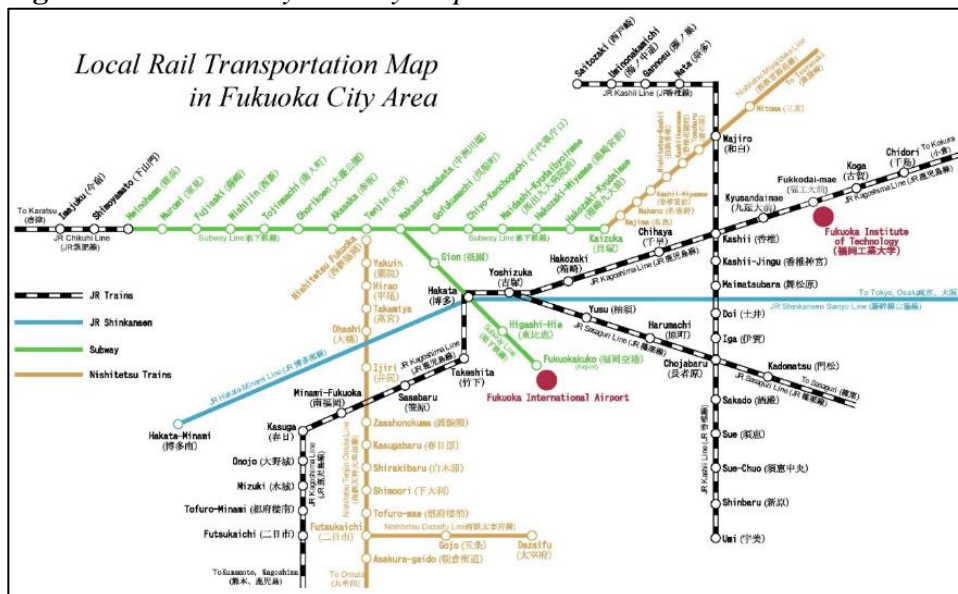


(b) Applications to social networks

The subject of this part is the centrality of a vertex in a given graph, which has been frequently used in network science since the end of the last century.

Graphs can be applied to effectively describe the structure of many social situations. In these cases, the word “network” is used more often than a graph. (Thus, in this part, “network” and “graph” are almost equivalent. However, in the next part, “network” is used in a more restricted sense.) For example, Figure 7 shows the railway network map in the Fukuoka city area.

Figure 7. Fukuoka City Railway Map



Source: <https://ontheworldmap.com/japan/city/fukuoka/fukuoka-rail-map.html>.

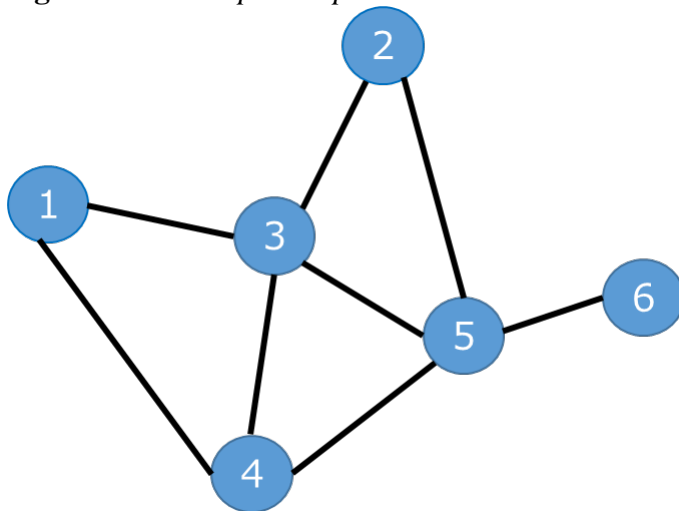
In this case, each vertex corresponds to a station and each edge corresponds to a railway path between two stations. We emphasized that we focused only on whether one station is (directly) connected to another station by a railway or not. The distance between the two stations does not matter, although, in reality, the distance or time it takes to move between them are sometimes important factors.

The centrality indicates the importance of a vertex in the graph: the higher the centrality, the higher the importance of the vertex. The meaning of “important” depends on the purpose and context. Thus, many different definitions of centrality have been proposed, and choosing, or sometimes developing, a centrality suitable for a specific purpose is a substantial problem.

We examined three types of centrality in this lecture: degree, closeness, and betweenness centralities. These are easily evaluated, and the ideas on which they are based are easy to understand.

We began by introducing two quantities required to define the centralities: the degree of a vertex and the distance between the two vertices. For simplicity, we assumed that each edge was undirected and unweighted. As an example, consider the graph shown in Figure 8.

Figure 8. *The Sample Graph Used to Evaluate the Centralities in the Lecture*



As explained in the second section, the degree of a vertex is the number of edges connected to it. For example, the degree of vertex 1 is 2 because two edges are connected to it. The degree counts the number of vertices directly connected to the vertex. Subsequently, we consider a path from one vertex to another along the edges of the graph. The distance is the least number of edges necessary to start from one vertex and move to another; such a path is called the shortest path. For example, the distance between vertices 1 and 5 is 2, since the shortest paths are $1 \rightarrow 3 \rightarrow 5$ and $2 \rightarrow 4 \rightarrow 5$, and each of them consists of two edges.

The degree centrality is defined as the degree of a vertex. This can be easily evaluated and understood. Typical social examples include large stations with many connected railway lines and influencers on social networking services with

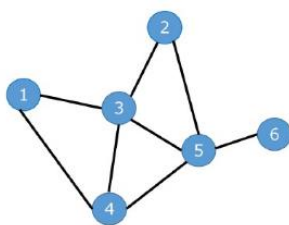
many followers. A vertex with a high degree centrality is important in the graph because it can directly affect many other vertices.

Briefly, a vertex with high closeness centrality is close to any other vertex. The closeness centrality is defined as the reciprocal of the average distance between a vertex and all other vertices in the graph. For example, we evaluated the closeness centrality of vertex 1 in the graph shown in Figure 8. The distances from vertex 1 to vertices 2, 3, 4, 5, and 6 were 2, 1, 1, 2, and 3, respectively. Thus, the average distance was $9/5$, and the closeness centrality was $5/9 \doteq 0.556$. The closeness centrality is useful in transportation, communication, etc.

The betweenness centrality measures the extent to which a vertex is involved in the (indirect) connection with other vertices in the graph. In other words, if a vertex with high betweenness centrality is removed, many pairs of connections will be cut off or become relatively long. For example, it is utilized in traffic and information exchanges. It is defined as the proportion of the shortest paths between two vertices that include the vertex. For example, we evaluate the betweenness centrality of vertex 3 in the graph shown in Figure 2. The number of pairs of vertices, excluding vertex 3, is 10. The shortest path between vertices 1 and 2 includes vertex 3; however, the shortest paths between vertices 1 and 4, 2 and 5, 2 and 6, 4 and 5, 4 and 6, and 5 and 6 do not include vertex 3. For pairs 1 and 5, 1 and 6, and 2 and 4, there are two shortest paths between them, and in each pair, one includes vertex 3. Thus, each pair is considered to contribute half. Therefore, the betweenness centrality was $(1 + 1/2 + 1/2)/10 = 0.25$.

The participants tried to evaluate these three types of centralities of all the vertices of the graph shown in Figure 8 with the help of KOSEN students. It seems that it took many participants some time to find the shortest paths of a given pair of vertices, which are necessary for the evaluation of the closeness and betweenness centralities. The results are shown in Figure 9.

Figure 9. The Three Centralities of all the Vertices of the Graph Shown Leftward (the Same Graph as Shown in Figure 8) the Maximum Values in Each Centrality are Colored Red

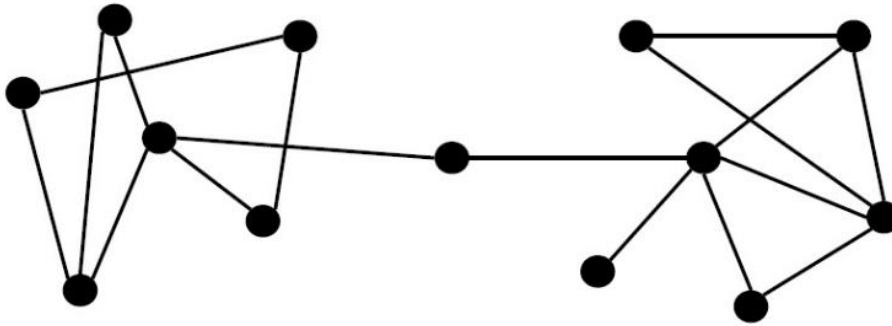


vertex	Degree Centrality	Closeness Centrality	Betweenness centrality
1	2	0.556	0
2	2	0.625	0
3	4	0.833	0.25
4	3	0.714	0.1
5	4	0.833	0.45
6	1	0.5	0

In this example, no matter which type of centrality we considered, vertices with a high centrality almost coincide. This is because the size and order of the graph (i.e., the numbers of vertices and edges) is small and the shape of the graph is “typical” for the social network. We commented that for a graph with an extreme shape, which is a vertex with high centrality depends on the type of centrality (see

Figure 10). This is sometimes the case. Finally, we commented that as the size of the graph increases, it will be a terrible task to evaluate the centralities by hand; therefore, we use a computer to analyze a real social network.

Figure 10. *An Example of a Graph with an Extreme Shape. The Vertex at the Center Bridges the Left and Right Clusters. The Degree Centrality of the Center Vertex is not Considerably High; however, the Closeness and Betweenness Centralities are High*



(c) *Applications of the graph theory: Maximum flow problems and Ford–Fulkerson algorithm*

In this lecture, participants analyzed transportation capacity using algorithms related to graph theory to understand the practical applications of graph theory. We constructed the contents of the lecture as follows:

- Maximum flow problem and networks
- Flow networks
- Max-flow min-cut theorem
- Residual networks
- Ford–Fulkerson algorithm
- Exercises

The following are the descriptions of each content.

Maximum Flow Problem and Networks

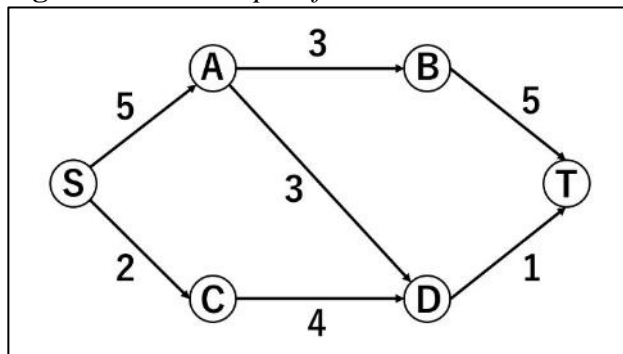
The maximum flow problem is the problem of determining the maximum amount that can be transported from the starting point to the terminal point on a graph.

For example, the problem of determining the maximum amount of goods that can be transported from point S to point T in the graph shown in Figure 11 is called the maximum flow problem.

The edges of the graph shown in Figure 11 indicate the direction in which the goods can be transported. Additionally, the maximum quantity of goods that can be transported is indicated at the edge. As shown in Figure 11, a graph with starting

and terminal points and with a defined direction and non-negative integer values on the edges is called a network.

Figure 11. An Example of Networks



Flow Networks

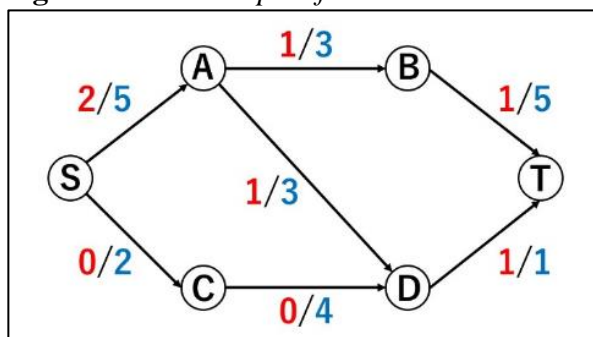
A correspondence that assigns a nonnegative integer to each edge of the network under the following constraints is called a flow.

- **Capacity constraints:** The non-negative integer assigned to each edge is less than or equal to the maximum amount that can be transported.
- **Balance constraints:** The inflows and outflows at each point are the same, except for the starting and terminal points.

In other words, flow is a method of transporting everything from the starting point to the terminal point, maintaining the upper limit of the amount that can be transported at each edge.

The graph shown in Figure 12, which is called a flow network, is based on the network shown in Figure 11, with capacity constraints represented by blue numbers and flows represented by red numbers. The flow network shown in Figure 12 shows how to transport two goods from point S to point T.

Figure 12. An Example of Flows



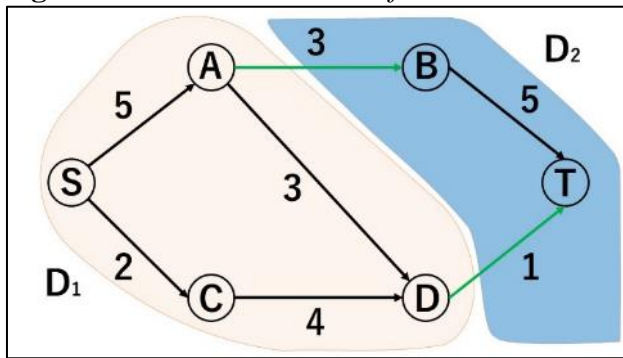
Max-flow Min-cut Theorem

The division of all points into two regions, D_1 containing the starting point and D_2 containing the terminal point, is called a cut. The sum of the capacities of the edges flowing from region D_1 to region D_2 is referred to as the capacity of the cut. The cut with the smallest capacity is referred to as the minimum cut.

Figure 13 shows an example of a cut in the network shown in Figure 11. The capacity of the cut is $3 + 1 = 4$, which is the minimum cut.

The max-flow min-cut theorem states that the maximum quantity that can be transported from the starting point to the terminal point is equal to the capacity of the minimum cut. For example, in the network shown in Figure 11, the maximum amount that can be transported from the starting point to the terminal point has a minimum cut capacity of 4. This indicates that the number of goods that can be transported is higher than the flow, as shown in Figure 12.

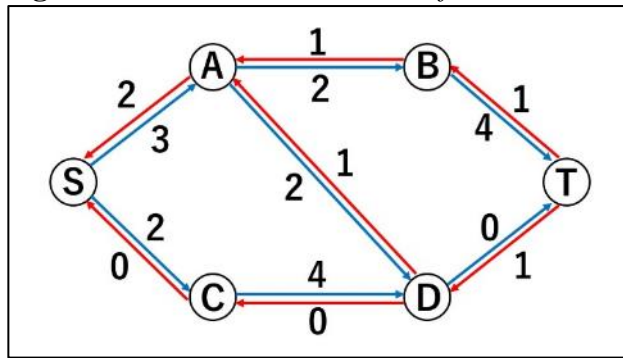
Figure 13. *The Minimum Cut of the Network in Figure 11*

*Residual Networks*

The residual network is a graph representing how much more flow can be added and how much more flow can be returned. For example, Figure 14 shows the residual network created based on the network shown in Figure 12.

The flow from point A to point B is 1, whereas the maximum amount that can be transported is 3. Therefore, it is possible to flow an additional 2 from point A to point B. Since the flow is 1 from point A to point B, it is possible to return 1 from point B to point A. Notably, if 1 is returned from point B to point A in the residual network, the flow from point A to point B becomes 0 in the flow network.

Similarly, by representing how much flow can be added and how much flow can be returned, a residual network can be created, as shown in Figure 14.

Figure 14. The Residual Network of the Network in Figure 12

Ford–Fulkerson Algorithm

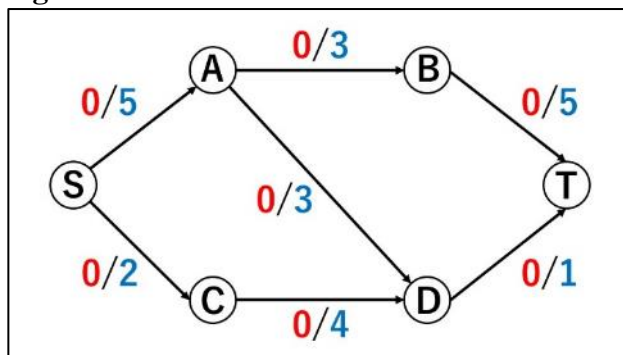
The following algorithm can be used to find the maximum amount that can be transported and the transport method. This is known as the Ford–Fulkerson algorithm.

– Algorithm (Ford–Fulkerson algorithm)

- Step 0:** The initial flow is set to 0.
- Step 1:** Create the residual network from the flow network.
- Step 2:** Find a path in the residual network from the starting point to the terminal point, consisting of edges assigned to positive integers. If there is no such path, stop the process.
- Step 3:** Update the flow in the flow network corresponding to the path found in Step 2 and return to Step 1.

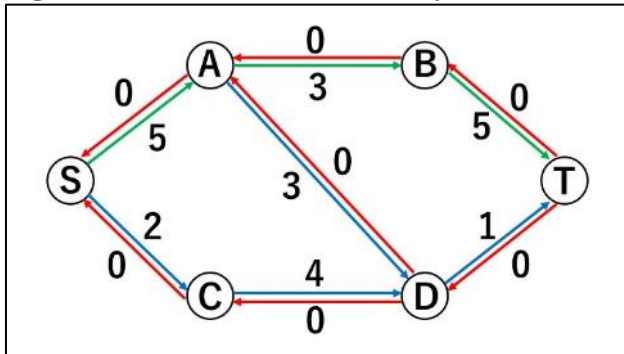
– Ford–Fulkerson example

The first flow is set to zero.

Figure 15. The Network with Zero Flow

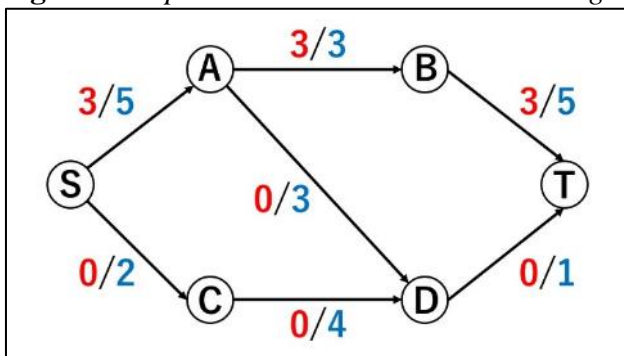
Subsequently, a residual network is created. From this, we find a path consisting of edges to which a positive integer is assigned. Here, we select path $S \rightarrow A \rightarrow B \rightarrow T$.

Figure 16. The Residual Network of the Network in Figure 15



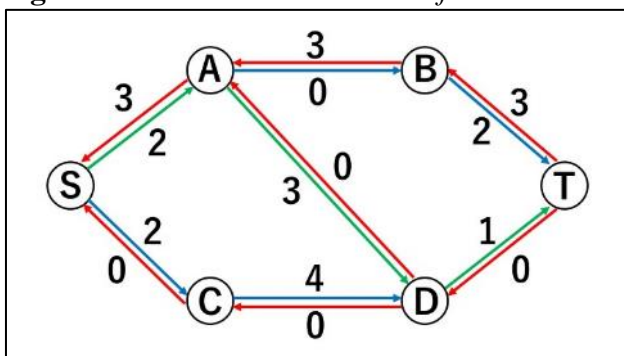
The minimum capacity among the three edges is 3 (A→B). Accordingly, the flow network is updated.

Figure 17. Updated Flow Network Based on Figure 16

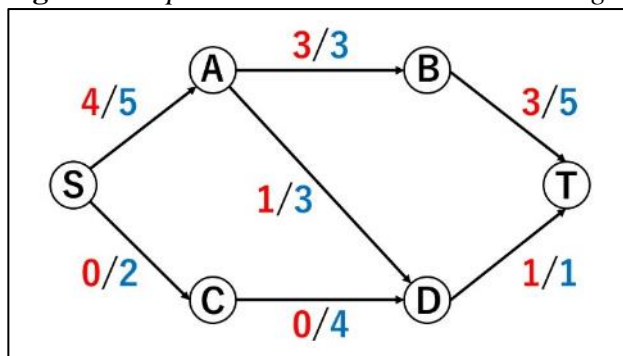


Afterward, we create the residual network again and select another path: S→A→D→T.

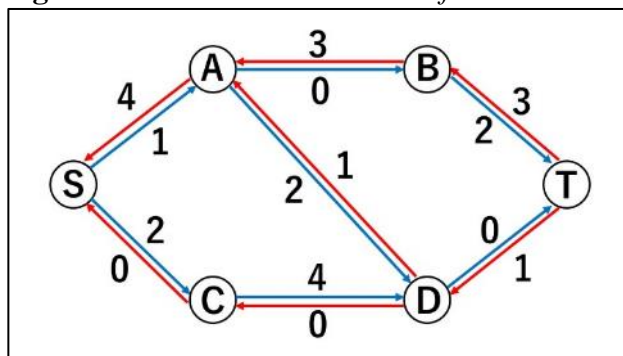
Figure 18. The Residual Network of the Network in Figure 17



The minimum capacity of the three edges is 1 (D→T). Accordingly, the flow network is updated (this is the maximum flow according to the max-flow min-cut theorem).

Figure 19. Updated Flow Network Based on Figure 18

We create a residual network; however, there is no path from the starting point to the terminal point consisting of edges that are assigned positive integers. Thus, the procedure is discontinued.

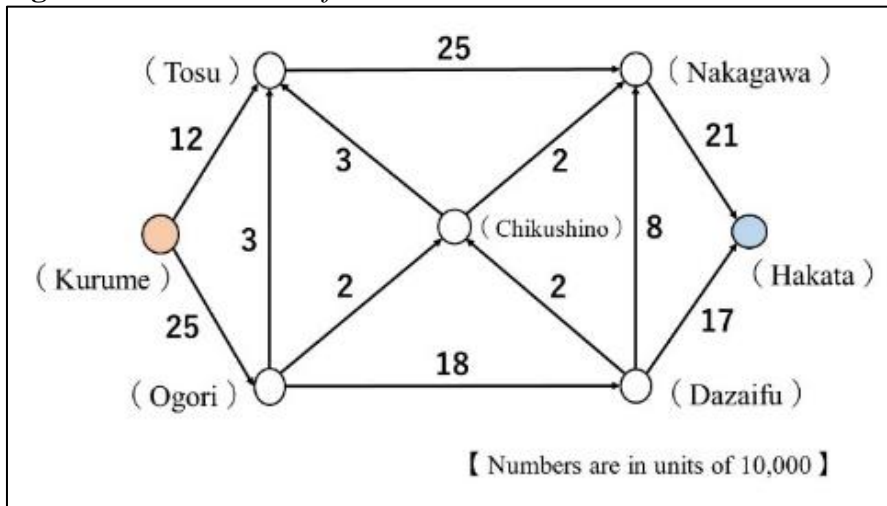
Figure 20. The Residual Network of the Network in Figure 19

Exercises

At the end of the lecture, participants analyzed their transportation capacity during the following exercise.

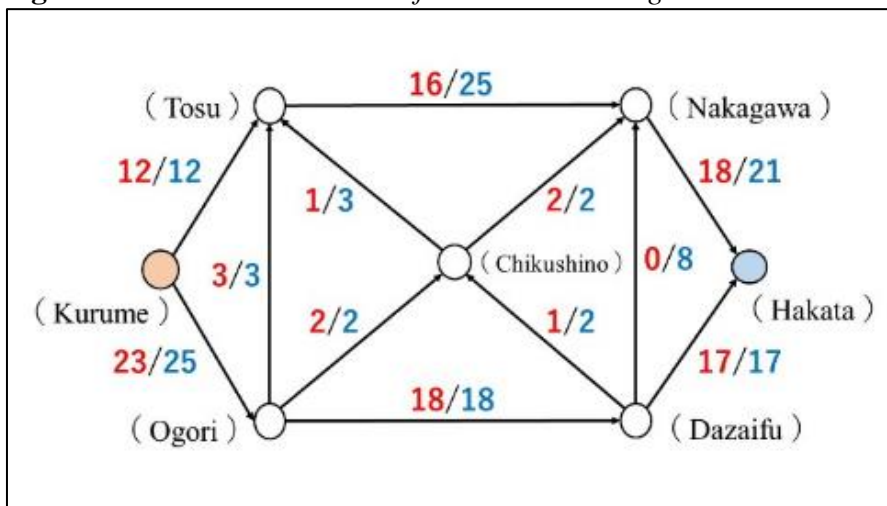
- **Exercise:** Kurume is crowded with people returning to Hakata after the Chikugo River Fireworks Festival. How many people should be transported between each route to deliver as many people as possible to Hakata? Note that the arrows indicate passable routes, and the numbers indicate the number of people that can be transported.

Figure 21. *The Network of the Above Exercise*



– **Sample answer:** It can transport up to 350,000 people, which can be achieved by transporting as shown in the figure below.

Figure 22. *The Maximum Flow of the Network in Figure 21*



The participants completed this exercise using the Ford–Fulkerson algorithm. They attempted this exercise with advice from teaching assistants, and almost all participants could arrive at the correct answer. We believe that the participants could understand that graph theory could be used in familiar situations by completing this exercise.

Some Scenes from the Open Course

In this section, we review the scenes of an open course using photos. A teacher and student showed slides on the screen, and the participants solved the exercise (see Figure 23).

Figure 23. *The Scene of Open Course*

Discussion of Practice Results

To verify the achievement of the project's objectives of “providing opportunities for our students who have studied graph theory to teach what they have learned (as TAs)” and “providing participants with opportunities to arouse their interest in mathematics and to gain multiple perspectives,” we interviewed TAs and conducted a survey of participants after the course was over. The results of the interviews for the TAs are as follows.

- “I was nervous, but it was a good experience.”
- “I was glad to have the opportunity to teach”.
- “It was difficult to teach what I had studied to others, but it was a good experience.”
- “Preparing for the class was difficult, but teaching it helped me understand it even better”.
- “I would like to use this experience to teach others in the future.”

According to interviews, the TAs not only had the opportunity to teach what they had learned, but also experienced the challenges and joys of teaching.

The items and the order of them in the survey to the participants are as follows, where items 1 and 3 are “agree,” “somewhat agree,” “somewhat disagree,” and “disagree;” item 2 is “difficult,” “somewhat difficult,” “neither,” “somewhat easy,” and “easy;” and item 4 is “agree,” “somewhat agree,” “neither,” “somewhat disagree,” and “disagree”.

Item 1: Did you understand this course?

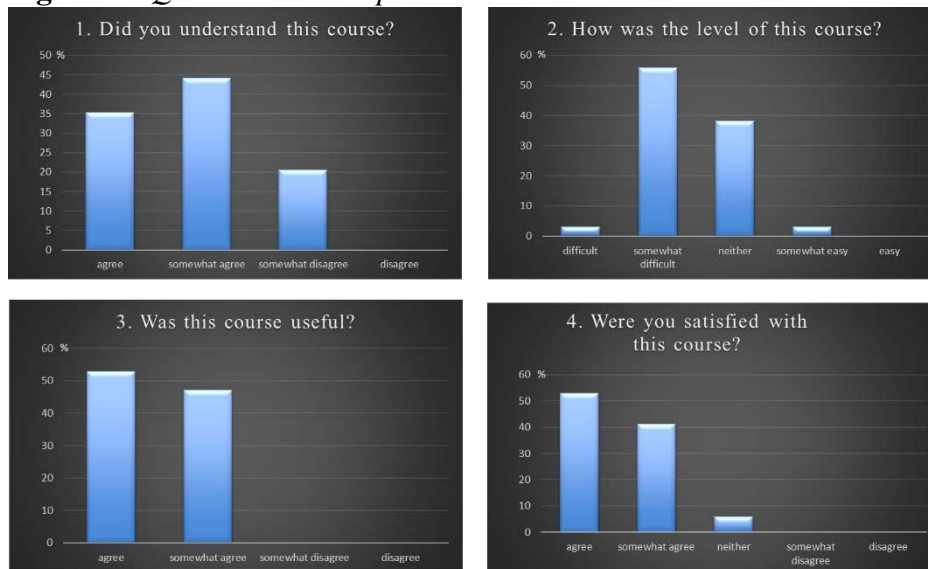
Item 2: What was the level of this course?

Item 3: Was this course useful?

Item 4: Were you satisfied with this course?

The results of the responses to the questionnaire items are shown in the figures below, with the numbers representing the response rate for each option.

Figure 24. Questionnaire Responses



The total percentages of “agree” and “somewhat agree” were more than 80% for all the questions except for 2. On the other hand, we found that more than 60% of participants found the classes difficult or somewhat difficult.

We included an open-ended section in our questionnaire. Some of the comments are as follows.

- Because it was the content I had not learned in junior high school, the class was interesting and I enjoyed solving the problems.
- I was glad that the course was useful in the future and that practice problems were provided. The applications were a little difficult, but I could roughly understand them. It was good to learn about theories and theorems that I didn't know.
- It was a little difficult, but thanks to the TAs teaching me, I understood a little.
- I enjoyed learning many things that I did not learn in junior high school.

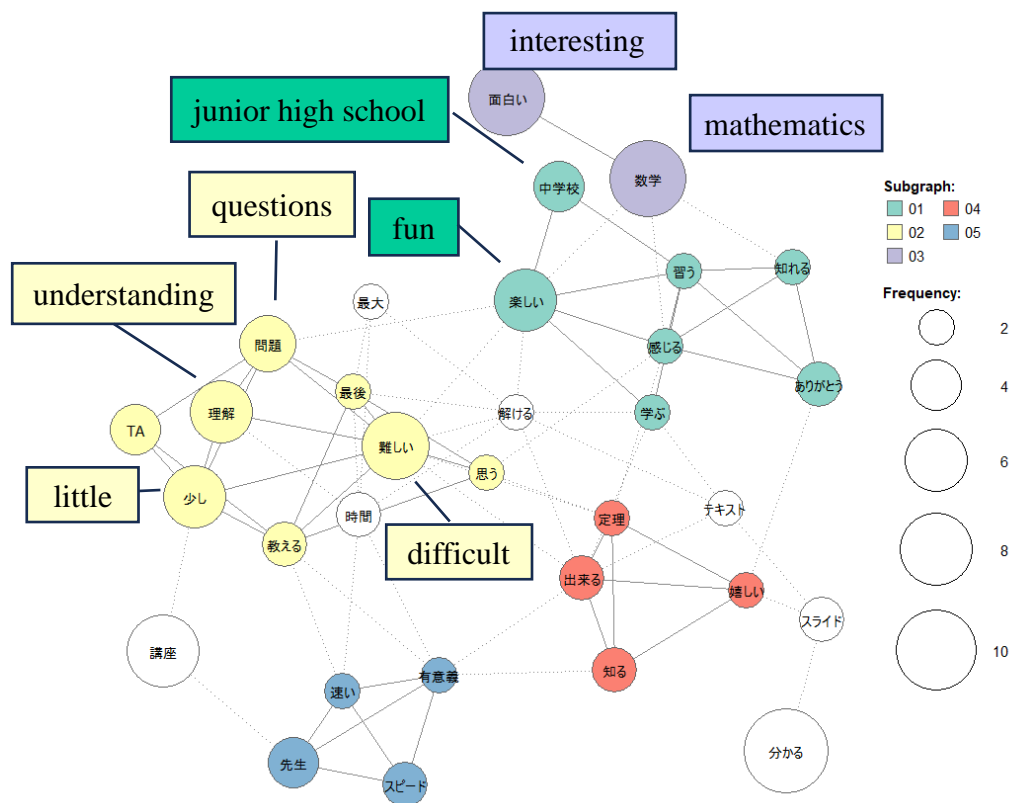
I felt again that mathematics is interesting.

- It was good to feel the fun of mathematics. I thought the theorems of mathematicians such as Euler were very difficult, but they were surprisingly easy to understand.
- It was a good experience for me to experience the mathematics I learn in high school. It was difficult, but it was fun when I could solve it.

- It was very meaningful to learn how math and science knowledge are used in society.

Moreover, we used KH Coder which was a software for text mining to analyze the comments in detail. In KH Coder, the size of the circle represents the frequency of occurrence of a word, the line represents words that appear simultaneously, and the color represents groups of related words. The comments were written in Japanese, and to avoid the influence of language structure we performed a KH Coder analysis in Japanese. The output results are as follows.

Figure 25. Co-occurrence Network of Comments by KH Coder



Subgroup 01 can be assumed to represent “I enjoy learning new knowledge,” 02 “The exercises were difficult, but I could understand them with the support of the TA,” 03 “Mathematics is interesting,” 04 “I am glad to know new theorems,” and 05 “The teacher’s class is fast paced but meaningful”.

The comments and their analysis indicate that the course has achieved some degree of success in arousing interest in mathematics. Moreover, the last comment may suggest that the survey provided an opportunity to have a multifaceted viewpoint. In addition, we observed the activities of TAs and participants in class. During the exercise-solving time, we saw participants eagerly solving exercises and TAs actively approaching them and answering their questions.

Conclusion of this Effort and a Future Subject

Our goal is to “provide our students with the opportunity to teach what they have learned about graph theory”, and to “spark participants’ interest in mathematics and give them the opportunity to gain multiple perspectives” through this course.

Interviews with TAs, survey responses, KH coder analysis of the comments, and observations indicate that the course is somewhat effective for learners and has potential as a STEAM resource. On the other hand, we found that participants did not fully understand the relevance of mathematics to other fields or acquire multiple perspectives. Moreover, since the participants were interested in mathematics to begin with, the survey was not sufficient to determine changes in their attitudes about it.

In the future, we would like to create STEAM teaching materials that emphasize the relationship between mathematics and other fields, and conduct surveys before and after the class to investigate changes in participants' awareness and verify the effectiveness of our efforts.

We now describe STEAM teaching materials for future use. “*Knot theory*” is easy for beginners to understand because it is not necessary to know its background well, and there are various teaching materials from which they can learn visually. The knot theory is associated with various fields, such as quantum field theory in physics, molecular design in chemistry, and DNA in biology. In the future, we intend to create STEAM teaching materials related to physics and chemistry.

The next material is the “*L-S category*” (Cornea et al. 2003, Sakai and Miyaji 2013), which is an invariant for various figures. We find it easy to begin because we can learn it visually as a knot theory and because we need little preliminary knowledge of it. For these reasons, this theme would be interesting to students. For example, the L–S categories of a torus and a Klein bottle are both two. There is a *fibrewise* version of the L–S category, which is known to have a possibly different value from the ordinary L–S category. As a simple example, a torus has a value of two as its ordinary L–S category but one as its fibrewise version (Cornea et al. 2003). Using the property of the fibrewise A_∞ -structure, one of the authors states that the fibrewise L–S category of the Klein bottle has a value of two (Sakai 2010). Additionally, it is known that the fibrewise L–S category is related to “topological complexity,” a field of research involved in the motion planning of robot arms (Iwase and Sakai 2010).

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References

- Cornea O, Lupton G, Operea J, Tanre D (2003) *Lusternik-Schniremann Category*. American Mathematical Society.
- Iwase N, Sakai M (2010) Topological complexity is a fibrewise L-S category. *Topology and its Applications* 157: 10–21.
- Sakai M (2010) A_∞ -spaces and L-S category in the category of fibrewise spaces. *Topology and its Applications* 157(13): 2131–2135.
- Sakai M, Miyaji T (2013) *Various Efforts to Improve Motivation to Learn Mathematics*. In *ATINER'S Conference Paper Series*, 5–13. MAT2013-0572.
- Yakman G (2008) STEAM education: An overview of creating a model of integrative education. In *Pupils' Attitudes Towards Technology (PATT-19) Conference: Research on Technology, Innovation, Design and Engineering Teaching*. Salt Lake City, UT.

